second Semester
chapter two

Gyroscopic Couple and Precessional Motion

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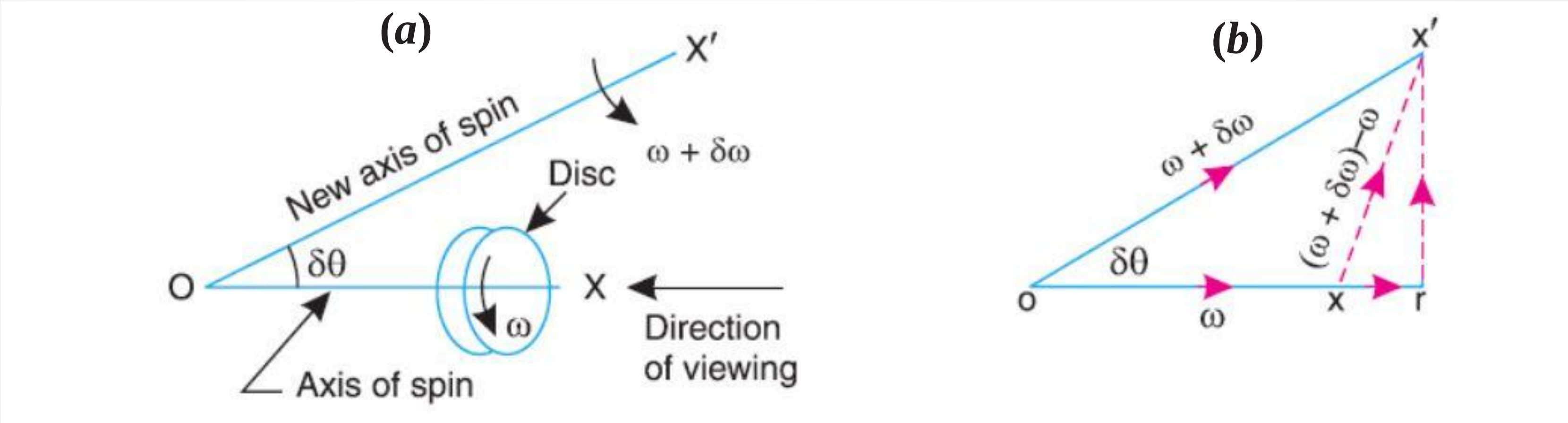
Precessional Angular Motion

We have already discussed that the angular acceleration is the rate of change of angular velocity with respect to time. It is a vector quantity and may be represented by drawing a vector diagram with the help of right hand screw rule

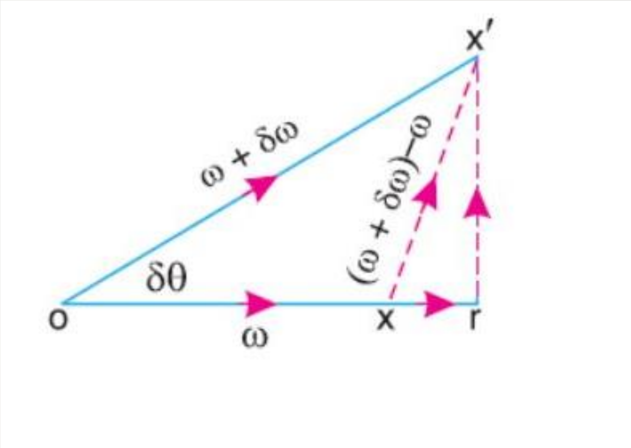


Consider a disc, as shown in **Figure (*a*)**, revolving or spinning about the axis *OX* (known as **axis of spin**) in anticlockwise when seen from the front, with an angular velocity **ω** in a plane at right angles to the paper.

After a short interval of time **δ*t***, let the disc be spinning about the new axis of spin ***OX*** (at an angle δθ) with an angular velocity (ω + δω). Using the right hand screw rule, initial angular velocity of the disc (ω) is represented by vector *ox*; and the final angular velocity of the disc (ω+δω) is represented by vector *ox* as shown in **Figure (*b*)**. The vector *xx* represents the change of angular velocity in time **δ*t*** *i.e.* the angular acceleration of the disc. This may be resolved into two components, one parallel to *ox* and the other perpendicular to *ox*.

Component of angular acceleration in the direction of *ox*,

$$\alpha_t = \frac{xr}{\delta t} = \frac{or - ox}{\delta t} = \frac{ox'\cos\delta\theta - ox}{\delta t} = \frac{(\omega+\delta\omega)\cos\delta\theta - \omega}{\delta t} = \frac{\omega\cos\delta\theta + \delta\omega\cos\delta\theta - \omega}{\delta t}$$



Since δθ is very small, therefore substituting cos δθ = 1, we have

$$\alpha_t = \frac{\omega + \delta\omega - \omega}{\delta t} = \frac{\delta\omega}{\delta t}$$

In the limit, when $\delta t \to 0$, $\alpha_t = Lt_{\delta t\to 0}\left(\frac{\delta\omega}{\delta t}\right) = \frac{d\omega}{dt}$

Component of angular acceleration in the direction perpendicular to *ox*,

$$\alpha_c = \frac{rx'}{\delta t} = \frac{ox'\sin\delta\theta}{\delta t} = \frac{(\omega+\delta\omega)\sin\delta\theta}{\delta t} = \frac{\omega\sin\delta\theta + \delta\omega\sin\delta\theta}{\delta t}$$

Since δθ in very small, therefore substituting sin δθ = δθ, we have

$$\alpha_c = \frac{\omega\delta\theta + \delta\omega\delta\theta}{\delta t} = \frac{\omega\delta\theta}{\delta t}$$

In the limit, when $\delta t \to 0$,

$$\alpha_c = Lt_{\delta t\to 0}\left(\frac{\omega\delta\theta}{\delta t}\right) = \omega\frac{d\theta}{dt} = \omega\,\omega_P$$

Gyroscopic Couple

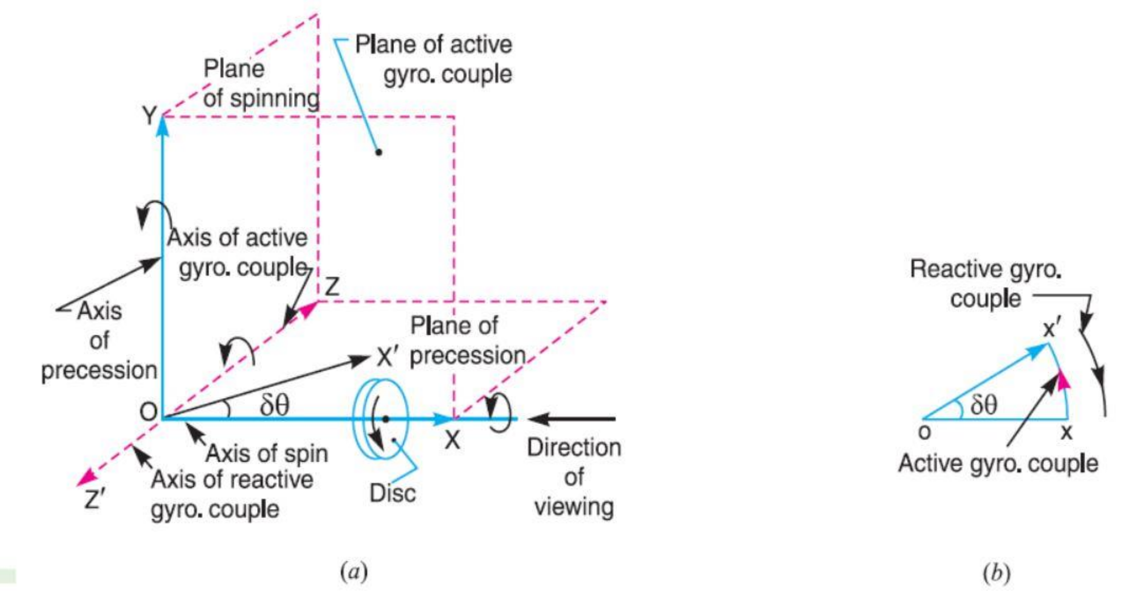
Consider a disc spinning with an angular velocity ω rad/s about the axis of spin *OX*, in anticlockwise direction when seen from the front, as shown in **Figure (*a*)**. Since the plane in which the disc is rotating is parallel to the plane *YOZ*, therefore it is called ***plane of spinning.*** The plane *XOZ* is a horizontal plane and the axis of spin rotates in a plane parallel to the horizontal plane about an axis *OY*. In other words, the axis of spin is said to be rotating or processing about an axis *OY*. In other words, the axis of spin is said to be rotating or processing about an axis *OY* (which is perpendicular to both the axes *OX* and *OZ*) at an angular velocity ω_P rap/s. This horizontal plane *XOZ* is called ***plane of precession*** and *OY* is the ***axis of precession.***

I = Mass moment of inertia of the disc about *OX*, and
ω = Angular velocity of the disc.

∴ Angular momentum of the disc = *I*.ω

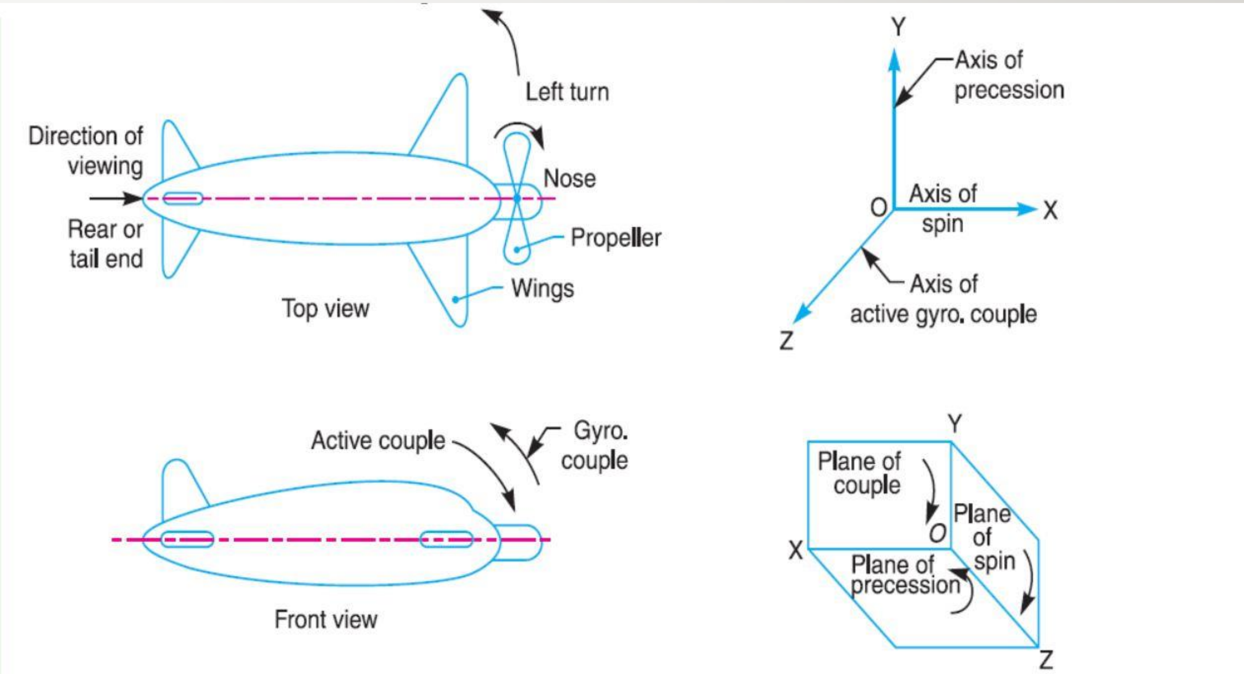
Since the rate of change of angular momentum will result by the application of a couple to the disc, therefore the couple applied to the disc causing precession,

$$C = I\,\omega\,\omega_P$$



Effect of the Gyroscopic Couple on an Aeroplane

The top and front view of an aeroplane are shown in Figure. Let engine or propeller rotates in the clockwise direction when seen from the rear or tail end and the aeroplane takes a turn to the left.



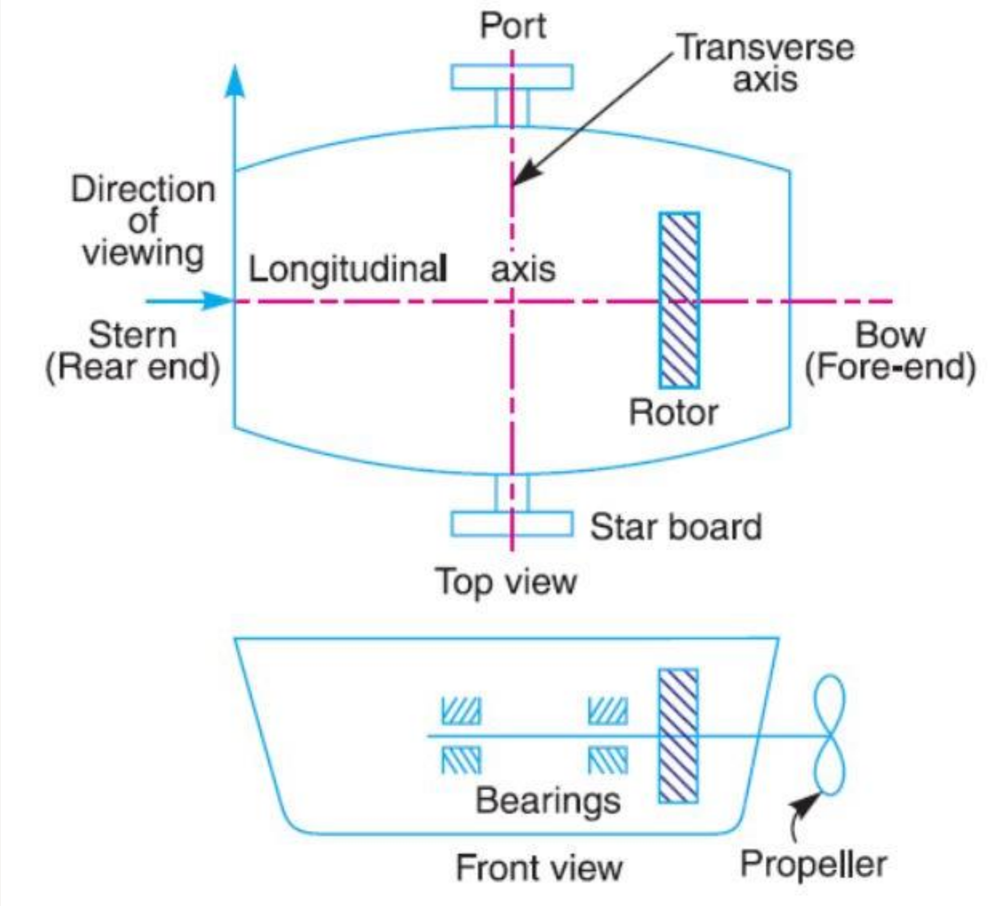
ω = Angular velocity of the engine in rad/s,
m = Mass of the engine and the propeller in kg,
k = Its radius of gyration in metres,
I = Mass moment of inertia of the engine and the propeller in **kg.m²** $= m.k^2$,
v = Linear velocity of the aeroplane in m/s,
R = Radius of curvature in metres, and
ω_P = Angular velocity of precession, $\omega_P = \dfrac{v}{R}$

$\therefore$ Gyroscopic couple acting on the aeroplane,

$$C = I.\omega.\omega_P$$

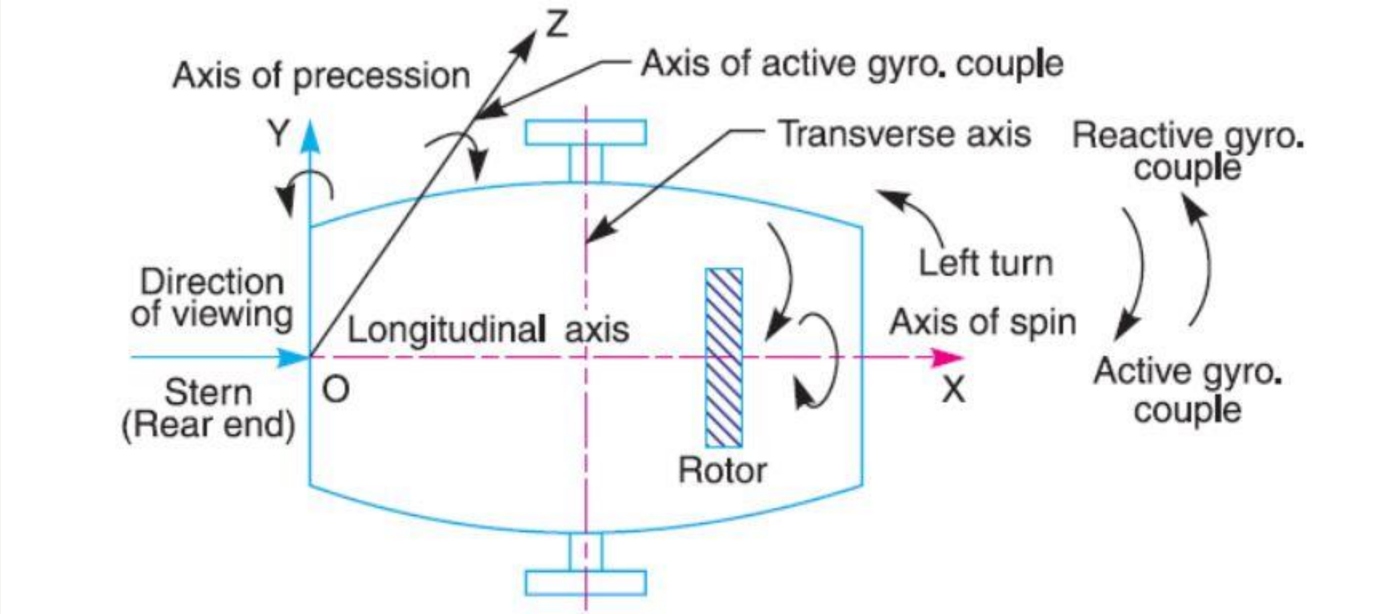
Terms Used in a Naval Ship

The top and front views of a naval ship are shown in Figure. The fore end of the ship is called ***bow*** and the rear end is known as ***stern*** or ***aft***. The left hand and right hand sides of the ship, when viewed from the stern are called ***port*** and ***star-board*** respectively. We shall now discuss the effect of gyroscopic couple on the naval ship in the following three cases: **1.** Steering, **2.** Pitching, and **3.** Rolling.



Effect of Gyroscopic Couple on a Naval Ship during Steering

Steering is the turning of a complete ship in a curve towards left or right, while it moves forward. Consider the ship taking a left turn, and rotor rotates in the clockwise direction when viewed from the stern, as shown in Figure. The effect of gyroscopic couple on a naval ship during steering taking left or right turn may be obtained in the similar way as for an aeroplane as discussed .



Effect of Gyroscopic Couple on a Naval Ship during Pitching

Pitching is the movement of a complete ship up and down in a vertical plane about transverse axis, as shown in Figure . In this case, the transverse axis is the axis of precession. The pitching of the ship is assumed to take place with simple harmonic motion *i.e.* the motion of the axis of spin about transverse axis is **simple harmonic.**

$\therefore$ Angular displacement of the axis of spin from mean position after time *t* seconds,

$$\theta = \phi \sin \omega_1 . t$$

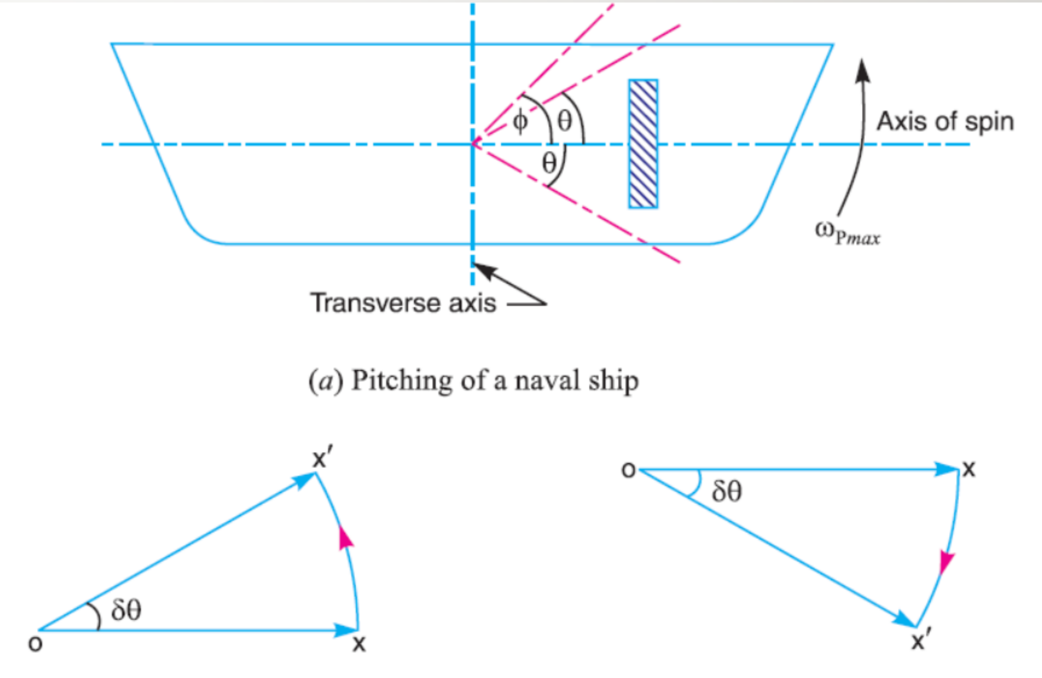
ϕ = Amplitude of swing *i.e.* maximum angle turned from the mean position in radians, and

ω_1 = Angular velocity of S.H.M.

$$\omega_1 = \frac{2\pi}{Time\ Period\ S.H.M\ in\ second} = \frac{2\pi}{t_P} \quad rad/sec$$

Angular velocity of precession,

$$\omega_P = \frac{d\theta}{dt} = \frac{d}{dt}(\phi \sin \omega_1 .t) = \phi\omega_1 \cos \omega_1 .t$$



(*a*) Pitching of a naval ship

(*b*) Pitching upward

(*c*) Pitching downward

The angular velocity of precession will be maximum, if cos $\omega_1.t = 1$.

∴ Maximum angular velocity of precession, $\omega_{Pmax} = \phi.\omega_1 = \phi \times 2\pi / t_p$

I = Moment of inertia of the rotor in kg.m^2 , and
ω = Angular velocity of the rotor in rad/s.

∴ Mamimum gyroscopic couple, $C_{max} = I.\ \omega.\ \omega_{Pmax}$

The angular acceleration during pitching,

$$\alpha = \frac{d^2\theta}{dt^2} = \phi\,\omega_1^2 \sin\omega_1.t$$

The angular acceleration is maximum, if sin $\omega_1 t = 1$.
∴ Maximum angular acceleration during pitching,

$$\alpha_{max} = (\omega_1)^2$$

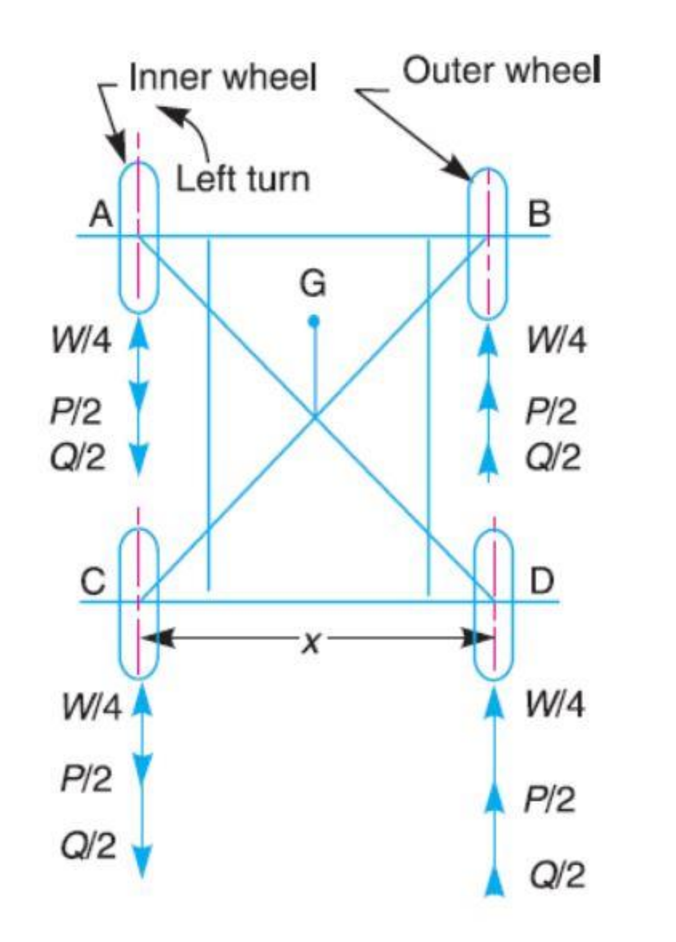
Effect of Gyroscopic Couple on a Naval Ship during Rolling

We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. If, however, the axis of precession becomes parallel to the axis of spin, there will be no effect of the gyroscopic couple acting on the body of the ship.

In case of rolling of a ship, the axis of precession (*i.e.* longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the gyroscopic couple acting on the body of a ship.

Stability of a Four Wheel Drive Moving in a Curved Path

Consider the four wheels ***A***, ***B***, ***C*** and ***D*** of an automobile locomotive taking a turn towards left as shown in Figure. The wheels *A* and *C* are inner wheels, whereas *B* and *D* are outer wheels. The centre of gravity (*C.G.*) of the vehicle lies vertically above the road surface



m = Mass of the vehicle in kg,

W = Weight of the vehicle in newtons = $m.g$,

r_W = Radius of the wheels in metres,

R = Radius of curvature in metres ($R > r_W$),

h = Distance of centre of gravity, vertically above the road surface in metres,

x = Width of track in metres,

I_W = Mass moment of inertia of one of the wheels in kg-m^2.

ω_W = Angular velocity of the wheels or velocity of spin in rad/s,

I_E = Mass moment of inertia of the rotating parts of the engine in kg-m$_2$,

ω_E = Angular velocity of the rotating parts of the engine in rad/s,

G = Gear ratio = ω_E / ω_W,

v = Linear velocity of the vehicle in m/s = $\omega_W . r_W$

A little considereation will show, that the weight of the vehicle (*W*) will be equally distributed over the four wheels which will act downwards. The reaction between each wheel and the road surface of the same magnitude will act upwards. Therefore

Road reaction over each wheel **= $W/4 = m.g/4$ newtons**

Let us now consider the effect of the gyroscopic couple and centrifugal couple on the vehicle.

1. *Effect of the gyroscopic couple*

Since the vehicle takes a turn towards left due to the precession and other rotating parts, therefore a gyroscopic couple will act. We know that velocity of precession,

$\omega_P = v/R$

$\therefore$ Gyroscopic couple due to 4 wheels,

$C_W = 4\, I_W.\omega_W.\omega_P$

and gyroscopic couple due to the rotating parts of the engine,

$C_E = I_E.\omega_E.\omega_P = I_E.G.\omega_W.\omega_P$

$\therefore$ Net gyroscopic couple,

$C = C_W \pm C_E = 4\, I_W.\omega_W.\omega_P \pm I_E.G.\omega_W.\omega_P = \omega_W.\omega_P\,(4\, I_W \pm G.I_E)$

The ***positive*** sign is used when the wheels and rotating parts of the engine rotate in the same direction. If the rotating parts of the engine revolves in opposite direction, then ***negative*** sign is used.

Due to the gyroscopic couple, vertical reaction on the road surface will be produced. The reaction will be vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer or inner wheels be P newtons. Then

$$P \times x = C \text{ or } P = C/x$$

∴ Vertical reaction at each of the outer or inner wheels, $P/2 = C/2x$

when rotating parts of the engine rotate in opposite directions, then –ve sign is used, *i.e.* net gyroscopic couple, $C = C_W - C_E$

When $C_E > C_W$, then C will be –ve. Thus the reaction will be vertically downwards on the outer wheels and vertically upwards on the inner wheels.

2. *Effect of the centrifugal couple*

Since the vehicle moves along a curved path, therefore centrifugal force will act outwardly at the centre of gravity of the vehicle. The effect of this centrifugal force is also to overturn the vehicle. We know that centrifugal force,

$$F_c = \frac{m \times V^2}{R}$$

∴ The couple tending to overturn the vehicle or overturning couple, $C_o = F_c \times h = \frac{m \times V^2}{R} \times h$