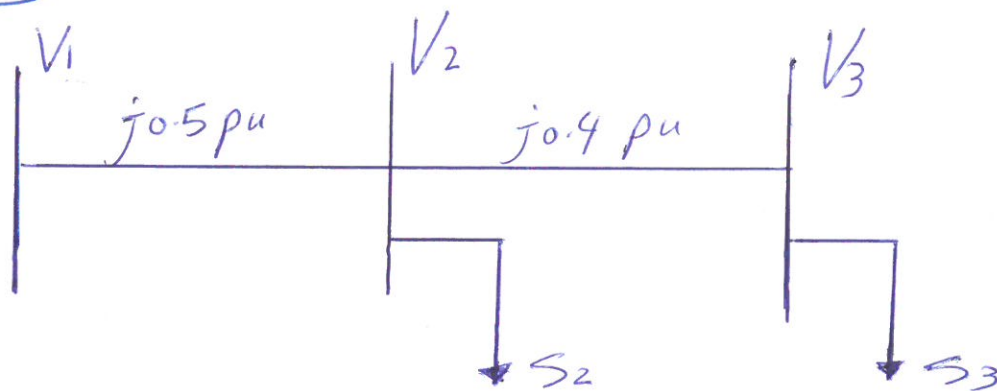


ex 9) The one-line diagram of a 3- $\phi$  system in the <sup>4-24</sup>  
fig. below



Impedance are marked in per unit on 100 MVA, 400 kV base. The load at bus 2 is

$$S_2 = 15.93 \text{ MW} + j 33.4 \text{ Mvar} \text{ and at bus 3}$$

$$S_3 = 77 \text{ MW} + j 14 \text{ Mvar} \text{ . It is required to hold the voltage at bus 3 at } 400 \angle 0 \text{ kV}$$

working in per unit, determine the voltage at bus 2 and 1

Sol)  $V_B = 400$  (in all point because there is no Transformer)  
 $S_B = 100 \text{ MVA}$

$$S_2 = \frac{15.9 - j 33.4}{100} = 0.159 - j 0.334 \text{ pu}$$

$$S_3 = \frac{77 + j 14}{100} = 0.77 + j 0.14 \text{ pu}$$

$$S_3^{\text{pu}} = V_3^{\text{pu}} I_3^{\text{pu}*}$$

$$I_3 = \frac{S_3^{\text{pu}}}{V_3^{\text{pu}}}$$

$$V_3^{\text{pu}} = \frac{\text{actual}}{\text{Base}} = \frac{400}{400}$$

$$V_3^{\text{pu}} = 1 \angle 0 \text{ pu}$$

$$\frac{I_3}{pu} = \frac{S_3^*}{V_3^*} = \frac{0.77 - j0.14}{1 \angle 0} = 0.77 - j0.14 \text{ pu} \quad (1-25)$$

$$V_2 = V_3 + I_3(j0.4)$$

$$= 1 \angle 0 + (0.77 - j0.14) * (j0.4)$$

$$= 1 + j0.308 + 0.056 = 1.056 + j0.308 \text{ pu}$$

$$= 1.1 \angle 16.26^\circ$$

$$V_2 = 1.1 \text{ pu} * 400 \text{ kV}_{\text{Base}} = \underline{440 \text{ kV}}_{\text{Line to Line}}$$

actual

$$\frac{I_2}{pu} = \frac{S_2^*}{V_2^*} = \frac{0.1593 + j0.334}{1.1 \angle -16.26^\circ} = 0.054 + j0.332 \text{ pu}$$

$$I_1 = I_2 + I_3$$

$$= (0.054 + j0.332) + (0.77 - j0.14)$$

$$I_1 = 0.824 + j0.192 \text{ pu}$$

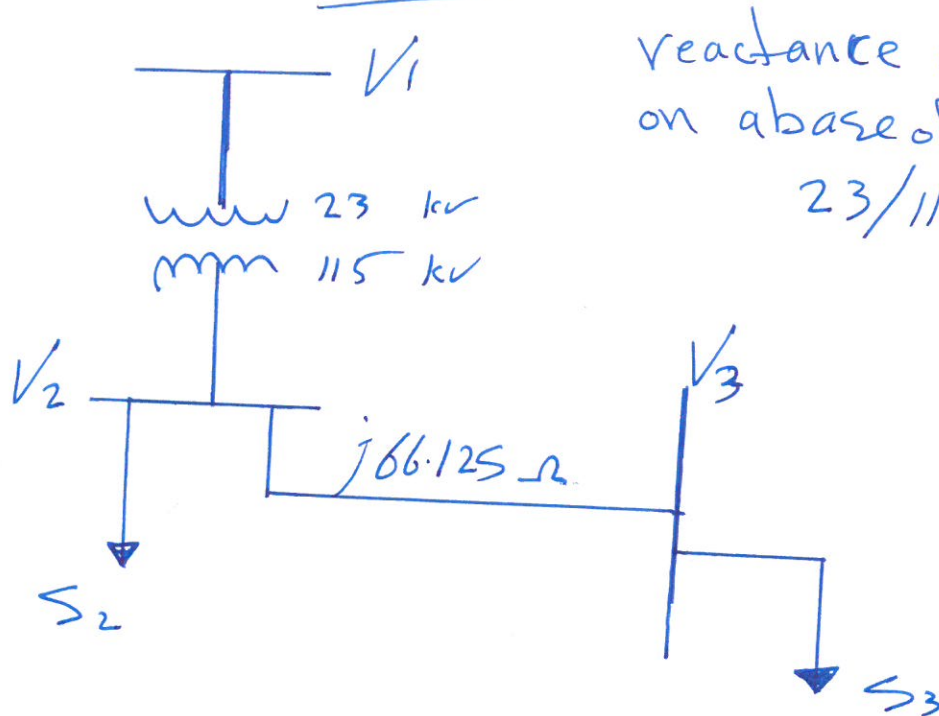
$$V_1 = V_2 + I_1(j0.5)$$

$$= 1.1 \angle 16.26^\circ + (0.824 + j0.192)(j0.5)$$

$$= 1.2 \angle 36.87^\circ$$

$$V_1 = 400 * 1.2 = \underline{480 \text{ kV}}_{\text{Line to Line}}$$

Ex 10) The one-line diagram of a 3- $\phi$  power system (1-26) is shown in the fig. below. The transformer



reactance is 20 percent on a base of 100 MVA, 23/115 kV.

The line impedance  $Z = j66.125 \Omega$ .

The load  $S_2 = 184.8 \text{ MW} + j6.6 \text{ MVar}$

$S_3 = 0 \text{ MW} + j20 \text{ MVar}$

It is required to hold the voltage at bus 3 at 115 kV. Working in per unit, find the voltage  $V_2, V_1$

Sol

$S_B = 100 \text{ MVA}$

$V_{B1} = 23 \text{ kV}$

$V_{B2} = 115 \text{ kV}$

(1-27)

$$S_2 = \frac{184.8 + j6.6}{100} = 1.848 + j0.066 \text{ pu}$$

$$S_3 = \frac{0 + j20}{100} = 0 + j0.2 \text{ pu}$$

$$V_3 = \frac{\text{actual}}{\text{Base}} = \frac{115 \text{ kV}}{115} = 1.0 \text{ pu}$$

$$V_2 = V_3 + I_3 Z$$

$$I_3 \text{ pu} = \frac{S_3^*}{V_3^*} = \frac{-j0.2}{1.0} = -j0.2 \text{ pu}$$

$$Z \text{ pu} = \frac{\text{actual}}{\text{Base}} \quad \left| \quad Z_B = \frac{V_B^2}{S_B} = \frac{115^2}{100}$$

$$Z_B = 132.25 \Omega$$

$$Z = \frac{66.125}{132.25} = j0.5 \text{ pu}$$

$$V_2 = 1.0 + (-j0.2)(j0.5)$$

$$= 1 + 0.1 = 1.1 \text{ pu} \quad \neq$$

$$V_2 = 1.1 \times 115 = 126.5 \text{ kV} \quad \text{line to line}$$

$$I_2 = \frac{S_2^*}{V_2^*} = \frac{1.848 - j0.066}{1.1} = 1.68 - j0.26$$

$$I_1 = I_3 + I_2$$

$$I_1 = (-j0.2) + (1.68 - j0.06) = 1.68 - j0.26 \text{ pu}$$

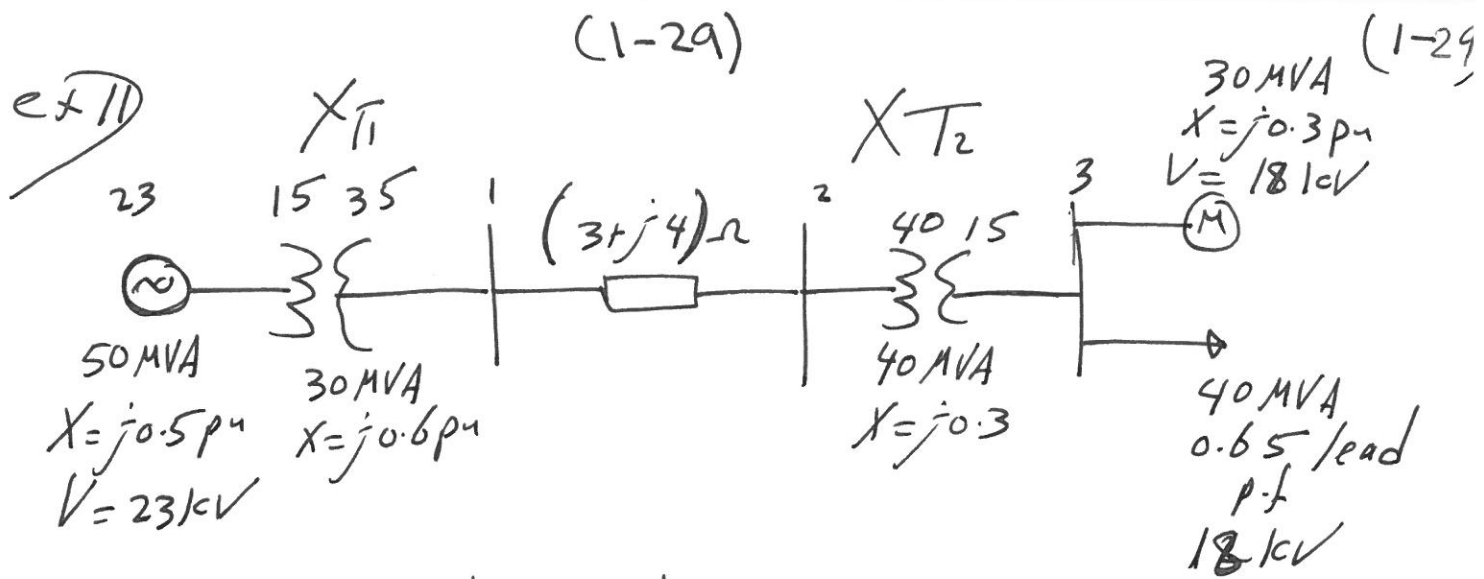
$$V_1 = I_1 X_{\text{Trans}} + V_2$$

$$= (1.68 - j0.26) * (j0.2) + 1.1 \angle 0^\circ$$

$$= 1.2 \angle 16.26^\circ$$

$$V_1 = 1.2 * 23 = \underline{\underline{27.6 \text{ kV}}}$$

Line to Line



- ① Find per unit impedance
- ② Find  $E_m$  of the motor

Take  $V_B = 22 \text{ kV}$  at bus bar 1  
 $S_B = 100 \text{ MVA}$

Sol)

$$V_B = 22 \times \frac{15}{35} = 9.4 \text{ kV (base)}$$

$$V_m = 22 \times \frac{15}{40} = 8.25 \text{ kV (base)}$$

$V_{Load}$

$$X_{G1} = 0.5 \left( \frac{100}{50} \right) \left( \frac{23}{9.4} \right)^2 = j5.98 \text{ pu}$$

$$X_{T1} = 0.6 \left( \frac{100}{30} \right) \left( \frac{35}{22} \right)^2 = j5.0619 \text{ pu}$$

Secondary

$$X_{T2} = 0.3 \left( \frac{100}{40} \right) \left( \frac{40}{22} \right)^2 = j2.47 \text{ pu}$$

primary

$$X_m = 0.3 \left( \frac{100}{30} \right) \left( \frac{18}{8.25} \right)^2 = j4.76 \text{ pu}$$

(1-30)

$$Z_{B \text{ Line}} = \frac{V_B^2}{S_B} = \frac{22^2}{100} = 4.84 \Omega$$

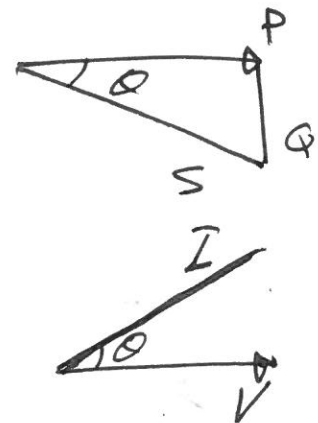
$$Z_{\text{Line pu}} = \frac{\text{actual}}{\text{Base}} = \frac{3+j4}{4.84} = 0.62 + j0.826 \text{ pu}$$

$$Z_{\text{Load}} = \frac{V_{LL}^2}{S^*_{L}} \Rightarrow S_L = 40 \angle -\cos^{-1} 0.65$$

$$= 40 \angle -49.45^\circ$$

$$Z_{\text{Load}} = \frac{18^2}{40 \angle 49.45^\circ} = 8.1 \angle -49.45^\circ$$

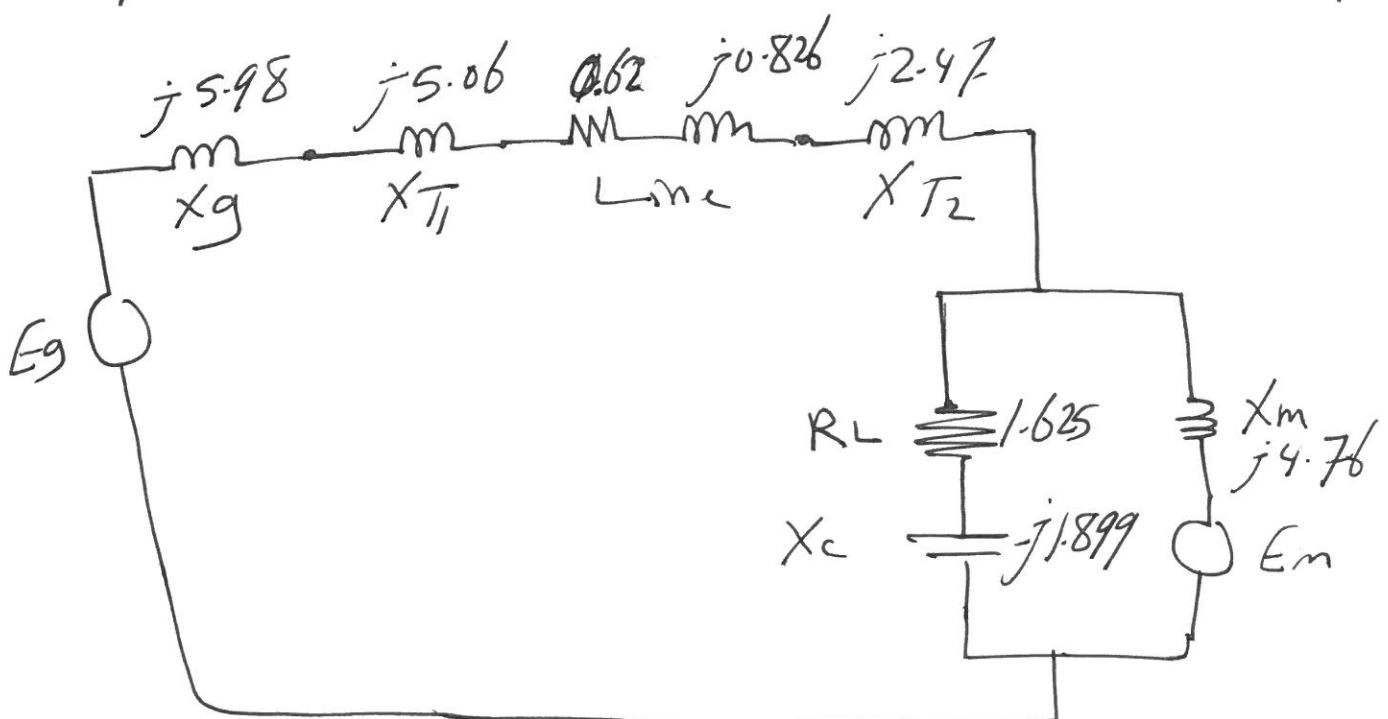
$$Z_{\text{Load}} = (5.265 - j6.154) \Omega$$



leading p.f

$$Z_{B \text{ Load}} = \frac{V_B^2}{S_B} = \frac{18^2}{100} = 3.24 \Omega$$

$$Z_{\text{Load pu}} = \frac{\text{actual}}{\text{Base}} = \frac{5.265 - j6.154}{3.24} = 1.625 - j1.899 \text{ pu}$$



$$E_m = V_m - \bar{I}_m X_m$$

(1-31)

(1-31)

$$V_m = \frac{\text{actual}}{\text{Base}} = \frac{18}{8.25} = 2.18 \text{ pu} = \sqrt{3}$$

$$S_m = V_m \bar{I}_m^* \Rightarrow \bar{I}_m = \frac{S_m^*}{V_m^*}$$

$$\bar{I}_m = \frac{S_m^*}{V_m^*}$$

$$S_m = 30 \angle -90$$

because it has only  
reactance  $X_m = j0.476$   
(leading P.F.)

$$S_m = \frac{30 \angle 90}{100}$$

$$= 0.3 \angle 90 = j0.3 \text{ pu}$$

$$\bar{I}_m = \frac{S_m^*}{V_m^*} = \frac{-j0.3}{2.18} = -j0.1376 \text{ pu}$$

$$E_m = V_m - \bar{I}_m X_m$$

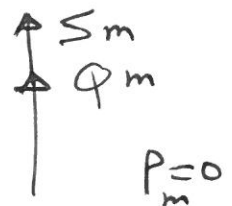
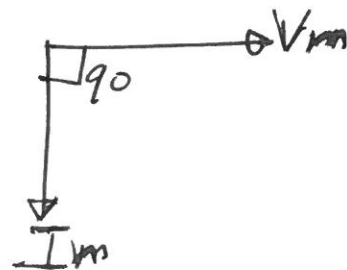
$$= 2.18 \angle 0 - (-j0.1376)^* (j4.76)$$

$$= 2.18 - 0.654$$

$$= 1.526 \text{ pu}$$

OR  $E_m = V_2 - (\bar{I}_m + \bar{I}_L) X_{T2} - \bar{I}_m X_m$

$$V_2 = V_m + (\bar{I}_m + \bar{I}_L) X_{T2}$$



$$E_m = V_2 - (\hat{I}_m + \hat{I}_L) X_T - \hat{I}_m X_m$$

(1-32)

$$V_2 = (\hat{I}_m + \hat{I}_L) X_T + V_m$$

$$E_m = (\hat{I}_m + \hat{I}_L) X_T + V_m - (\hat{I}_m + \hat{I}_L) X_T - \hat{I}_m X_m$$

$$E_m = V_m - \hat{I}_m X_m$$

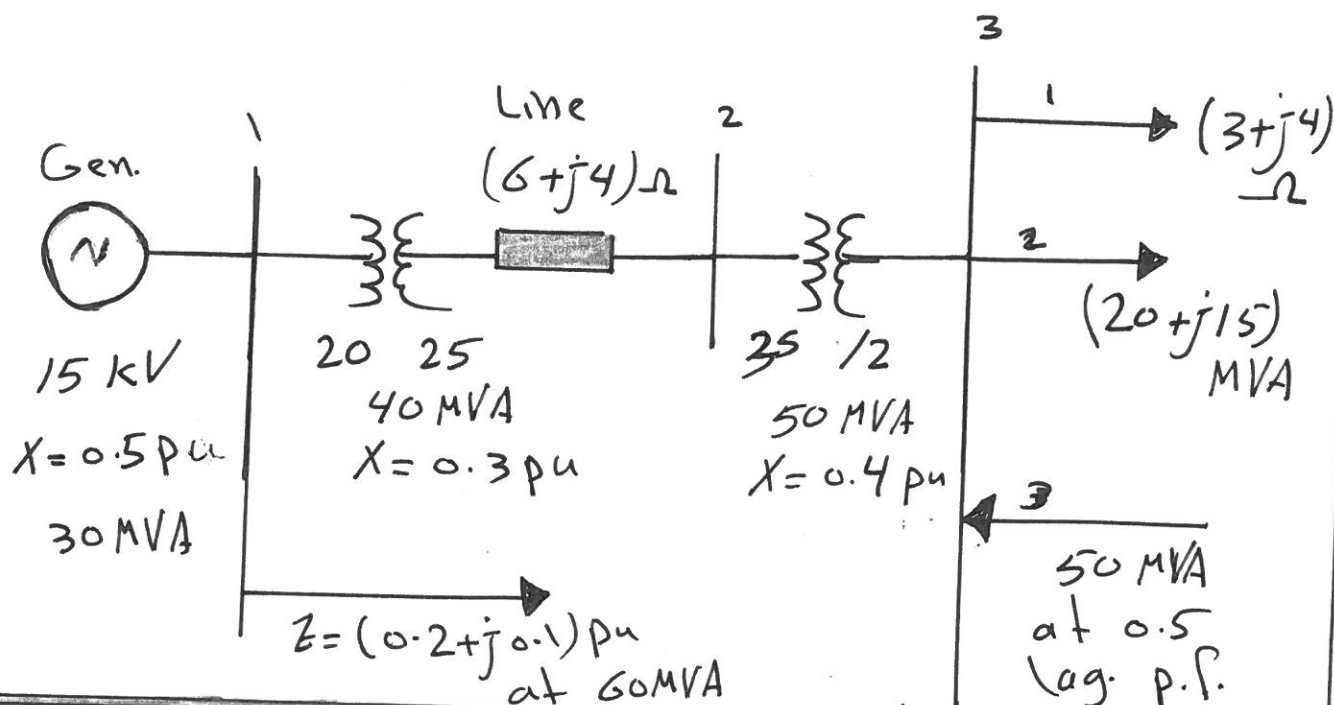
$$E_G = V_m + (\hat{I}_m + \hat{I}_L) * (Z + X_G)$$

$$Z = X_{T2} + X_{Line} + X_{T1}$$

Q)

1-3

In the Fig. below



The base MVA is 100 MVA  
and the base voltage is at  
the line circuit is 40 kV

The terminal voltage at  
bus bar 3 is 15 kV

Find

1- The per unit impedance

2- The line current (per unit and actual)

3- The actual generator current

4- The emf of the generator and motor

5- Draw the per unit ckt.

Sol)

$$V_{B1} = V_{BG} = 40 \times \frac{20}{25} = 32 \text{ kV}$$

$$V_{B3} = 40 \times \frac{12}{35} = 13.71 \text{ kV}$$

$$X_{G_{\text{new}}} = X_{\text{old}} \left( \frac{S_{B_{\text{new}}}}{S_{B_{\text{old}}}} \right) \left( \frac{V_{B_{\text{old}}}}{V_{B_{\text{new}}}} \right)^2$$

$$X_G = 0.5 \left( \frac{100}{30} \right) \left( \frac{15}{32} \right)^2 = j 0.3662 \text{ pu}$$

$$X_{T1_{\text{pri}}} = 0.3 \left( \frac{100}{40} \right) \left( \frac{20}{32} \right)^2 = j 0.293 \text{ pu}$$

$$X_{T2_{\text{pri}}} = 0.4 \left( \frac{100}{50} \right) \left( \frac{35}{40} \right)^2 = j 0.6125 \text{ pu}$$

$$Z_{B_{\text{line}}} = \frac{V_B^2}{S_B} = \frac{40^2}{100} = 16 \Omega$$

$$Z_{\text{Line pu}} = \frac{\text{actual}}{\text{Base}} = \frac{(6+j4)}{16} = 0.375 + j0.25 \text{ pu}$$

$$Z_{\text{Load } 3-1} = (3+j4) \Omega$$

$$Z_B = \frac{V_B^2}{S_B} = \frac{13.71^2}{100} = 1.879 \Omega$$

$$Z_{\text{Load } 3-1 \text{ pu}} = \frac{\text{actual}}{\text{Base}} = \frac{3+j4}{1.879} = 1.596 + j2.128 \text{ pu}$$

$$Z_{\text{Load } 3-2} = \frac{V_L^2}{S_L^*} = \frac{15^2}{(20+j15)^*} = \frac{15^2}{20-j15}$$

$$= 7.2 + j5.4 \Omega$$

$$Z_{\text{Load } 3-2 \text{ Base}} = \frac{V_B^2}{S_B} = \frac{13.71^2}{100} = 1.879 \Omega$$

$$Z_{\text{Load } 3-2 \text{ pu}} = \frac{\text{actual}}{\text{Base}} = \frac{7.2+j5.4}{1.879}$$

$$= 3.83 + j2.87 \text{ pu}$$

$$Z_{Load 3-3} = \frac{V_L^2}{\sum_{3\phi}^*} = \frac{15^2}{(50 \angle \cos^{-1} 0.5)^*}$$

$$= \frac{15^2}{(50 \angle 60^\circ)^*} = \frac{15^2}{50 \angle -60^\circ}$$

$$= 4.5 \angle 60^\circ \Omega$$

$$Z_{Load 3-3} = \frac{\text{actual}}{\text{Base}} = \frac{4.5 \angle 60^\circ}{1.879}$$

$$= 2.395 \angle 60^\circ$$

$$= 1.197 + j2.074 \text{ pu}$$

$$Z_{Load 3-4} = j0.4 \left( \frac{100}{25} \right) \left( \frac{15}{13.71} \right)^2$$

$$= j1.915 \text{ pu}$$

$$X_m = 0.3 \left( \frac{100}{15} \right) \left( \frac{15}{13.71} \right)^2 =$$

$$= \underline{\underline{j2.3941 \text{ pu}}}$$

$$I_G = I_{Line} = I_{L3-1} + I_{L3-2} + I_{L3-3} + I_{L3-4} + I_m$$

$$\frac{I_{L3-1}}{pu} = \frac{V_L}{Z_{L3-1}} \frac{pu}{pu}$$

$$\frac{V_L}{pu} = \frac{V_{L actual}}{V_L Base}$$

$$= \frac{15}{13.71} = 1.094 \angle 0^\circ$$

$$\frac{I_{L3-1}}{pu} = \frac{1.094}{1.596 + j2.128}$$

$$I_{L3-1} = 0.246 - j0.329 \quad (\text{lagging})$$

$$= 0.4113 \angle -53.13^\circ \text{ pu}$$

$$\frac{I_{L3-2}}{pu} = \frac{V_L}{Z_{L3-2}} = \frac{1.094 \angle 0^\circ}{7.2 + j5.4} =$$

$$= 0.0972 - j0.0729 \text{ pu}$$

$$\frac{I_{L3-3}}{pu} = \frac{V_L}{Z_{L3-3}} = \frac{1.094 \angle 0^\circ}{1.197 + j2.074}$$

$$= 0.2284 - j0.3957$$

$$\text{pu}$$

$$\begin{aligned}\bar{I}_{L3-4} &= \frac{VL}{Z_{3-4}} = \frac{1.094 \angle 0}{j1.915} = - \\ &= -j0.5713 \text{ pu}\end{aligned}$$

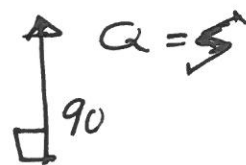
$$\bar{I}_m = \frac{\sum_m^*}{V_m^*} \quad \text{where} \quad S_m = V_m \bar{I}_m^*$$

$$\sum_m \text{ pu} = \frac{S_m \text{ actual}}{S_m \text{ Base}} = \frac{15}{100} = 0.15 \text{ pu}$$

$$\bar{I}_m = \frac{(0.15 \angle +90^\circ)^*}{1.094 \angle 0} = 0.1371 \angle -90^\circ$$

$$\text{where } X_m = j2.394$$

means that this motor have an inductive effect (lagging p.f.) pure inductive in this case.



$$\begin{aligned}I_G = I_{Lme} = & (0.246 - j0.329) + (0.0972 - j0.0729) \\ & - (0.2284 - j0.3957) + (-j0.5713) \\ & + (-j0.1371) =\end{aligned}$$

$$\underline{I}_G = \underline{I}_{Line} = 0.1148 - j0.7146 \text{ pu}$$

$$V_1 = V_3 + \underline{I}_{Line} \times \underbrace{(X_{T2} + Z_{Line} + X_{T1})}_Z$$

$$V_3 = 1.094 \angle 0$$

$$\underline{I}_{Line} = 0.1148 - j0.7146$$

$$Z = (j0.6125) + (0.375 + j0.25) + (j0.293)$$

$$Z = 0.375 + j1.155 \text{ pu}$$

$$V_1 = 1.094 + (0.1148 - j0.7146) \times (0.375 + j1.155)$$

$$= \underline{\underline{1.9628 - j0.135 \text{ pu}}} = 1.967 \angle -3.93^\circ \text{ pu}$$

$$= 1.967 \times 32 = \underline{\underline{62.95 \text{ kV}}}$$

$$Z_{L1-1} = (0.2 + j0.1) \times \left( \frac{100}{60} \right) \left( \frac{62.95}{32} \right)^2$$

$$= \underline{\underline{1.289 + j0.645 \text{ pu}}}$$

$$E_g = V_1 + I_G X_G$$

$$= (1.9628 - j0.135) + (0.1148 - j0.7146) \times j0.3662$$

$$E_g = 2.2245 - j0.093 \text{ p.u.}$$

$$2.2264 \angle -2.393^\circ \text{ p.u.}$$

$$E_{g \text{ actual}} = 2.264 \times 32$$

$$= 71.244 \text{ kV}$$

$$I_{G \text{ actual}} = (1.9628 - j0.135) \times I_B$$

$$I_B = \frac{S_B}{\sqrt{3} V_B} = \frac{100 \times 10^6}{\sqrt{3} \times 32 \times 10^3} \text{ A}$$

↑ at the generator ckt.

$$I_B = 1.80427 \times 10^3 \text{ A}$$

$$= 1804.27 \text{ A}$$

$$I_G = (1.9628 - j0.135) \times 1804.27$$

$$= 3541.4 - j243.57 \text{ A}$$

$$\bar{I}_{\text{Line actual}} = (1.9628 - j0.135) * \bar{I}_B$$

$$\bar{I}_{\text{actual}} = I_{\text{pu}} * \bar{I}_B$$

$$\bar{I}_B = \frac{S_B}{\sqrt{3} V_{B \text{ line}}} = \frac{100 * 10^6}{\sqrt{3} * 40 * 10^3}$$

$$= 1443.41 \text{ A}$$

$$\bar{I}_{\text{Line actual}} = 1443.41 * (1.9628 - j0.135)$$

$$= 2833.1 - j194.86 \text{ A}$$

$$\text{emf}_{\text{motor pu}} = E_m = V_m - I_m X_m$$

$$= 1.094 \angle 0 - (-j0.1371) * (j2.394)$$

$$= 0.7657 \text{ pu}$$

$$\text{emf}_m = 0.7657 * 13.71 \text{ kV}$$

$$\text{actual} = 10.498 \text{ kV}$$

