Design of Steel Structure Civil Engineering / Fourth stage **Flexural Members**

CLASSIFICATION OF SHAPES

AISC classifies cross-sectional shapes as compact, non-compact, or slender, depending on the values of the width-to-thickness ratios. For I shapes, the ratio for the projecting flange (an unstiffened element) is $\frac{bf}{2tf'}$

and the ratio for the web (a stiffened element) is $\frac{h}{tw}$. The classification of shapes is found in Section B4 of the Specification, Member Properties," in Table B4.1b (Table B4.1a is for compression members). It can be summarized as follows

$$\begin{split} \lambda &= \text{width-to-thickness ratio} \left(\frac{b}{t}\right) \\ \lambda &p &= \text{upper limit for compact category} \\ \lambda r &= \text{upper limit for non-compact category} \\ \text{Then:-} \\ \text{if } \lambda &\geq \lambda p \qquad \text{the shape is compact} \\ \text{if } \lambda p &< \lambda &\geq \lambda r \qquad \text{the shape is non-compact} \\ \text{if } \lambda &> \lambda r \qquad \text{the shape is slender} \end{split}$$







The category is based on the worst width-to-thickness ratio of the cross section. For example, if the web is compact and the flange is non-compact, the shape is classified as non-compact. Table 1 has been

extracted from AISC Table B4.1b foe member subjected to flexural. The web criterion is met by all standard I and C shapes listed in the Manual for Fy \leq 65 ksi; therefore, in most cases only the flange ratio needs to be checked (note that built-up welded I shapes can have non-compact or slender webs). Most shapes will also satisfy the flange requirement and will therefore be classified as compact. The non-compact shapes are identified in the dimensions and properties table with a footnote (^f). Note that compression members have different criteria than flexural members, so a shape could be compact for flexure but slender for compression, shapes with slender compression elements are identified with a footnote such as (W21x48^f).

Shape		Element	λ	λр	λr
I-shaped	<u>i</u> +0+ +	Flange	$\frac{bf}{2tf}$	$0.38\sqrt{E/fy}$	$1\sqrt{E/fy}$
sections		Web	$\frac{h}{tw}$	$3.76\sqrt{E/fy}$	$5.7\sqrt{E/fy}$
C-shapes	$\frac{ \frac{b}{2} }{\frac{1}{2}}$	Flange	$\frac{bf}{tf}$	$0.56\sqrt{E/fy}$	$0.56\sqrt{E/fy}$
		Web	$\frac{h}{tw}$	$3.76\sqrt{E/fy}$	$5.7\sqrt{E/fy}$
WT-shapes	$\frac{ \cdot b \cdot }{\int \frac{1}{t}}$	Flange	$\frac{bf}{2tf}$	$0.38\sqrt{E/fy}$	$1\sqrt{E/fy}$
Outstanding legs of single angle	← b→	Unstiffened element	$\frac{b}{t}$	0.54√ <i>E/fy</i>	$0.91\sqrt{E/fy}$
Square or rectangular HSS	$\begin{vmatrix} \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \\ \bullet \bullet \bullet t \\ \hline$	Stiffened element	$\frac{h}{t}$	$2.42\sqrt{E/fy}$	$5.7\sqrt{E/fy}$
Round HSS or pipes		Stiffened element	$\frac{D}{t}$	$0.07 \frac{E}{Fy}$	$0.31 \frac{E}{Fy}$

Table 1 Summary of Table B 4.1 for nexural members	able 1 Summar	y of Table B 4.1 for flexur	al members
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Example Use A992 steel material to classify the steel section according to AISC limitation (W18x35, and W21x48^f) Solution Steel fu fy A992 50 65 bf/2tf h/tw Section W18x35 7.06 53.5 -Flange $\lambda = \frac{bf}{2tf} = 7.06 < 0.38 \sqrt{\frac{E}{fy}}$ $6.28 < 0.38 \sqrt{\frac{29000}{50}}$ 7.06<9.15 ($\lambda \ge \lambda p$) the flange is compact -Web $\lambda = \frac{h}{tw} = 53.5 < 3.76 \sqrt{\frac{E}{fy}}$ - The section is compact $53.5 < 3.76 \sqrt{\frac{29000}{50}}$ 53.5<90.55 ($\lambda \ge \lambda p$) the web is compact

W18x35: This shape can also be identified as compact because there is no footnote in the dimensions and properties tables.

Section bf/2tf h/tw
W21x48 9.47 53.6
- Flange
$$\lambda = \frac{bf}{2tf} = 9.47$$

 $\lambda p = 0.38 \sqrt{\frac{E}{fy}} = 9.15$
 $\lambda r = 1 \sqrt{\frac{E}{fy}} = 24$

 $\lambda p < \lambda \ge \lambda r$ the shape is non-compact



W21x48^f: This shape can also be identified as non-compact because there is a footnote in the dimensions and properties tables.

DESIGN AND ANALYSIS OF BEAMS FOR FLEXURAL REQUIREMENTS (CHAPTER F PP.44)

The Basic design and analysis consideration for beams must be checking the AISC limitation in:

- 1-Bending (flexural) requirements
- 2-shear requirements
- 3-deflection requirements



For the flexural requirements, the required and available strength are moment

load and resistance factor design (LRFD)	For allowable strength design (ASD)
$\begin{split} & Mu = 1.2M_{d,l} + 1.6M_{Ll} \\ & Mu \leq \varphi_{b}Mn \\ & Where: - \\ & Mu = required \text{ moment strength} = maximum \\ & moment \text{ applied} \\ & \varphi_{b} = resistance \text{ factor for bending (flexure)} \\ & = 0.9 \\ & M_n = nominal \text{ moment strength} \\ & Mu \leq 0.9Mn \end{split}$	$\begin{split} & Ma = M_{d,l} + M_{l,l} \\ & Ma \leq \frac{Mn}{\Omega b} \\ & Where: \\ & Ma = required \text{ moment strength} = maximum \\ & moment \text{ applied} \\ & \Omega_b = safety \text{ factor for bending} = 1.67 \\ & M_n = nominal \text{ moment strength} \\ & M_a \leq \frac{Mn}{1.67} \end{split}$

Lateral torsional buckling (LTB), laterally unsupported length (Lb)

The compression flange of a beam behaves like an axially loaded column. Thus, in beams covering long spans the compression flange may tend to buckle. Unlike a column, however, the compression portion of the cross section is restrained by the tension portion, and the outward deflection (flexural buckling) is accompanied by twisting (torsion). This form of instability is called lateral-torsional buckling (LTB).

Lateral-torsional buckling occurs when the distance between lateral brace points is large enough that the beam fails by lateral, outward movement in combination with a twisting action (Δ and θ , respectively, in Figure below).



Lateral torsional buckling may be prevented through the following provisions:

1. Increases the Lateral supports at intermediate points in addition to lateral supports at the vertical supports

2. Using torsionally strong sections (for example, box sections) prevents twist directly; it can be either nodal or continuous, and it can take the form of either cross frames or diaphragms



3. I-sections with relatively wide flanges: Beams with wider flanges are less susceptible to lateral-torsional buckling because the wider flanges provide more resistance to lateral displacement

Lateral-torsional buckling is fundamentally similar to the flexural buckling or flexuraltorsional buckling of a column subjected to axial loading. The differences are that lateral torsional buckling is caused by flexural loading (M), and the buckling deformations are coupled in the lateral and torsional directions. Also there is one very important difference. For a column, the axial load causing buckling remains constant along the length. But, for a beam, usually the lateral-torsional buckling causing bending moment M(x) varies along the unbraced length.

If the maximum bending stress is less than the proportional limit when buckling occurs, the failure is said to be elastic. Otherwise, it is inelastic.

For convenience, we first categorize beams as compact, non-compact, or slender, and then determine the moment resistance based on the degree of lateral support.

L_b= laterally unsupported length

 L_p = Plastic length: maximum unbraced length at which the nominal bending strength equals the plastic moment capacity and at which inelastic lateral-torsional buckling occurs.

L_r= unbraced length at which **elastic** lateral– torsional buckling occurs.



Where:-

Iy = moment of inertia about the weak axis of the cross section (in.⁴) G = shear modulus of structural steel = 11,200 ksi h_0 =distance between the flange centroids=d-tf J = torsional constant (in.⁴) C_w = warping constant (in.⁶) The nominal bending strength, M_n , is a function of the following:

- 1. Lateral-torsional buckling (LTB),
- 2. Flange local buckling (FLB), and
- 3. Web local buckling (WLB).



If the beam is compact and has continuous lateral support, or **if the unbraced length is very short**, the nominal moment strength, Mn, is the full plastic moment capacity of the shape, Mp. For members with inadequate lateral support, the moment resistance is limited by the lateral-torsional buckling strength, either in elastic or elastic

Laterally unsupported length (Lb)

1-when Lb≤Lp (Zone 1)

 M_n = The moment strength of compact shapes is a function of the unbraced length, L_b, defined as the distance between points of lateral support, or bracing indicated as "×". The relationship between the nominal strength, M_n , and the unbraced length is shown in Figure. If the unbraced length is no greater than L_p , the beam is considered to have full lateral support, and

 $M_p = FyZ$. The first category, laterally supported compact beams, is quite common and is the simplest case. For a doubly-symmetric, compact I- or C-shaped section bent about its major axis, AISC F2.1 gives the nominal strength as

Mn = Mp (AISC Equation F2-1)
Where

$$Mp = FyZ$$

Lb = the full length (Lu) if there is no intermediate laterally supported between the supports

Lb = 0 if the beams has full laterally supported beams



2-when Lp<Lb≤Lr (Zone 2)

If L_b is greater than L_p but less than or equal to L_r , the strength is based on inelastic LTB.

$$M_{n} = C_{b} \left[M_{p} - (M_{p} - 0.7F_{y}S_{x}) \left(\frac{L_{b} - L_{p}}{L_{r} - L_{p}} \right) \right] \le M_{p}$$
 (AISC Equation F2-2)

Cb =laterally torsional buckling modification factor for non-uniform moments diagrams when both ends of the un- supported segment are brace

 $Cb = \frac{12.5Mmax}{2.5Mmax + 3MA + 4MB + 3MC} Rm \le 3$ Where

 M_{max} = absolute value of maximum moment in the unbraced segment, kip-in. M_A = absolute value of moment at quarter point of the unbraced segment, kip-in. M_B = absolute value of moment at centerline of the unbraced segment, kip-in. M_C = absolute value of moment at three-quarter point of the unbraced segment, kip-in. Rm = cross-section monosymmetry parameter = 1.0, doubly symmetric members

= 1.0, singly symmetric member's subjected to single curvature bending = $0.5 + 2\left(\frac{Iyc}{Iy}\right)^2$

 I_y = moment of inertia about the principal y-axis, in.⁴

lyc = moment of inertia about y-axis referred to the compression flange, or if reverse curvature bending, referred to the smaller flange, in.⁴

Cb: is permitted to be conservatively taken as 1.0 for all cases. For cantilevers or overhangs where the free end is unbraced, Cb = 1.0.

Table 3–1 Values for C_b for Simply Supported Beams							
Load	Lateral Bracing Along Span	C _b					
P	None Load at midpoint	1.52					
t t	At load point	1.67 1.67 ¥					
PP	None Loads at third points	¥ 1.74 ¥					
<u>†</u>	At load points	1.67 1.00 1.67					
	None Loads at quarter points	F 1.34					
† †	At load points	1.67 7.11 5.11 1.67 F					
	None	1.24					
	At midpoint	1.30 1.30					
	At third points	1.45 1.01 1.45					
1 1	At quarter points	1.52 1.06 1.06 1.52					
	At fifth points	1.56 1.12 1.00 1.12 1.56					

Determine the lateral torsional factor for the beam shown below



or use table (3 - 1)Cb = 1.14

Homework Determine the lateral torsional factor for the beam shown below



Flexural Members

3- When Lb>Lr(zone 3)

The AISC Specification gives a different, but equivalent, form for the elastic buckling stress F_{cr} . AISC gives the nominal moment strength as

$$Mn = FcrSx \le Mp$$
 (AISC Equation F2-3)

$$F_{cr} = \frac{C_b \pi^2 E}{(L_b / r_{ls})^2} \sqrt{1 + 0.078 \frac{J_c}{S_x h_0} \left(\frac{L_b}{r_{ls}}\right)^2}$$

(AISC Equation F2-4)

Where the yield stress has been reduced by 30% to account for the effect of residual stress



Summary of Nominal Flexural Strength

The nominal bending strength for compact I and C-shaped sections can be summarized as follows:

For $L_b \leq L_p$, $M_n = M$

$$= M_p \qquad (AISC Equation F2-1)$$

For $L_p < L_b \leq L_r$,

$$M_n = C_b \left[M_p - (M_p - 0.7F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \le \boldsymbol{M}_p \qquad \text{(AISC Equation F2-2)}$$

For $L_b > L_r$,

$$M_n = F_{cr} S_x \le M_p \tag{AISC Equation F2-3}$$

where

$$F_{cr} = \frac{C_b \pi^2 E}{(L_b/r_{ls})^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_0} \left(\frac{L_b}{r_{ls}}\right)^2}$$
(AISC Equation F2-4)



Zone 1: Plastic behavior, full plastic moment Zone 2: Inelastic lateral torsion buckling Zone 3: Elastic lateral torsion buckling

The beam shown in Figure below is a W16 \times 31 of A992 steel. It supports a reinforced concrete floor slab that provides continuous lateral support of the compression flange. The service dead load is 450 lb/ft. This load is superimposed on the beam; it does not include the weight of the beam itself. The service live load is 550 lb/ft. Does this beam have adequate moment strength?

Solution					D.L = 450 lb/ft
<u>Steel</u>	fy	<u>fu</u>			L.L = 550 lb/ft
A992	50	65			F **** ** ** ** ** ** ** ** ** ** ** **
<u>Section</u>	bf,	/2tf	<u>h/tw</u>	<u>Zx</u>	
W16x31	6.	28	51.6	54	30 ft

First, determine the nominal flexural strength. -Check the local buckling for

-flange

$$\lambda = \frac{bf}{2tf} = 6.28 < 0.38 \sqrt{\frac{E}{fy}}$$

6.28<0.38 $\sqrt{\frac{29000}{50}}$

6.28<9.15 ($\lambda \ge \lambda p$) the flange is compact -web

 $λ = \frac{h}{tw} = 51.6 < 3.76 \sqrt{\frac{E}{fy}}$ 5.16<3.76 $\sqrt{\frac{29000}{50}}$ 51.6<90.55 (λ≥ λp) the web is compact

The section is compact



Lb = 0 the beams has full laterally supported

Since the beam is compact and laterally supported, the nominal flexural strength is

Mn = Mp = FyZx = 50(54.0) = 2700 in.-kips = 225.0 ft kips.

Compute the maximum bending moment.

The total service dead load, including the weight of the beam, is

W_D = 450 + 31 = 481 lb/ft=0.481 k/ft

W_l=550 lb/ft =0.55 k/ft

W_u=1.2W_d+1.6W₁=1.2x0.481+1.6x0.55=1.456 k/ft

For a simply supported, uniformly loaded beam, the maximum bending moment occurs at mid span and is equal to where w is the load in units of force per unit length, and L is the span length.

 $Mu = \frac{wul^2}{8} = \frac{1.456 \times 30^2}{8} = 164 \text{ ft.kip}$ Alternatively, the loads can be factored at the outset: The design strength is ϕ Mn = 0.90(225.0) = 203 ft-kips > 164 ft-kips (OK) The design moment is greater than the factored-load moment, So the W16 × 31 is satisfactory (adequate)

Homework

Re solve the above example by using ASD method

Use LRFD method to determine the flexural strength of a W14 \times 68 of A992 steel material if a. Continuous lateral support.

- b. An unbraced length of 20 ft with Cb = 1.0.
- c. An unbraced length of 30 ft with Cb = 1.0.

Solution

<u>Steel</u>	<u>fy</u>	<u>fu</u>										
A992	50	65										
Section W14x68	<u>bf/2tf</u> 6.97	<u>h/tw</u> 27.5	<u>Sx</u> 103	<u>Zx</u> 115	<u>rts</u> 2.8	<u>ho</u> 13.3	<u>J</u> 3.01	<u>cw</u> 5380	<u>ry</u> 2.46	<u>ly</u> 121		
Check the -Flange	e section											
$\lambda = \frac{bf}{2tf} = 6.97$	v < 0.38 []	E fy										
$5.97 < 0.38 \sqrt{\frac{29000}{50}}$												
6.97<9.15 -Web	5 (λ≥λp)) the f	lange	e is c	отр	act						
$\lambda = \frac{h}{tw} = 27.5$	$< 3.76 \sqrt{\frac{E}{f_{s}}}$	y y					— т	he sect	ion is co	ompact	: Use eq	u. F ₂
27.5<3.76	$\sqrt{\frac{29000}{50}}$											
27.5<90.5	55(λ≥λ	p) the	web	is co	тра	ct _						

a. Continuous lateral support

 L_b = 0 the beams has full laterally supported

 $L_b < L_p$ — Zone 1

Since the beam is compact and laterally supported, the nominal flexural strength is Mn = Mp = FyZx = 50(115) = 5750in.-kips = 479.16 ft kips. $\phi Mn = \phi Mp = \phi FyZx = 0.9x 479.16 = 431.25 ft. kip$

b. An unbraced length of 20 ft with Cb = 1.0.

L_b=20ft

 $Lp = 1.76ry \sqrt{\frac{E}{fy}} = Lp = 1.762.46 \sqrt{\frac{2900}{50}} = 104.27 \text{ in} = \frac{104.27}{12} = 8.689 \text{ ft}$ or use table (3.2) Lp=8.69 ft

$$L_{r} = 1.95 r_{ts} \frac{E}{0.7 F_{y}} \sqrt{\frac{J c}{S_{x} h_{o}}} \sqrt{1 + \sqrt{1 + 6.76 \left(\frac{0.7 F_{y}}{E} \frac{S_{x} h_{o}}{J c}\right)^{2}}$$

$$\mathbf{r}_{\mathsf{ts}} = \left(\frac{\sqrt{\mathbf{I}_{\mathsf{y}} \, \mathbf{C}_{\mathsf{w}}}}{\mathbf{S}_{\mathsf{x}}}\right)^{1/2}$$

rts=2.798 in rt=2.8 h₀=d-tf=13.28 c=1 I shape Design of Steel Structure Civil Engineering / Fourth stage

Lr= 351.3 in $=\frac{351.3}{12}$ = 29.28 ft Or use table (3.2) Lr= 29.3 ft

 $Lp < Lb \le Lr \longrightarrow$ Zone 2 8.69< 20 \le 29.3

 $Mn = Cb[Mp - (Mp - 0.7FySx)] [\frac{lb - lp}{lr - lp}] \le Mp$ Cb=1,Mp = FyZX=50x115=5750 in.kip Mn = 1[570 - (5750 - 0.7x50x103)[$\frac{20 - 8.69}{29.3 - 8.69}$] \le 5750 =4572< 5750 Use Mn=4572 in.kip ϕ Mn=0.9x4572=4114.8 in.kip =342.9 ft.kip

c. An unbraced length of 30 ft with Cb = 1.0.

$$\begin{aligned} \text{lb>Lr} &\longrightarrow \text{Zone 3} \\ 30> 29.3 \\ Mn &= FcrxSx < MP \\ Fcr &= \frac{cb\pi^2 E}{(\frac{lb}{rts})^2} \sqrt{(1+0.078\frac{Jc}{Sxho}(\frac{lb}{rts})^2)} \\ Fcr &= \frac{1x\pi^2 29000}{(\frac{30x12}{2.8})^2} \sqrt{(1+0.078\frac{3.01x1}{103x13.3}(\frac{30x12}{2.8})^2)} \\ &= 33.9 \text{ ksi} \\ Mn &= 33.9x103 < 5750 \\ &= 3492 < 5750 \\ &= \frac{3492}{12} = 291 \text{ ft.kip} \end{aligned}$$

φMn=0.9x291=261.9 ft. kip

Homework

Re solve the above example by use ASD method

Use ASD method to determine the flexural strength of a W14 \times 74 of A992 steel material if

- a. Continuous lateral support.
- b. An unbraced length of 15 ft.
- c. An unbraced length of 35 ft.

Solution

<u>Steel</u>	<u>fy</u>	<u>fu</u>								
A992	50	65								
Section W14x74	<u>bf/2tf</u> 6.41	<u>h/tw</u> 25.4	<u>S_</u> 112	<u>Z</u> <u>x</u> 126	<u>r_{ts}</u> 2.82	<u>h</u> ₀ 13.4	<u>J</u> 3.87	<u>c_w</u> 5990	<u>r_v</u> 2.48	<u>l_v</u> 134
Check the	e section)								
-Flange										
$\lambda = \frac{bf}{2tf} = 6.97$	$\lambda = \frac{bf}{2tf} = 6.97 < 0.38 \sqrt{\frac{E}{fy}}$									
$5.41 < 0.38 \sqrt{\frac{29000}{50}}$										
6.41<9.1	5 (λ≥ λ p)) the f	lange	e is c	отра	ct –]			
-Web										
$\lambda = \frac{h}{tw} = 25.4$	Web $x = \frac{h}{tw} = 25.4 < 3.76 \sqrt{\frac{E}{fy}}$ The section is compact Use equ. F ₂									
25.4<3.76	$\sqrt{\frac{29000}{50}}$									
25.4<90.5	55(λ≥λ	.p) the	web	is coi	npact		J			

a. Continuous lateral support

L_b = 0 the beams has full laterally supported

 $L_b < L_p$ — Zone 1

Since the beam is compact and laterally supported, the nominal flexural strength is Mn = Mp = FyZx = 50(126) = 6300 in.-kips = 525 ft kips. $\frac{Mn}{\Omega b} = \frac{Mp}{\Omega b} = \frac{FyZx}{\Omega b} = \frac{525}{\Omega b} = 314.4 \text{ ft. kip}$

b. An unbraced length of 15 ft.

L_b=15 ft Cb=1.3 table 3.1 use table (3.2) Lp=8.76 ft Lr = 31 ft Lp < Lb \leq Lr \longrightarrow Zone 2 8.76< 15 \leq 31 $Mn = Cb[Mp - (Mp - 0.7FySx)[\frac{lb - lp}{lr - lp}] \leq Mp$ Cb=1.3,Mp = 6300 in.kip $Mn = 1.3x[6300 - (6300 - 0.7x50x112)[\frac{15 - 8.76}{31 - 8.76}] \leq 6300$ Design of Steel Structure Civil Engineering / Fourth stage Flexural Members

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=7321.8> 6300 Use Mn=6300 in.kip $\frac{Mn}{\Omega b} = \frac{Mp}{\Omega b} = \frac{6300}{1.67} = \frac{3772.5}{12} = 314.44 \, ft. \, kip$

c. An unbraced length of 35 ft.

$$\begin{aligned} \text{lb>Lr} &\longrightarrow \text{Zone 3} \\ Mn &= FcrxSx < MP \\ Fcr &= \frac{cb\pi^2 E}{(\frac{lb}{rts})^2} \sqrt{(1+0.078\frac{Jc}{Sxho}(\frac{lb}{rts})^2)} \\ Fcr &= \frac{1.3x\pi^2 29000}{(\frac{35x12}{2.82})^2} \sqrt{(1+0.078\frac{3.87x1}{112x13.4}(\frac{35x12}{2.82})^2)} \\ &= 39.2 \text{ ksi} \\ Mn &= 39.2x112 < 6300 \\ &= 4390.52 < 6300 \\ &= \frac{3949.2}{12} = 365.87 \text{ ft.kip} \\ \frac{Mn}{\Omega b} &= \frac{Mp}{\Omega b} = \frac{365.87}{1.67} = 219 \text{ ft. kip} \end{aligned}$$



Homework

Re solve the above example by use LRFD method