Beam-Columns

INTRODUCTION

Columns in practice rarely experience concentric axial compression alone. Since columns are usually parts of a frame, they experience both bending moment and axial force. The frames, in which columns are members, may be classified as braced or unbraced. In braced frames the resistance to lateral loads at floor levels is provided either by bracings, or shear walls. In case of unbraced frames, the resistance to lateral loads is obtained from the members of the frames with moment resisting connections between them. Thus the relative translation between the ends of a column in a braced frame is prevented, whereas in unbraced frames the columns are free to sway causing relative translation between their ends. More details on classification of frames as braced and unbraced are given in the chapter on frames. Thus columns in practice experience bending about one or both axis in addition to axial compression, due to one or more of the following reasons.

• The compressive force may be eccentrically transferred to the column, when this eccentric force is transferred to the center line of the column, an equivalent axial compression and bending moment act on the column.

• When the beams in braced rigid portal frames are subjected to gravity loads, the rotation of the beams at their intersection with the column causes rotation of the column and the beam due to rigid connection. Hence beam transfers bending moments to the column in addition to axial load.

• When a multi-story multi-bay un-braced frame is subjected to gravity loads and lateral loads due to wind or earthquake, the columns are subjected to sway deflection and bending. In such cases, the columns experience axial compression as well as bending moments. Beams may frame from two orthogonal directions in corner columns in buildings. In such cases the columns may be subjected to bending about both principal axes in addition to axial compression.

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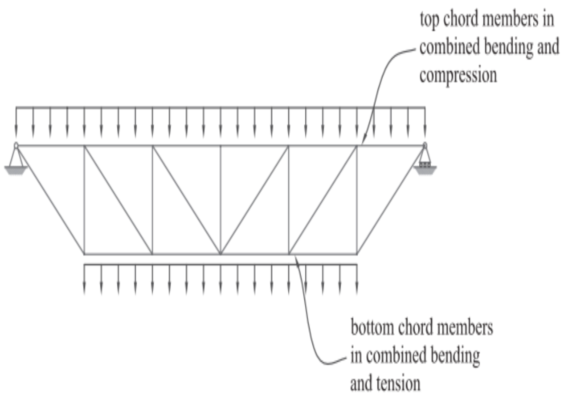
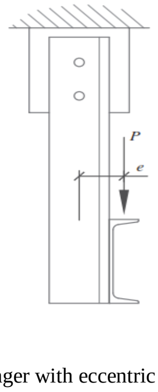
• Moments in tension members are not as serious as those in compression members, because tension tends to reduce lateral deflections while compression increases them. Increased lateral deflections in turn results in large moments, which cause larger lateral deflection.

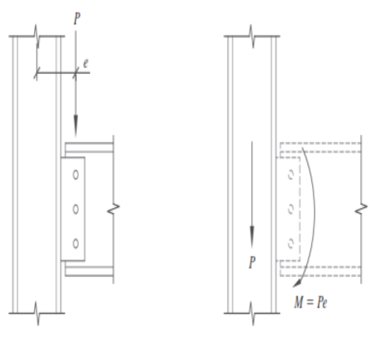
Type of beam column



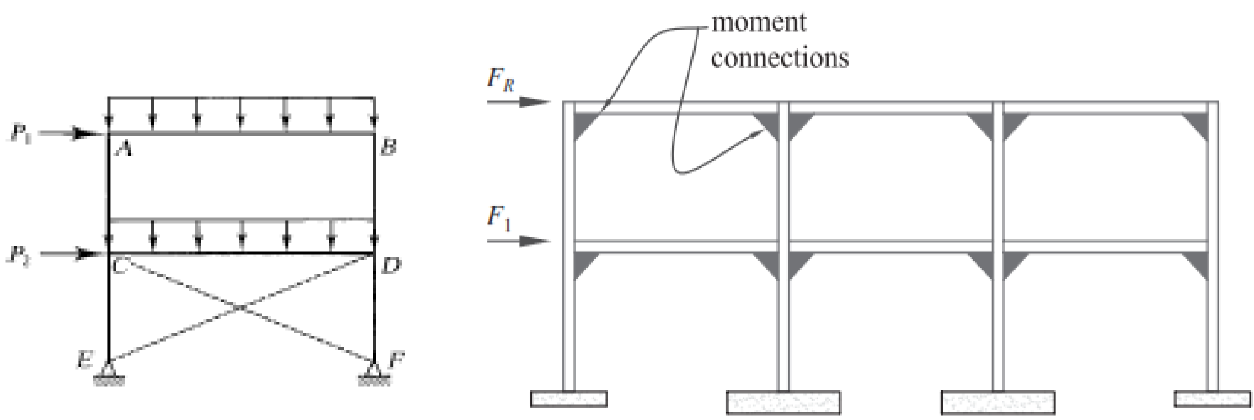
Top and bottom chords of trusses



Hanger with eccentric axial load



Columns in braced frames



Moment for braced or unbraced, frames

Interaction formulas

The relationship between required and available strengths may be expressed as

$$\frac{required\ strength}{available\ strength} \leq 1.0 \qquad (1)$$

For compression members, the strengths are axial forces. For example, for LRFD,

$$\frac{Pu}{\phi Pn} \leq 1.0$$

and for ASD,

$$\frac{Pa}{Pn/\Omega} \leq 1.0$$

These expressions can be written in the general form

$$\frac{Pr}{Pc} \leq 1.0$$

Where
Pr = required axial strength
Pc = available axial strength
If more than one type of resistance is involved, Equation (1) can be used to form the basis of an interaction formula. As we discussed in beam formula (Chapter F) in conjunction with biaxial bending, the sum of the load-to-resistance ratios must be limited to unity. For example, if both bending and axial compression are acting. The interaction formula would be

$$\frac{Pr}{Pc} + \frac{Mr}{Mc} \leq 1.0$$

Where:-
Mr = required moment strength
=Mu for LRFD
= Ma for ASD

Mc = available moment strength
=ϕMn for LRFD
= Mn/Ω for ASD

For biaxial bending, there will be two moment ratios:

$$\frac{Pr}{Pc} + \left[\frac{Mrx}{Mcx} + \frac{Mry}{Mcy}\right] \leq 1.0 \qquad \text{.............(2)}$$

Where the x and y subscripts refer to bending about the x and y axes. Equation .2 is the basis for the AISC formulas for members subject to bending plus axial compressive load. Two formulas are given in the Specification: one for small axial load and one for large axial load. If the axial load is small, the axial load term is reduced. For large axial load, the bending term is slightly reduced.

The AISC requirements are given in Chapter H, "Design of Members for Combined Forces and Torsion," and are summarized as follows:

For $\frac{Pr}{Pc} \geq 0.2$

$$\frac{Pr}{Pc} + \frac{8}{9}\left[\frac{Mrx}{Mcx} + \frac{Mry}{Mcy}\right] \leq 1.0 \qquad \text{(AISC Equation H1-1a)}$$

For $\frac{Pr}{Pc} < 0.2$

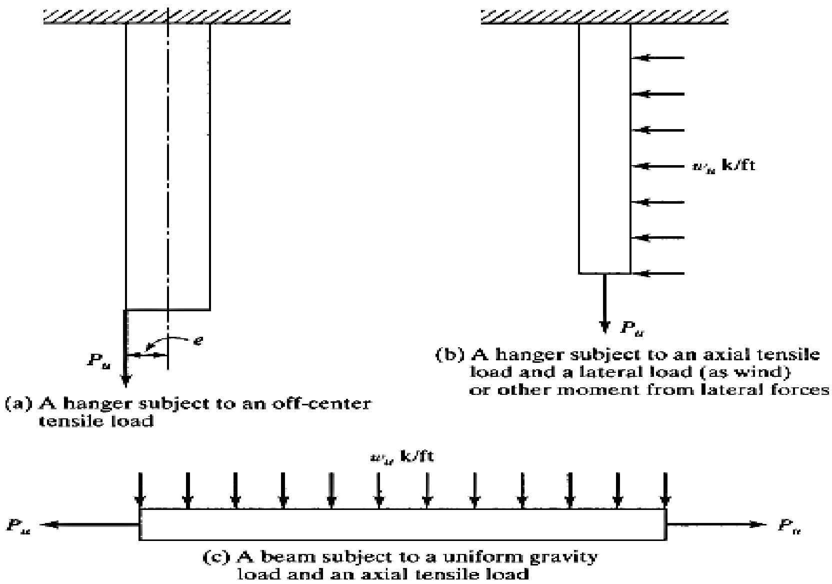
$$\frac{Pr}{2Pc} + \left[\frac{Mrx}{Mcx} + \frac{Mry}{Mcy}\right] \leq 1.0 \qquad \text{(AISC Equation H1-1b)}$$

These requirements may be expressed in either LRFD or ASD as follows

LRFD	ASD
For $\frac{Pr}{Pc} = \frac{Pu}{\phi Pn} \geq 0.2$ $\frac{Pr}{Pc} + \frac{8}{9}\left[\frac{Mrx}{\phi Mnx} + \frac{Mry}{\phi Mny}\right] \leq 1.0$ (H1-1 a)	For $\frac{Pr}{Pc} = \frac{Pa}{\frac{Pn}{\Omega}} \geq 0.2$ $\frac{Pr}{Pc} + \frac{8}{9}\left[\frac{Mrx}{Mnx/\Omega} + \frac{Mry}{Mny/\Omega}\right] \leq 1.0$ (H1-1 a)
For $\frac{Pu}{\phi Pn} < 0.2$ $\frac{Pr}{2Pc} + \left[\frac{Mrx}{\phi Mnx} + \frac{Mry}{\phi Mny}\right] \leq 1.0$ (H1-1 b)	For $\frac{Pa}{Pn/\Omega} < 0.2$ $\frac{Pr}{2Pc} + \left[\frac{Mrx}{Mnx/\Omega} + \frac{Mry}{Mny/\Omega}\right] \leq 1.0$ (H1-1 b)

Members subjected to bending and axial tension

A few type of member subjected to both bending and axial tension as shown in the Fig. below. In section H1 of the AISC Specification, the interaction equations that follow are given for symmetric shapes subjected simultaneously to bending and axial tensile forces. When the beam–columns subjected to axial tensile loads and bending moments, the factored moments about the x–x and y–y axes (i.e., M_{ux} and M_{uy}), respectively were applied directly without any modification.



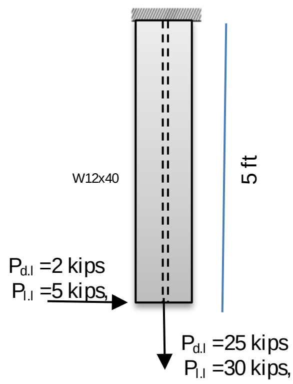
(a) A hanger subject to an off-center tensile load

(b) A hanger subject to an axial tensile load and a lateral load (as wind) or other moment from lateral forces

(c) A beam subject to a uniform gravity load and an axial tensile load

Example

A992 steel material W12x40 tension member with no holes is subjected to the axial loads $P_{d.l}$ =25 kips and $P_{l.l}$ =30 kips, as well as the horizontal force $P_{d.l}$= 2 kips and $P_{l.l}$= 5 kips.is the member satisfactory, use LRFD method



Solution

<u>Steel</u>	<u>fy</u>	<u>fu</u>
A992	50	65

<u>Sect</u>.	<u>Ag</u>
W12 x40	11.7

1- Determine the applied load and moments

$P_u = 1.2 \times P_{d.l} + 1.6 \times P_{l.l}$

$= 1.2 \times 25 + 1.6 \times 30 = 78$ kip

$M_{d.l.y} = 2 \times 5 = 10$ ft.kip

$M_{l.l.y} = 5 \times 5 = 25$ ft.kip

$M_{uy} = 1.2 \times M_{d.ly} + 1.6 \times M_{l.ly}$

$= 1.2 \times 10 + 1.6 \times 25 = 52$ ft. kip

Mux= 0 no load subjected on X-axis

2- Determine the tensile strength

Since the tension member has no hole (that mean depending on yielding strengths or gross area only)

$\phi P_n = \phi A_g F_y$

$= 0.9 \times 11.7 \times 50 = 52.6$ kip

3- Select the beam column formula

For $\frac{Pu}{\phi Pn} < 0.2$

$\frac{78}{52.6} = 0.14 < 0.2$ Use equation H1-1b

4- Find the flexural strength of the section (ϕMnx)
Mux=0 no load subjected to the major axis then no need to find (ϕMnx)

5- Find the flexural strength of the section (ϕMny)

ϕMny= 63.0 ft. Kip from table (3-4)

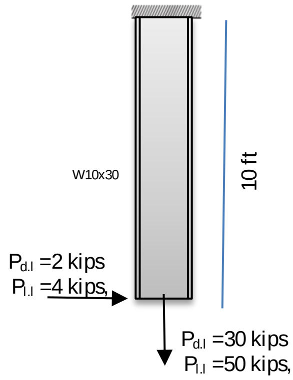
Shape	Z_y	M_{ny}/Ω_b	$\phi_b M_{ny}$	Shape	Z_y	M_{ny}/Ω_b	$\phi_b M_{ny}$	Shape	Z_y	M_{ny}/Ω_b	$\phi_b M_{ny}$
		kip-ft	kip-ft			kip-ft	kip-ft			kip-ft	kip-ft
	in.³	ASD	LRFD		in.³	ASD	LRFD		in.³	ASD	LRFD
W10×60	**35.0**	**87.3**	**131**	**W8×40**	**18.5**	**46.2**	**69.4**	**W8×24**	**8.57**	**21.4**	**32.1**
W30×90	34.7	86.6	130	W21×55	18.4	45.9	69.0	W12×26	8.17	20.4	30.6
W21×93	34.7	86.6	130	W14×43	17.3	43.2	64.9	W18×35	8.06	20.1	30.2
W27×84	33.2	82.8	125	**W10×39**	**17.2**	**42.9**	**64.5**	W10×26	7.50	18.7	28.1
W14×61	32.8	81.8	123	W12×40	16.8	41.9	63.0	W16×31	7.03	17.5	26.4
W8×67	32.7	81.6	123	W18×50	16.6	41.4	62.3	**W10×22**	**6.10**	**15.2**	**22.9**
W24×84	32.6	81.3	122	W16×50	16.3	40.7	61.1	**W8×21**	**5.69**	**14.2**	**21.3**
W12×58	32.5	81.1	122								

$$\frac{Pu}{2\phi Pn} + [\frac{Mux}{\phi Mnx} + \frac{Muy}{\phi Mny}] \leq 1.0$$

$0.148 + (0 + \frac{52}{63}) = 0.899 < 1.0$ OK

The section is adequate

Example
A992 steel material W10x30 tension member with no holes is subjected to the axial loads $P_{d.l}$ =30 kips and $P_{l.l}$ =50 kips, as well as the horizontal force $P_{d.l}$= 2 kips and $P_{l.l}$= 4 kips , assuming Cb=1. Is the member satisfactory, use LRFD method



Solution

Steel	fy	fu
A992	50	65

Sect.	Ag	Zx	ϕMpx	BF	Lp	Lr
W10 x30	8.84	36.6	137	4.62	4.84	16.1

1- Determine the applied load and moments

$Pu= 1.2x\ P_{d.l} + 1.6x\ P_{l.l}$
= 1.2x30 +1.6x50 = 116kip
$M_{d.l.x}$ = 2x10 =20 ft.kip
$M_{l.l.x}$ = 4x10 = 40 ft.kip
$Mux=1.2x\ M_{d.l\,x} + 1.6x\ M_{l.l\,x}$
=1.2x20 +1.6x40= 88 ft. kip
Muy =0 no load subjected to the minor axis

2- Determine the tensile strength

Since the tension member has no hole (that mean depending on yielding strengths or gross area only)

$\phi Pn = \phi Ag\ Fy$

$= 0.9x8.84x50 = 397.8$ kip

3- Select the beam column formula

For $\frac{Pu}{\phi Pn} > 0.2$

$\frac{116}{397.8} = 0.292 > 0.2$ Use equation H1-1 a

W14×30	47.3	118	177	73.4	110	4.63	6.95	5.26	14.9	291	74.5	112
W10×39	46.8	117	176	73.5	111	2.53	3.78	6.99	24.2	209	62.5	93.7
W16×26[v]	44.2	110	166	67.1	101	5.93	8.98	3.96	11.2	301	70.5	106
W12×30	43.1	108	162	67.4	101	3.97	5.96	5.37	15.6	238	64.0	95.9
W14×26	40.2	100	151	61.7	92.7	5.33	8.11	3.81	11.0	245	70.9	106
W8×40	39.8	99.3	149	62.0	93.2	1.64	2.46	7.21	29.9	146	59.4	89.1
W10×33	38.8	96.8	146	61.1	91.9	2.39	3.62	6.85	21.8	171	56.4	84.7
W12×26	37.2	92.8	140	58.3	87.7	3.61	5.46	5.33	14.9	204	56.1	84.2
W10×30	36.6	91.3	137	56.6	85.1	3.08	4.61	4.84	16.1	170	63.0	94.5

4- Find the flexural strength of the section (ϕMnx)

Find ϕM_{nx}

Lb= 10 ft ,LP=4.84 ft,Lr=16.1 ft

LP <Lb< Lr

4.84< 10 < 16.1 zone 2

$\phi Mnx = cb(\phi MPx - BF(Lb - LP) \leq \phi MPx$

= 1x(137- 4.62x(10-4.84))≤ 137

= 113 < 137 OK

ϕMnx= 113 ft.kip

5- M_{ny}= 0 no load subjected to the Mainor axis then no need to find (ϕMny)

For $\frac{Pu}{\phi Pn} > 0.2$

$\frac{116}{397.8} = 0.292 > 0.2$ Use equation H1-1 a

$$\frac{Pu}{\phi Pn} + \frac{8}{9}\left[\frac{Mux}{\phi Mnx} + \frac{Muy}{\phi Mny}\right] \leq 1.0$$

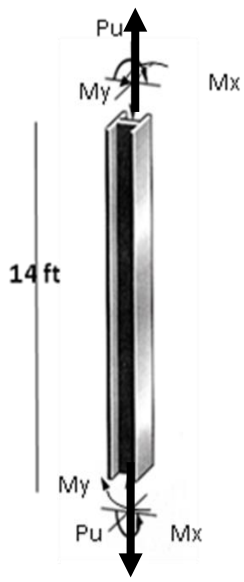
$$0.292 + \frac{8}{9}\left[\frac{88}{113} + 0\right] \leq 1$$

=0.942 < 1.0 OK

The section is adequate

Homework

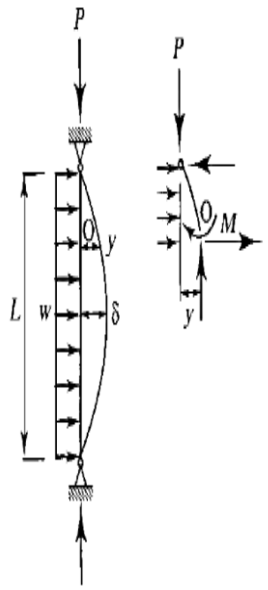
A992 steel material W10x100 tension member with no holes is subjected to the axial loads $P_{d.l}$ =30 kips and $P_{l.l}$ =50 kips, as well as the $M_{d.lx}$ = 20 ft.kips and $M_{l.lx}$=30ft. kips and $M_{d.ly}$= 10 ft.kips and $M_{l.ly}$= 20 ft. kips, assuming Cb=1. Is the member satisfactory, use LRFD method



Members subjected to bending and axial compression

When the beam–columns subjected to axial compression loads and bending moments, the factored moments about the x–x and y–y axes (i.e., M_{ux} and M_{uy}, respectively) must include the effect of the slenderness of the compression member (i.e., the so-called P-delta effects).

The presence of substantial axial loads in flexural members causes secondary moments that must be considered in design. This is illustrated in the figure below



At point O in the figure, the total moment is caused by the transverse load w and the axial load P operating through a lateral displacement y. The secondary moment, Py, is largest where the lateral deflection is largest, namely, at the centerline of the beam column. At this location the total moment is:

$$Mt = M + P\delta$$

Because the total deflection cannot be found directly, this problem is nonlinear.

Standard structural analysis methods do not take the displaced geometry into account and are termed first-order methods.

Iterative numerical techniques can account for displaced geometry by incrementally applying the transverse and axial loads and reformulating the stiffness matrix at each time step. These methods, often termed second-order methods, can find the deflections and secondary moments, but are usually implemented in a computer program.

Current design codes, including the LRFD Specification, permit the use of second-order analysis or the moment-amplification method.

The moment amplification method or p-delta effects

The moment amplification method entails computing the maximum bending moment resulting from flexural loading (transverse loads or member end moments) by a first order analysis, then multiplying by a moment amplification factor to account for the secondary moment.

There are two types of secondary moments: P-δ (caused by member deflection) and P-Δ (caused by the effect of sway when the member is part of an unbraced frame [moment frame]). Because of this, two amplification factors must be used.

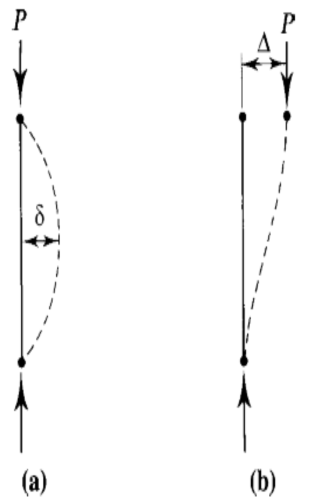


Figure below illustrates these two components of deflection. Figure (a) show the member is restrained against side sway, and the maximum secondary moment is P δ, which is added to the maximum moment within the member. If the frame is actually unbraced, there is an additional component of the secondary moment, as shown in Figure (b) that is caused by side sway. This secondary moment has a maximum value of PΔ, which represents an amplification of the end moment

To approximate these two effects, two amplification factors,

B_1 and B_2, are used for the two types of moments. The amplified moment to be used in design is computed from the loads and moments as follows:

$Mr = B_1 M_{nt} + B_2 M_{\ell t}$

M_r= required moment strength

M_{nt} = maximum moment assuming that no sides way occurs, whether the frame is actually braced or not (the subscript $_{nt}$ is for "no translation"). These moments are caused by gravity loads.

$M_{\ell t}$= maximum moment caused by side sway (the subscript lt is for "lateral translation"). This moment can be caused by lateral loads or by unbalanced gravity loads. M_{lt} will be zero if the frame is actually braced. For most reasonably symmetric moment frames, these moments are caused only by lateral wind or seismic loads.

For braced frames, there are no lateral translation moments; therefore, $M_{\ell t}$ =0.

B_1 = amplification factor for the moments occurring in the member when it is braced against side sway (P-δ moments).

B_2 = amplification factor for the moments resulting from side sway (P-Δ moments).

For brace (Non sway) beam Moment Magnification Factor, *B*1

$$B_1 = \frac{cm}{1-\frac{\alpha Pr}{pe1}} \geq 1.0$$

Where

Pr= required axial compressive strength

=Pu , α= 1 for LRFD method

= Pa , α= 1.6 for ASD method

$$P_{el} = \frac{\pi^2\ EI}{(kl)^2}$$

P_{el}= Euler buckling load

$$P_{ex} = \frac{\pi^2\ EIx}{(klx)^2}$$

Pex= Euler buckling load for x-axis

$$P_{ey} = \frac{\pi^2\ EIy}{(kly)^2}$$

Pey= Euler buckling load for Y-axis

KL is the slenderness ratio about the axis of bending.

K ≤ 1.0 (a practical value of K for columns in braced frames = 1.0), and

The moment reduction coefficient, C_m , accounts for the effect of moment gradient in the column, and is obtained as follows:

$$Mr = B_1 Mnt$$

Cm values
Depending on transvers load acting on the member

If there are not transvers load acting on the member

Cm=0.6-0.4 $\frac{M_1}{M_2}$

Where

$\frac{M_1}{M_2}$= Absolute ratio of bending moment at the end of member

M_1= Smaller moment

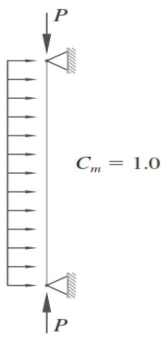
M_2= Larger moment

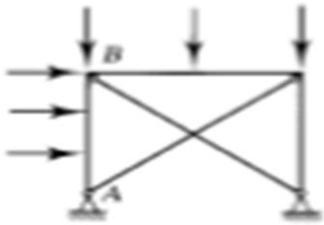
$\frac{M_1}{M_2}$= -ve for single curvatuer

$\frac{M_1}{M_2}$= +ve for single curvatuer

If there are transvers load acting on the member

- Cm=1.0



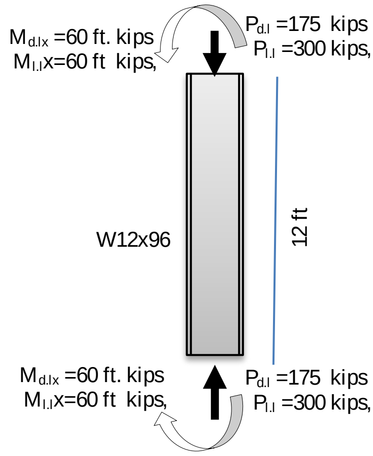


(c) Braced (with transverse loads)

Example

Check the adequacy of 12 ft W12 x96 of A752 G50 steel material beam column in a brace frame bent in single curvature, and not subjected to intermediate transverse loads. Subjected to $P_{d.l}$ =175 kips and $P_{l.l}$ =300 kips, and first order, M_{dx} =60 ft.kip, M_{lx}=60 ft.kip at both ends, assuming Cb=1.0 and use ASD method

Solution



Steel	fy	fu
A572G 50	50	65

Sect.	Zx	Mpx/Ω	BF	Lp	Lr	Ixx
W12 x96	147	367	3.85	10.9	46.7	833

1- Determine the applied load and moments

$Pa= P_{d.l} + P_{l.l}$

$= 175 + 300 = 475$ kip

$Ma_{x\ top} = Ma_{x\ bottom} = M_{d.l\ x} + M_{l.l\ x}$

$= 60 + 60 = 120$ ft. kip

W12×120	186	464	698	285	428	3.94	5.95	11.1	56.5	1070	186	279
W24×68	**177**	**442**	**664**	**269**	**404**	**14.1**	**21.2**	**6.61**	**18.9**	**1830**	**197**	**295**
W16×89	175	437	656	271	407	7.76	11.6	8.80	30.2	1300	176	265
W14×99[f]	173	430	646	274	412	4.91	7.36	13.5	45.3	1110	138	207
W21×73	172	429	645	264	396	12.9	19.4	6.39	19.2	1600	193	289
W12×106	164	409	615	253	381	3.93	5.89	11.0	50.7	933	157	236
W18×76	163	407	611	255	383	8.50	12.8	9.22	27.1	1330	155	232
W21×68	**160**	**399**	**600**	**245**	**368**	**12.5**	**18.8**	**6.36**	**18.7**	**1480**	**181**	**272**
W14×90[f]	157	382	574	250	375	4.82	7.26	15.1	42.5	999	123	185
W24×62	**153**	**382**	**574**	**229**	**344**	**16.1**	**24.1**	**4.87**	**14.4**	**1550**	**204**	**306**
W16×77	150	374	563	234	352	7.34	11.1	8.72	27.8	1110	150	225
W12×96	147	367	551	229	344	3.85	5.78	10.9	46.7	833	140	210
W10×112	147	367	551	220	331	2.69	4.03	9.47	64.1	716	172	258
W18×71	146	364	548	222	333	10.4	15.8	6.00	19.6	1170	183	275
W21×62	**144**	**359**	**540**	**222**	**333**	**11.6**	**17.5**	**6.25**	**18.1**	**1330**	**168**	**252**
W14×82	139	347	521	215	323	5.40	8.10	8.76	33.2	881	146	219
W24×55[v]	**134**	**334**	**503**	**199**	**299**	**14.7**	**22.2**	**4.73**	**13.9**	**1350**	**167**	**252**
W18×65	133	332	499	204	307	9.98	15.0	5.97	18.8	1070	166	248
W12×87	132	329	495	206	310	3.81	5.73	10.8	43.1	740	129	193
W16×67	130	324	488	204	307	6.89	10.4	8.69	26.1	954	129	193
W10×100	130	324	488	196	294	2.64	4.00	9.36	57.9	623	151	226
W21×57	129	322	484	194	291	13.4	20.3	4.77	14.3	1170	171	256

2- Determine the column strength

Kx=ky= 1 brace member

Lx=Ly =12 ft
Kly= 1x12= 12
Sect. W12x 96
Fy=50
ASD

Go to table 4.1 Pn/Ω= 720 kip

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

F_y = 50 ksi — W12

Shape		W12×									
lb/ft		96		87		79		72		65	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
least radius of gyration, r_y	0	844	1270	766	1150	695	1040	632	949	572	859
	6	811	1220	736	1110	667	1000	606	911	549	825
	7	800	1200	726	1090	657	988	597	898	540	812
	8	787	1180	714	1070	646	971	587	883	531	798
	9	772	1160	700	1050	634	953	576	866	521	783
	10	756	1140	685	1030	620	932	564	847	510	766
	11	739	1110	670	1010	606	910	550	827	497	747
	12	720	1080	653	981	590	887	536	806	484	728
	13	701	1050	635	954	574	862	521	783	470	707
	14	680	1020	616	925	556	836	505	759	456	685
	15	659	990	596	896	538	809	489	735	441	663

3- Select the beam column formula

$$\frac{Pr}{Pc}=\frac{Pa}{Pn/\Omega}=\frac{475}{720}=0.66>0.2\ used\ equatuion\ H\ 1-1\ a$$

$$\frac{Pr}{Pc}+\frac{8}{9}\left[\frac{Mrx}{Mnx/\Omega}+\frac{Mry}{Mny/\Omega}\right]\leq 1.0 \quad \text{(H1-1 a)}$$

4- Find the flexural strength of the section (Mnx/Ω)
Lb= 12ft, Lp=10.9ft, Lr =46.7 ft

Lp< Lb<Lr

10.9< 12 <49.6 Zone 2

$$Mnx/\Omega = Cb[Mpx/\Omega - BF(Lb-Lp)] \leq Mpx/\Omega$$

Cb= 1

$$\mathrm{Mnx}/\Omega = 1\mathrm{x}[367-3.85(12-10.9)] \leq 367$$

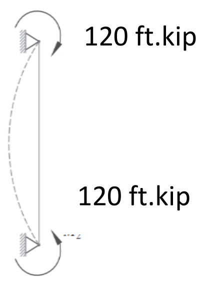
=362.8 < 367

Use $\mathrm{Mnx}/\Omega = 362.8$ ft.kip

$$Mr = B1Mnt$$

$$\mathrm{Mnt} = 120\ \mathrm{ft\,.\,kip}$$

$$B1x = \frac{cm}{1-\frac{\alpha Pr}{pex}} \geq 1.0$$



$$Cm = 0.6-0.4\left(\frac{M1}{M2}\right)$$

$$= 0.6-0.4\left(-\frac{120}{120}\right) = 1.0$$

$$Pex = \frac{\pi^2\, EIx}{(klx)^2}$$

$$Pex = \frac{\pi^2 x29000x833}{(1x12x12)^2} = 11497.88\ kip$$

α= 1.6 ASD

$$B1x = \frac{cm}{1-\frac{\alpha Pr}{pex}} \geq 1.0$$

$$B1x = \frac{1}{1-\frac{1.6x475}{11497.88}} = 1.071 > 1 \ \ ok$$

$$\text{Mr} = 1.071\text{x}\,120 = \ \ 128.5\ \text{ft. kip}$$

5- M_{ny}= 0 no load subjected to the Mainor axis then no need to find (ϕMny)

$$\frac{Pr}{Pnc} + \frac{8}{9}\left[\frac{Mrx}{Mnx/\Omega} + \frac{Mry}{Mny/\Omega}\right] \leq 1.0 \ \ \text{(H1-1 a)}$$

$$0.66 + \frac{8}{9}\left[\frac{128.5}{362.8} + 0\right] = \ 0.975 \leq 1.0 \qquad \text{OK}$$

The section W12x96 satisfied according to AISC specification

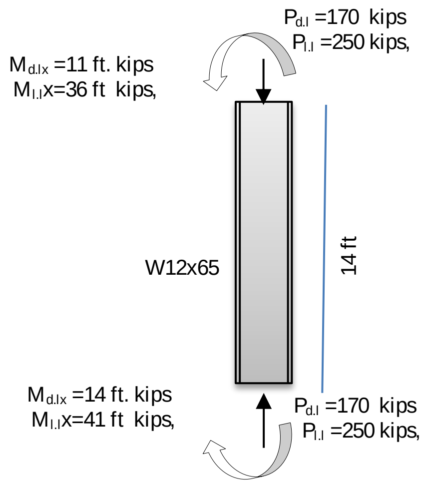
Example

A992 steel material W12x79 is used as abeam column in a brace frame, it bent in single curvature with equal opposite end moment and not subjected to intermediate transverse loads. Is the section satisfactory if $P_{d.l}$ =170 kips and $P_{l.l}$ =250 kips, and first order, M_{dx} =11 ft. Kip, M_{lx}=36 ft.kip at top end and M_{dx} =14 ft.kip, M_{lx}=41 ft. Kip at bottom end assuming Cb=1.0 and use LRFD method

Solution

Steel	fy	fu
A992	50	65

Sect.	Zx	ϕMpx	BF	Lp	Lr	Ixx
W12 x79	119	446	5.67	10.8	39.9	662



1- Determine the applied load and moments

$Pu = 1.2x\ P_{d.l} + 1.6x\ P_{l.l}$

$= 1.2x170 + 1.6x250 = 604$ kip

$Mu_{xtop} = 1.2x\ M_{d.l}\,x + 1.6x\ M_{.l.l\,x}$

$= 1.2x11 + 1.6x36 = \ 70.8$ ft. kip

$Mu_{xbott} 1.2x\ M_{d.l\,x} + 1.6x\ M._{l.l\,x}$

$= 1.2x14 + 1.6x41 = \ 82.4$ ft. kip

F_y = 50 ksi

Table 3-2 (continued)
W-Shapes
Selection by Z_x

Z_x

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF/Ω_b	$\phi_b BF$	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.³	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.⁴	ASD	LRFD
W21×55	126	314	473	192	289	10.8	16.3	6.11	17.4	1140	156	234
W14×74	126	314	473	196	294	5.31	8.05	8.76	31.0	795	128	192
W18×60	123	307	461	189	284	9.62	14.4	5.93	18.2	984	151	227
W12×79	119	297	446	187	281	3.78	5.67	10.8	39.9	662	117	175
W14×68	115	287	431	180	270	5.19	7.81	8.69	29.3	722	116	174
W10×88	113	282	424	172	259	2.62	3.94	9.29	51.2	534	131	196
W18×55	112	279	420	172	258	9.15	13.8	5.90	17.6	890	141	212
W21×50	110	274	413	165	248	12.1	18.3	4.59	13.6	984	158	237
W12×72	108	269	405	170	256	3.69	5.56	10.7	37.5	597	106	159
W21×48[f]	107	265	398	162	244	9.89	14.8	5.86	16.5	959	144	216
W16×57	105	262	394	161	242	7.98	12.0	5.65	18.3	758	141	212

2- Determine the column strength

Kx=ky= 1 brace member

Lx=Ly =14 ft
Kly= 1x14= 14
Sect. W12x 79
Fy=50
LRFD

Go to table 4.1 ϕPn= 836 kip

F_y = 50 ksi

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

W12

Shape		W12×									
lb/ft		96		87		79		72		65	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
least radius of gyration, r_y	0	844	1270	766	1150	695	1040	632	949	572	859
	6	811	1220	736	1110	667	1000	606	911	549	825
	7	800	1200	726	1090	657	988	597	898	540	812
	8	787	1180	714	1070	646	971	587	883	531	798
	9	772	1160	700	1050	634	953	576	866	521	783
	10	756	1140	685	1030	620	932	564	847	510	766
	11	739	1110	670	1010	606	910	550	827	497	747
	12	720	1080	653	981	590	887	536	806	484	728
	13	701	1050	635	954	574	862	521	783	470	707
	14	680	1020	616	925	556	836	505	759	456	685
	15	659	990	596	896	538	809	489	735	441	663

3- Select the beam column formula

$$\frac{Pr}{Pc} = \frac{Pu}{\phi Pn} = \frac{604}{836} = 0.7225 > 0.2\ used\ equatuion\ H\ 1-1\ a$$

$\frac{Pr}{Pc} + \frac{8}{9}\left[\frac{Mrx}{\phi Mnx} + \frac{Mry}{\phi Mny}\right] \leq 1.0$ (H1-1 a)

4- Find the flexural strength of the section (ϕMnx)
Lb= 14ft, Lp=10.8ft, Lr =39.9 ft

Lp< Lb<Lr

10.8< 14 <39.9 Zone 2

$\phi Mnx = Cb[\phi Mpx - BF(Lb - Lp)] \leq \phi Mpx$

Cb= 1

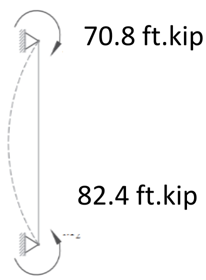
$\phi \text{Mnx} = 1\text{x}[446 - 5.67(14 - 10.8)] \leq 446$

=427.85 < 446

Use $\phi \text{Mnx} = 427.85$ ft. kip

$\text{Mr} = \text{B1Mnt}$

$\text{Mnt} = 82.4$ ft . kip (larger value)



B1 = $\frac{\text{cm}}{1-\frac{\alpha \text{Pr}}{\text{pe1}}} \geq 1.0$

Cm =0.6-0.4($\frac{\text{M1}}{\text{M2}}$)

= 0.6-0.4($-\frac{70.8}{82.4}$)= 0.9437

Pex = $\frac{\pi^2 \text{ EIx}}{(\text{klx})^2}$

Pex = $\frac{\pi^2 \text{x29000x662}}{(1\text{x14x12})^2} = 6713.32$ kip

α= 1.0 LRFD

B1x = $\frac{\text{cm}}{1-\frac{\alpha \text{Pr}}{\text{pex}}} \geq 1.0$

B1x = $\frac{0.9437}{1-\frac{1\text{x}604}{6713.32.}}$= 1.027 > 1 ok

$\text{Mu} = 1.027\text{x } 82.4 = 84.6$ ft. kip

$\frac{Pr}{Pc} + \frac{8}{9}\left[\frac{Mrx}{\phi Mnx} + \frac{Mry}{\phi Mny}\right] \leq 1.0$ (H1-1 a)

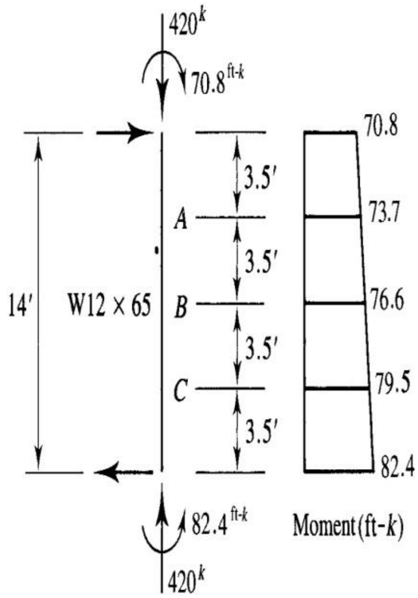
$0.7225 + \frac{8}{9}\left[\frac{84.6}{427.85} + 0\right] = 0.898 \leq 1.0$ OK

The section W12x79 satisfied according to AISC specification

Note

For the actual value of $\boldsymbol{Cb}$, refer to the moment diagram

$$Cb=\frac{12.5\ Mmax.}{2.5\ Mmax.+3MA+4MB+3Mc}$$



M max.= Maximum moment applied
M A= Moment at L/4= 3.5 ft
M B = Moment at L/2= 7.0 ft
M C= Moment at ¾ L= 10.5 ft

M max.= 82.4 ft.kip
M A= 73.7 ft. kip
M B = 76.6 ft. kip
M C=79.5 ft. kip

$$Cb=\frac{12.5\ x82.4}{2.5\ x\ 82.4\ +3x73.7+4x76.6+3x\ 79.5}$$

= 1.06

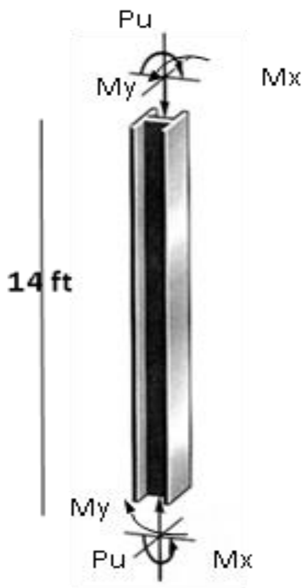
Example

Check the adequacy of (14 ft.), (W14x120), A992 steel material which it be used as abeam column for a part of brace frame its bents in single curvature with equal opposite end moment in both direction and it's not subjected to intermediate transvers load, if the service dead load 70 kips and service live load 100 kips, service dead moment (60 ft.K) and service moment load (80 ft.K) about X- axis and service dead moment (40 Ft.k) and service moment load (60 ft.K) about Y- axis . Assume the beam col. has lateral support at end span only, and (Cb=1) .

Solution

Steel	fy	fu
A992	50	65

Sect.	Zx	ϕMpx	BF	Lp	Lr	Ixx	Iyy	ϕMpy
W14 x120	212	795	7.65	13.2	51.9	1380	495	383



1- Determine the applied load and moments

$Pu= 1.2x\ P_{d.l} +1.6x\ P_{l.l}$

$= 1.2x70 +1.6x100 = 244$ kip

$Mux =1.2x\ M_{d.l\ x} +1.6x\ M._{l.l\ x}$

$=1.2x60+1.6x80= 200$ ft. kip

$Muy=1.2x\ M_{d.l\ x} +1.6x\ M_{.l.l\ x}$

$=1.2x40+1.6x60= 144$ ft. kip

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF/Ω_b	$\phi_b BF$	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W24×84	**224**	**559**	**840**	**342**	**515**	**16.2**	**24.2**	**6.89**	**20.3**	**2370**	**227**	**340**
W21×93	221	551	829	335	504	14.6	22.0	6.50	21.3	2070	251	376
W12×136	214	534	803	325	488	4.02	6.06	11.2	63.2	1240	212	318
W14×120	212	529	795	332	499	5.09	7.65	13.2	51.9	1380	171	257
W18×97	211	526	791	328	494	9.41	14.1	9.36	30.4	1750	199	299
W24×76	**200**	**499**	**750**	**307**	**462**	**15.1**	**22.6**	**6.78**	**19.5**	**2100**	**210**	**315**
W16×100	198	494	743	306	459	7.86	11.9	8.87	32.8	1490	199	298
W21×83	196	489	735	299	449	13.8	20.8	6.46	20.2	1830	220	331
W14×109	192	479	720	302	454	5.01	7.54	13.2	48.5	1240	150	225
W18×86	186	464	698	290	436	9.01	13.6	9.29	28.6	1530	177	265
W12×120	186	464	698	285	428	3.94	5.95	11.1	56.5	1070	186	279
W24×68	**177**	**442**	**664**	**269**	**404**	**14.1**	**21.2**	**6.61**	**18.9**	**1830**	**197**	**295**
W16×89	175	437	656	271	407	7.76	11.6	8.80	30.2	1300	176	265
W14×99^f	173	430	646	274	412	4.91	7.36	13.5	45.3	1110	138	207
W21×73	172	429	645	264	396	12.9	19.4	6.39	19.2	1600	193	289
W12×106	164	409	615	253	381	3.93	5.89	11.0	50.7	933	157	236
W18×76	163	407	611	255	383	8.50	12.8	9.22	27.1	1330	155	232
W21×68	**160**	**399**	**600**	**245**	**368**	**12.5**	**18.8**	**6.36**	**18.7**	**1480**	**181**	**272**

F_y = 50 ksi

Table 3-4 (continued)
W-Shapes
Selection by Z_y

Z_y

Shape	Z_y	M_{py}/Ω_b	$\phi_b M_{py}$	Shape	Z_y	M_{py}/Ω_b	$\phi_b M_{py}$	Shape	Z_y	M_{py}/Ω_b	$\phi_b M_{py}$
		kip-ft	kip-ft			kip-ft	kip-ft			kip-ft	kip-ft
	in.^{3}	ASD	LRFD		in.^{3}	ASD	LRFD		in.^{3}	ASD	LRFD
W18×175	106	264	398	W24×104	62.4	156	234	W10×88	40.1	100	150
W40×211	105	262	394	W40×149	62.2	155	233	W27×94	38.8	96.8	146
W24×162	105	262	394	W21×101	61.7	154	231	W30×99	38.6	96.3	145
W14×120	102	254	383	W10×100	61.0	152	229	W24×94	37.5	93.6	141
				W18×106	60.5	151	227	W14×88	36.9	92.1	138

Table 3-5
W-Shapes
Selection by I_y

I_y

Shape	I_y	Shape	I_y	Shape	I_y	Shape	I_y
	in.^{4}		in.^{4}		in.^{4}		in.^{4}
W14×730^h	**4720**	**W14×283^h**	**1440**	**W14×193**	**931**	**W14×132**	**548**
		W40×372^h	1420	W40×249	926	W21×201	542
W14×665^h	**4170**	W36×330	1420	W44×262	923	W24×192	530
		W30×357^h	1390	W24×306^h	919	W36×256	528
W14×605^h	**3680**	W40×362^h	1380	W27×258	859	W40×278	521
		W27×368^h	1310	W30×235	855	W12×170	517
W14×550^h	**3250**	W36×302	1300	W33×221	840	W27×161	497
W36×652^h	3230	W33×318	1290				
				W14×176	**838**	**W14×120**	**495**
W14×500^h	**2880**	**W14×257**	**1290**	W12×252^h	828	W40×264	493

2- Determine the column strength

Kx=ky= 1 brace member

Lx=Ly =14 ft
Kly= 1x14= 14
Sect. W12x 79
Fy=50
LRFD

Go to table 4.1 ϕPn= 1370 kip

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

F_y = 50 ksi — W14

Shape		W14×											
lb/ft		145		132		120		109		99		90	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
least radius of gyration, r_y	0	1280	1920	1160	1750	1060	1590	958	1440	871	1310	793	1190
	6	1250	1880	1130	1700	1030	1550	932	1400	848	1270	772	1160
	7	1240	1860	1120	1680	1020	1530	923	1390	839	1260	764	1150
	8	1230	1840	1110	1660	1010	1510	913	1370	830	1250	755	1140
	9	1210	1820	1090	1640	994	1490	901	1350	819	1230	745	1120
	10	1200	1800	1080	1620	980	1470	888	1340	807	1210	735	1100
	11	1180	1770	1060	1600	965	1450	874	1310	794	1190	723	1090
	12	1160	1750	1040	1570	948	1430	859	1290	780	1170	710	1070
	13	1140	1720	1020	1540	931	1400	843	1270	766	1150	697	1050
	14	1120	1690	1000	1510	912	1370	826	1240	750	1130	682	1030
	15	1100	1650	982	1480	892	1340	808	1210	733	1100	667	1000

3- Select the beam column formula

$$\frac{Pr}{Pc} = \frac{Pu}{\phi Pn} = \frac{244}{1370} = 0.18 < 0.2 \ used\ equatuion\ H\ 1-1\ b$$

$$\frac{Pr}{2Pc} + \left[\frac{Mrx}{\phi Mnx} + \frac{Mry}{\phi Mny}\right] \leq 1.0 \quad \text{(H1-1 b)}$$

4- Find the flexural strength of the section (ϕMnx)

Lb= 14ft, Lp=13.2ft, Lr =51.9 ft

Lp< Lb<Lr

13.2< 14 <51.9 Zone 2

$\phi Mnx = Cb[\phi Mpx - BF(Lb - Lp)] \leq \phi Mpx$

Cb= 1

$\phi \text{Mnx} = 1\text{x}[795 - 7.65(14 - 13.2)] \leq 795$

=788.88 < 795

Use $\phi\text{Mnx} = 788.88$ ft. kip

$\text{Mrx} = \text{B1xMntx}$

$\text{Mnt} = 200 \text{ ft . kip}$

200 ft.kip

200 ft. kip

B1x = $\frac{\text{cm}}{1-\frac{\alpha \text{Pr}}{\text{pex}}}$ ≥1.0

Cm =0.6-0.4($\frac{\text{M1}}{\text{M2}}$)

= 0.6-0.4($-\frac{200}{200}$)= 1

Pex = $\frac{\pi^2 \text{ EIx}}{(\text{klx})^2}$

Pex = $\frac{\pi^2 \text{x29000x1380}}{(1\text{x}14\text{x}12)^2} = 13994.53 \text{ kip}$

α= 1.0 LRFD

B1x = $\frac{\text{cm}}{1-\frac{\alpha\text{Pr}}{\text{pex}}}$ ≥1.0

B1x = $\frac{1}{1-\frac{1\text{x}244}{13994.53}}$= 1.0178 > 1 ok

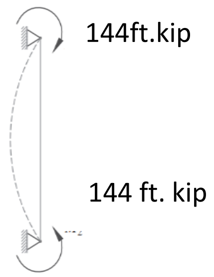
$\text{Mrx} = 1.0178\text{x } 200 = \; 203.55 \text{ ft. kip}$

5- Find the flexural strength of the section (ϕMny)

$\phi Mny = 383 \;\; \text{ft. kip}$

$\text{Mry} = \text{B1xMnty}$

$\text{Mnt} = 144 \text{ ft . kip}$



B1y = $\frac{\text{cm}}{1-\frac{\alpha\text{Pr}}{\text{pey}}}$ ≥1.0

Cm =0.6-0.4($\frac{\text{M1}}{\text{M2}}$)

= 0.6- 0.4($-\frac{144}{144}$) = 1

Pey = $\frac{\pi^2 \text{ EIy}}{(\text{kly})^2}$

Pey = $\frac{\pi^2 \text{x29000x 495}}{(1\text{x}14\text{x}12)^2} = 5019.78 \text{ kip}$

α= 1.0 LRFD

B1y = $\frac{\text{cm}}{1-\frac{\alpha\text{Pr}}{\text{pey}}}$ ≥1.0

B1y = $\frac{1}{1-\frac{1\text{x}244}{5019.78}}$= 1.051 > 1 ok

$$\text{Mry} = 1.051\text{x } 144 = \; 151.36 \text{ ft. kip}$$

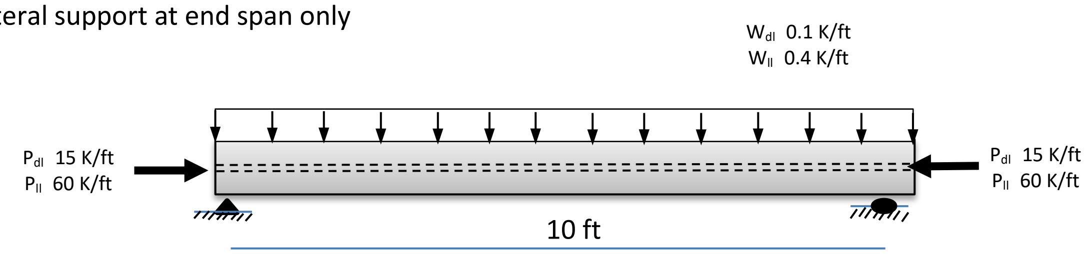
$\frac{244}{2x1370} + \left[\frac{203.55}{788.88} + \frac{151.36}{383}\right]$ ≤ 1.0 (H1-1 b)

0.7422 ≤ 1.0 OK

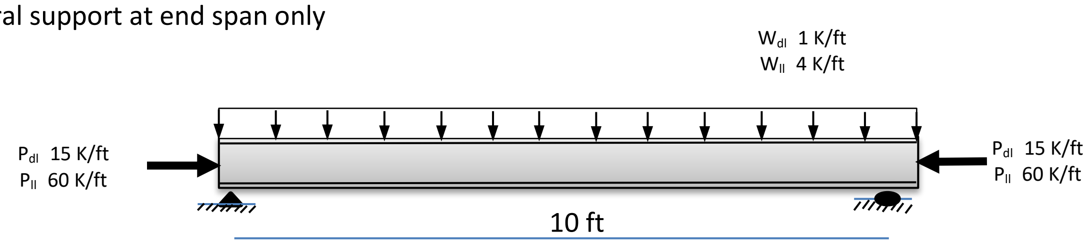
The section W14 x120 is adequate according to AISC specification

Homework

1-Investigate the adequacy of the W8x24 of A992 steel material to support axial compression force (P_{dl} =15 kip and P_{ll} =60 kip) and the uniformly distributed load (W_{dl} =0.1 k/ft and W_{ll} =0.4 k/ft) as shown in the Fig. below, use LRFD method. Assume the beam col. has lateral support at end span only



2- Investigate the adequacy of the W8x24 of A992 steel material to support axial compression force (P_{dl} =15 kip and P_{ll} =60 kip) and the uniformly distributed load (W_{dl} =1 k/ft and W_{ll} =4 k/ft) as shown in the Fig. below, use ASD method. Assume the beam col. has lateral support at end span only



3- What is the maximum total service load(W (k/ft)) that can be applied on a W10x30 of A992 steel material to support axial compression force (P_{dl} =25 kip and P_{ll} =75 kip) as shown in the Fig. below, use ASD method. Assume the beam col. has lateral support at end span only