# Design and Analysis of Concrete Gravity Dams

BY

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## Instructional objectives

On completion of this lesson, the student shall learn:

***1. The different components of concrete gravity dams and their layouts***

***2. Design steps for of concrete gravity dam sections***

***3. The expected loadings for gravity dams***

***4. Stability analysis of gravity dam sections***

## 1- Introduction

The dam is a barrier constructed across the river to store water on its upstream.

Dams may be classified with several ways as follows:

**1- Classification based on materials of construction.**

*a- Earth fill dams*
*b- Rock fill dams*
*c- Concrete dams*
*d- Masonary dams*

**2- Classification based on the using of dams.**

*a- Storage dams*
*b- Diversion dam*
*c- Detention dam*

**3- Classification based on the type of construction.**

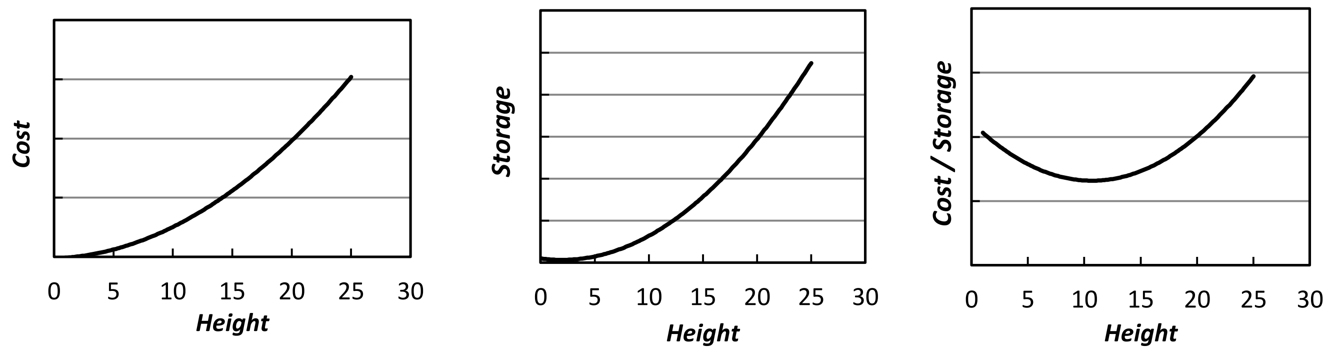
*a- Gravity dams*
*b- Buttress dams*
*c- Arch dams*

1-1 ***Factors that govern the selection of type of dam.***

*1- Materials of construction*
*2- Geological condition*
*3- Topography*

**1-2 Economic height of the dam**

It is the height of the dam, corresponding to which the cost per unit storage is minimum. Note the following figures:



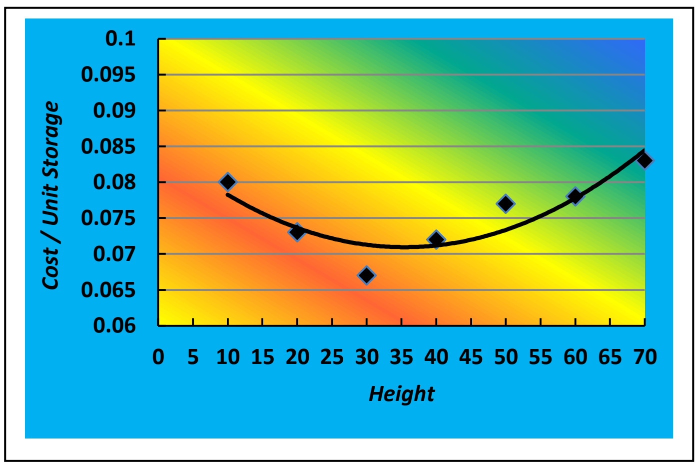

**_Example:_**

***The construction costs for certain possible heights of a dam have been estimated as follows:***

| ***Height ( m)*** | 10 | 20 | 30 | 40 | 50 | 60 | 70 |
|---|---|---|---|---|---|---|---|
| ***Cost (Millions US$ )*** | 4 | 8 | 12 | 18 | 27 | 39 | 50 |
| ***Storage (Millions $m^3$)*** | 50 | 110 | 180 | 250 | 350 | 500 | 600 |

***Solution: All the results are represented in the following Table and figure.***

| ***Height*** | ***Constructed Cost (Millions $)*** | ***Storage (Millions $m^3$)*** | ***Cost per Storage*** |
|---|---|---|---|
| *10* | *4* | *50* | ***0.08*** |
| *20* | *8* | *110* | ***0.073*** |
| *30* | *12* | *180* | ***0.067*** |
| *40* | *18* | *250* | ***0.072*** |
| *50* | *27* | *350* | ***0.077*** |
| *60* | *39* | *500* | ***0.078*** |
| *70* | *50* | *600* | ***0.083*** |




So, the optimum height may be considered as **(35 m ).**

***i.e, Choose h = 35 m***

## 2- Gravity dams

Dams constructed out of masonry or concrete and which rely solely on its self weight for stability fall under the nomenclature of gravity dams. Masonary dams have been in use in the past quite often but after independence, the last major masonry dam structure that was built was the N***agarjunsagar Dam*** on river ***Krishna*** which was built during 1958-69.

## 3- Concrete gravity dam and apparent structures-basic layout

The basic shape of a concrete gravity dam is triangular in section (Figure 1a), with the top crest often widened to provide a roadway (Figure 1b).

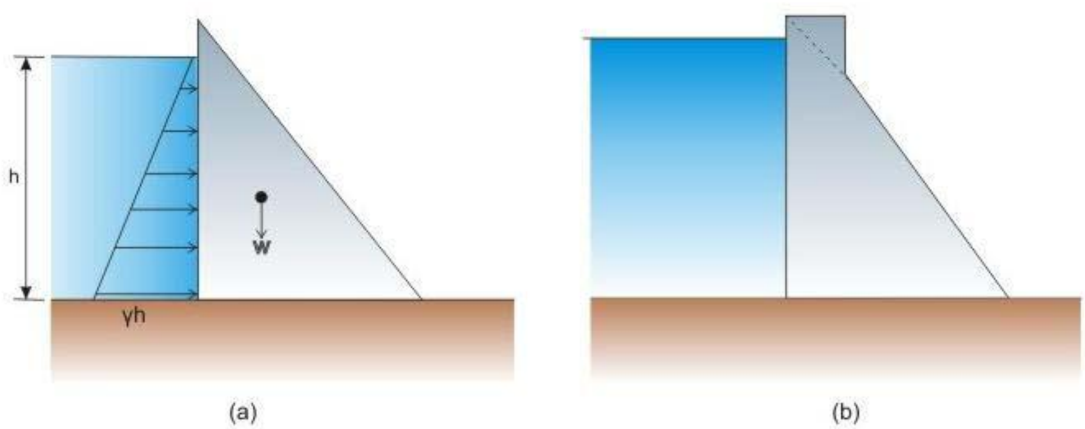



FIGURE 1 : Concrete gravity dam section (a) Basic triangular shape (b) Modified shape

## 4- Design of concrete gravity Dam sections

Fundamentally a gravity dam should satisfy the following criteria:

1. It shall be safe against overturning at any horizontal position within the dam at the contact with the foundation or within the foundation.

2. It should be safe against sliding at any horizontal plane within the dam, at the contact with the foundation or along any geological feature within the foundation.

3. The section should be so proportional that the allowable stresses in both the concrete and the foundation should not exceed.

Safety of the dam structure is to be checked against possible loadings, which may be classified as primary, secondary or exceptional. The classification is made in terms of the applicability and/or for the relative importance of the load.

1. Primary loads are identified as universally applicable and of prime importance of the load.

2. Secondary loads are generally discretionary and of lesser magnitude like sediment load or thermal stresses due to mass concreting.

3. Exceptional loads are designed on the basis of limited general applicability or having low probability of occurrence like inertial loads associated with seismic activity.

Technically a concrete gravity dam derives its stability from the force of gravity of the materials in the section and hence the name. The gravity dam has sufficient weight so as to withstand the forces and the overturning moment caused by the water impounded in the reservoir behind it. It transfers the loads to the foundations by cantilever action and hence good foundations are pre requisite for the gravity dam.

***The forces that give stability to the dam include:***

***1. Weight of the dam***
***2. Thrust of the tail water***

***The forces that try to destabilize the dam include:***

***1. Reservoir water pressure***

***2. Uplift***

***3. Forces due to waves in the reservoir***

***4. Ice pressure***

***5. Temperature stresses***

***6. Silt pressure***

***7. Seismic forces***

***8. Wind pressure***

The forces to be resisted by a gravity dam fall into two categories as given below:

1. Forces, such as weight of the dam and water pressure which are directly calculated from the unit weight of materials and properties of fluid pressure and

2. Forces such as uplift, earthquake loads, silt pressure and ice pressure which are assumed only on the basis of assumptions of varying degree of reliability. In fact to evaluate this category of forces, special care has to be taken and reliance placed on available data, experience and judgment. Figure 23 shows the position and direction of the various forces expected in a concrete gravity dam. Forces like temperature stresses and wind pressure have not been shown. Ice pressures being uncommon in Indian context have been omitted.

***For consideration of stability of a concrete dam, the following assumptions are made:***

1. That the dam is composed of individual transverse vertical elements each of which carries its load to the foundation without transfer of load from or to adjacent elements. However for convenience, the stability analysis is commonly carried out for the whole block.

2. That the vertical stress varies linearly from upstream face to the downstream face on any horizontal section.

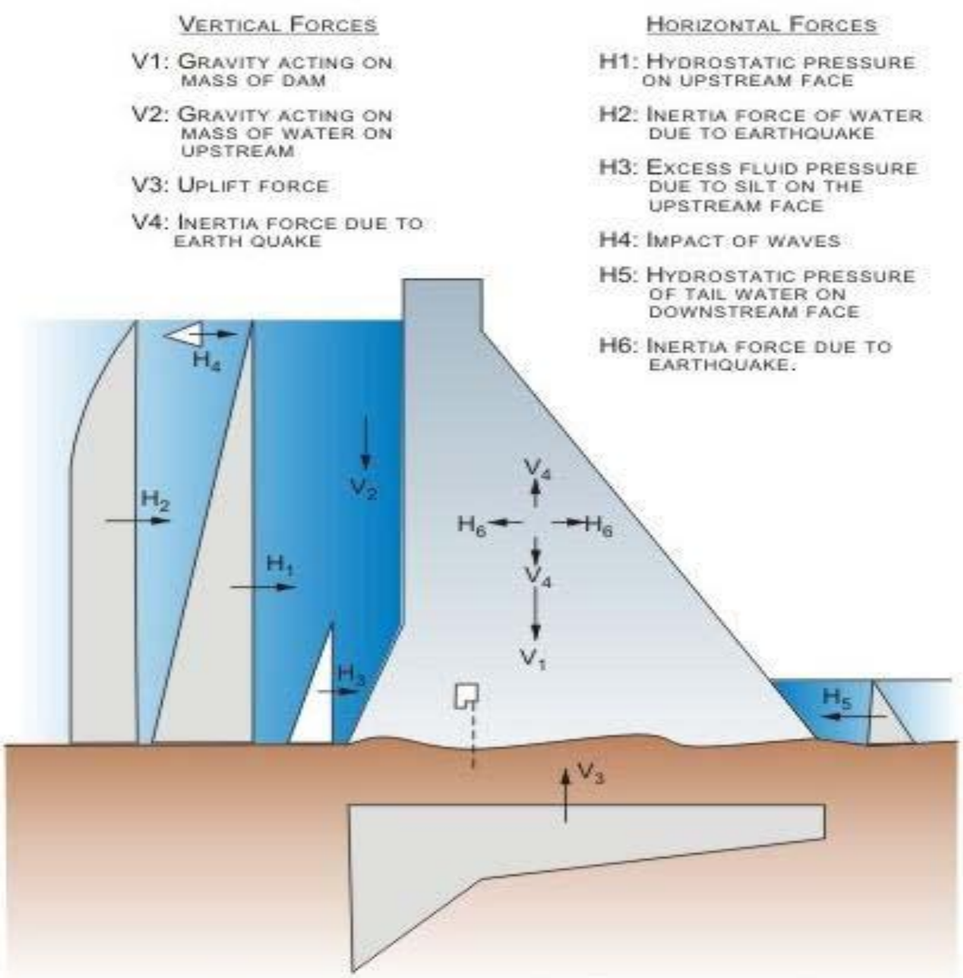



FIGURE 23: Different forces acting on a concrete gravity dam

The Bureau of Indian Standards code IS 6512-1984 "Criteria for design of solid gravity dams" recommends that a gravity dam should be designed for the most adverse load condition of the seven given type using the safety factors prescribed.

Depending upon the scope and details of the various project components, site conditions and construction programmer one or more of the following loading conditions may be applicable and may need suitable modifications. The seven types of load combinations are as follows:

***1. Load combination A (construction condition): Dam completed but no water in reservoir or tailwater.***

***2. Load combination B (normal operating conditions): Full reservoir elevation, normal dry weather tail water, normal uplift, ice and silt (if applicable).***

***3. Load combination C: (Flood discharge condition) - Reservoir at maximum flood pool elevation , all gates open, tailwater at flood elevation, normal uplift, and silt (if applicable).***

***4. Load combination D: Combination of A and earthquake.***

***5. Load combination E: Combination B, with earthquake but no ice.***

***6. Load combination F: Combination C, but with extreme uplift, assuming the drainage holes to be inoperative.***

***7. Load combination G: Combination E but with extreme uplift (drains inoperative).***

It would be useful to explain in a bit more detail the different loadings and the methods required to calculate them. These are explained in the following sections.

## 5- Loadings for concrete Gravity Dams

The significant loadings on a concrete gravity dam include the self-weight or dead load of the dam, the water pressure from the reservoir, and the uplift pressure from the foundation. There are other loadings, which either occur intermittently, like earthquake forces, or are smaller in magnitude, like the pressure exerted by the waves generated in the reservoir that hit the upstream of the dam face. These loadings are explained in the following section.

### 5.1 *Dead load*

The dead load comprises of the weight of the concrete structure of the dam body in addition to pier gates and bridges, if any over the piers. The density of concrete may be considered as ***2400 kg/m³.*** Since the cross section of a dam usually would not be simple, the analysis may be carried out by dividing the section into several triangles and rectangles and the dead load (self weight) of each of these sections (considering unit width or the block width) computed separately and then added up.

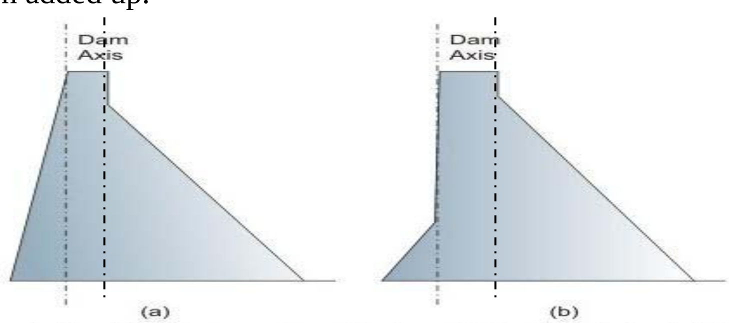



FIGURE 8. Upstream inclined face for concrete gravity dams. (a) Full face inclined; (b) Partly inclined

## 5.2 *Water pressure on dam*

The pressure due to water in the reservoir and that of the tailwater acting on vertical planes on the upstream and downstream side of the dam respectively may be calculated by the law of hydrostatics.

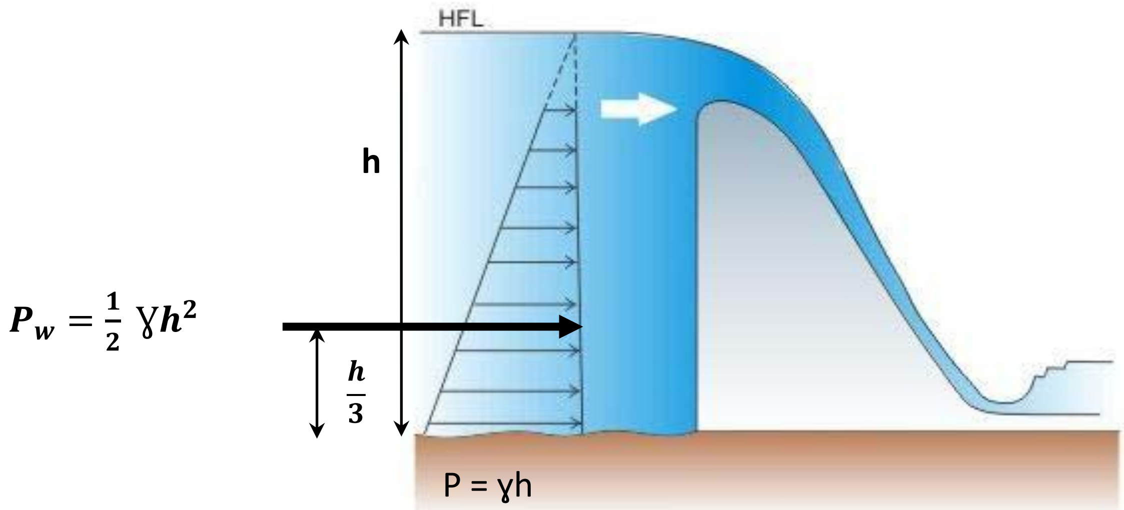



FIGURE 24. Horizontal water force on spillway block during flood water overflow

## 5.3 *Uplift pressures*

Uplift forces occur as internal pressure in pores, cracks and seams within the body of the dam, at the contact between the dam and its foundation and within the foundation.

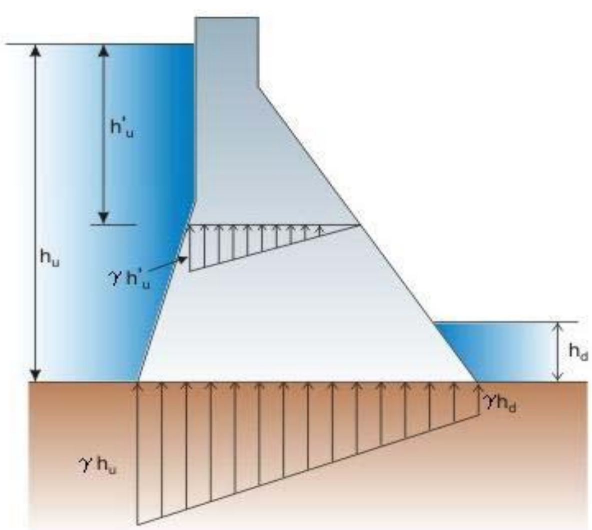



FIGURE 25. Uplift pressure at base and at any general plane in the dam body. Drainage holes are not considered

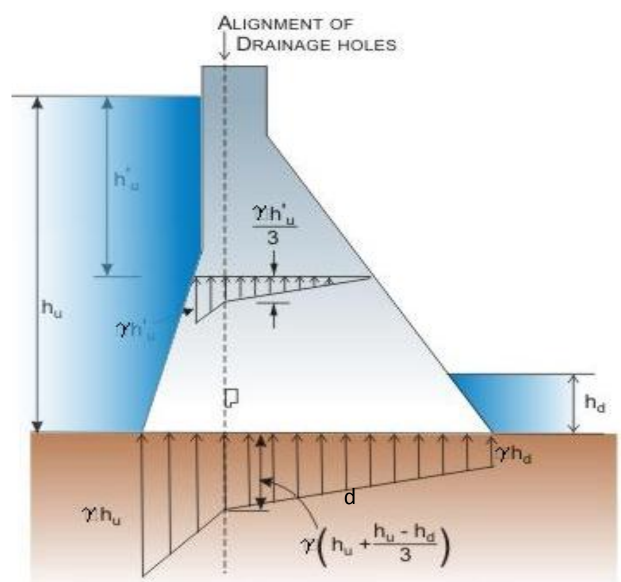



FIGURE 26: Assumed uplift pressure considering presence of drainage holes

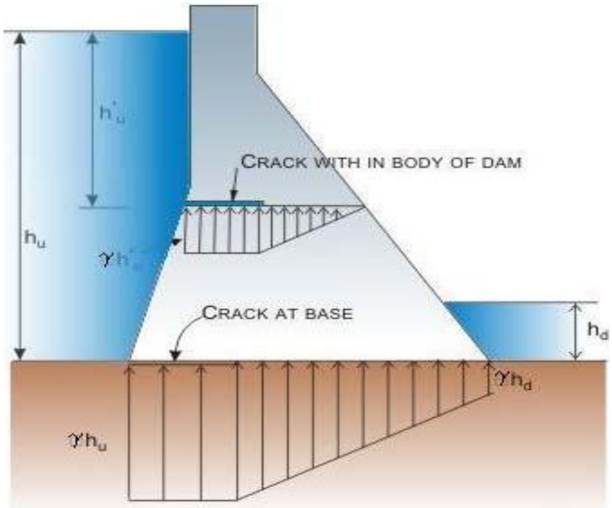
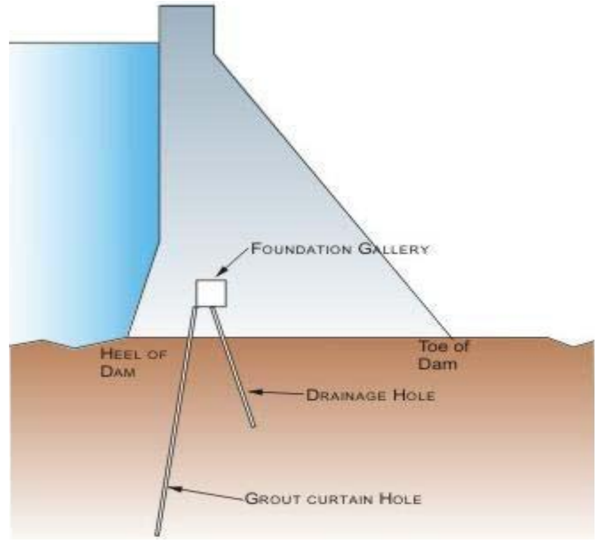



Figure 27: Uplift pressure diagrams considering horizontal cracks at any general plane/at the base.




## 5.4 *Silt pressure*

The weight and the pressure of the submerged silt are to be considered in addition to weight and pressure of water. The weight of the silt acts vertically on the slope and pressure horizontally, in a similar fashion to the corresponding forces due to water. So:

$$F_{silt} = \frac{1}{2}\,\gamma_s\,hs^2\,\frac{1 - sin\emptyset}{1 + sin\emptyset}$$

Where :

$\gamma_s$ = Submerged unit weight of silt

$h_s$ = height of silt deposited

$\emptyset$ = angle of internal friction

It is recommended that the submerged density of silt for calculating horizontal pressure may be taken as 1360 kg/m³. Equivalently, for calculating vertical force, the same may be taken as 1925 kg/m³.

Also, according to ***USBR*** recommendation which is mostly followed in the design the force due to silt could be estimated as:

$$F_{silt-horizantal} = \frac{1}{2}360\,h^2 = 180\;\,h^2\;\,(in\,Kg)\;\;at\;\;\frac{h}{3}$$

$$F_{silt-down} = \frac{1}{2}920\,h^2 = 460\,h^2\;\,(in\,Kg)$$

## *5.5 Wave pressure*

The reservoir behind a dam is prone to generation of waves produced by the shearing action of wind blowing over the surface. Of course, the pressure of the waves against massive dams of appreciable height is not of much consequence. The height of wave is generally more important in determination of the free board requirements of dams to prevent overtopping of the dam crest by wave splash. The force and dimensions of waves depend mainly on the extent and dimensions of waves depend mainly on the extent and configuration of the surface area of the reservoir, the depth of the reservoir, and the velocity of the wind. The procedure to work out the height of waves generated, and consequently derive the safe free board, may be done according to the method described in ***IS: 6512-1984 "Criteria for design of solid gravity dams".*** However, since it is a bit involved, a simpler method is prescribed as that given by the ***Stevenson formula (Davis and Sorenson 1969).***

$$h_w = 0.34\sqrt{F} + 0.76 - 0.26\sqrt[4]{F}$$

***Where :***

$h_w$ = Height of wave, crest to trough, in m

F = Fetch of the reservoir, that is, the longest straight distance of the reservoir from the dam up to the farthest point of the reservoir.

***When the fetch exceeds 20Km, the above formula can be approximated as***

$$h_w = 0.34\sqrt{F}$$

Since the height of the generated waves must be related to the wind velocity, the original formula has been modified to:

$$h_w = 0.032\sqrt{V * F} + 0.76 - 0.26\sqrt[4]{F}$$

Where V = wind speed along the fetch, in km/h

Stevenson's approximate formula is applicable for wind speeds of about 100km/hour, which is a reasonable figure for many locations. It is conservative for low wind speeds but under estimates waves for high wind speeds.

The pressure intensity due to waves ($\mathbf{P}_{wave}$, in $KN/m^2$) is given by the following expression for $\gamma w = 9.81\ Kn/m^3$ as:

$$P_{wave} = 2.4\,\gamma\,h_w = 23.544\,h_w \quad (\textit{pressure in } KN/m^2)$$

Where $h_w$ is the height of wave in m. and occurs at (1/8) $h_w$ above the still water level (Figure36). The total wave pressure $P_w$ per unit length (in KN/m) of the dam is given by the area of the triangle 1-2-3 as shown in Figure 36, and is given as:

$$P_w = \frac{1}{2}\, x\, P_w\, x\, \frac{5}{3}\, h_w$$

$$P_w = \frac{1}{2}\, x\, (2.4\,\gamma\, h_w) x\, \frac{5}{3}\, h_w$$

$$P_w = 2\,\gamma h_w^2 = 20\, h_w^2 \quad \textbf{(Force in Kn)}$$

Where the centre of application is at a height of (***3/8 $h_w$= 0.375 $h_w$*** ) above the still water level.

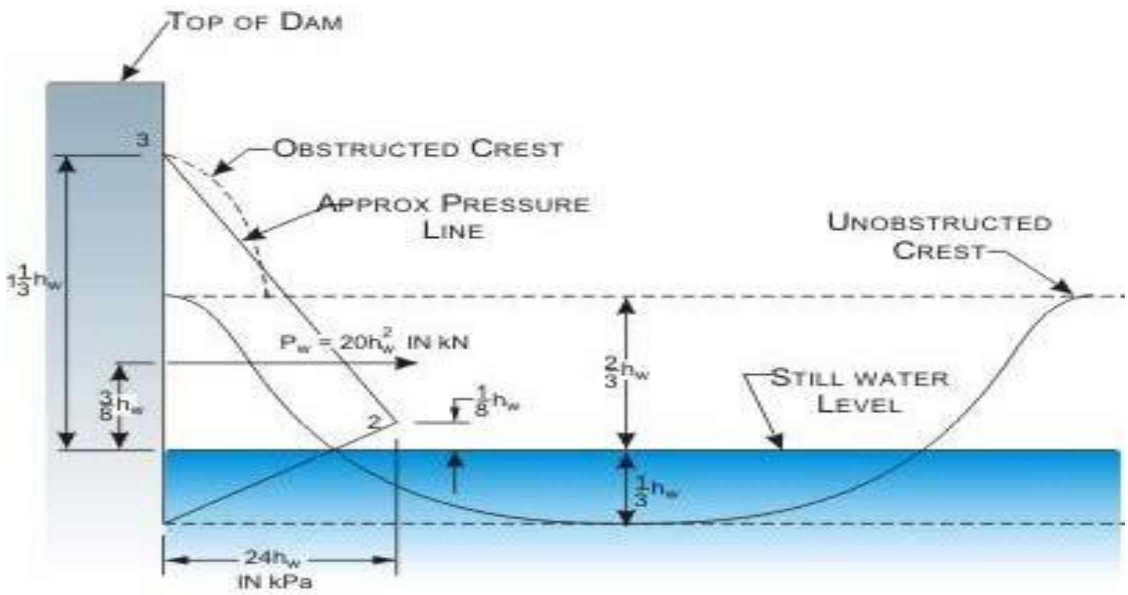



FIGURE. 36: Wave height, pressure and center of action

## 5.6 Free board

Free board is the vertical distance between the top of the dam and the sill water level. ***IS:6512-1984*** recommends that the free board shall be wind set-up plus 4/3 times wave height above normal pool elevation or above maximum reservoir level corresponding to design flood, whichever gives higher crest elevation. Wind set-up is the shear displacement of water towards one end of a reservoir by wind blowing continuously from one direction. The ***Zuider Zee formula*** (Thomas, 1976) and recommended by IS: 6512-1984 may be used as a guide for the estimation of set-up(S):

$$S = \frac{V^2\ F\ \cos A}{k\ D}$$

Where

S = Wind set-up, in m

V = Velocity of wind over water in m/s

F = Fetch, in km

D = Average depth of reservoir, in m, along maximum fetch

A = Angle of wind to fetch, may be taken as zero degrees for maximum set-up

K = A constant, specified as about 62000

The free-board shall not be less than 1.0 m above Maximum Water Level (MWL) corresponding to the design flood. If design flood is not same as Probable Maximum Flood (PMF), then the top of the dam shall not be lower than MWL corresponding to PMF.

## 5.7 *Earthquake (seismic) forces*

Earthquake or seismic activity is associated with complex oscillating patterns of acceleration and ground motions, which generate transient dynamic loads due to inertia of the dam and the retained body of water. Horizontal and vertical accelerations are not equal, the former being of greater intensity.

The earthquake acceleration is usually designated as a fraction of the acceleration due to gravity and is expressed as $\alpha \cdot g$, where $\alpha$ is the ***Seismic Coefficient***. The seismic coefficient depends on various factors, like the intensity of the earthquake, the part or zone of the country in which the structure is located, the elasticity of the material of the dam and its foundation, etc.

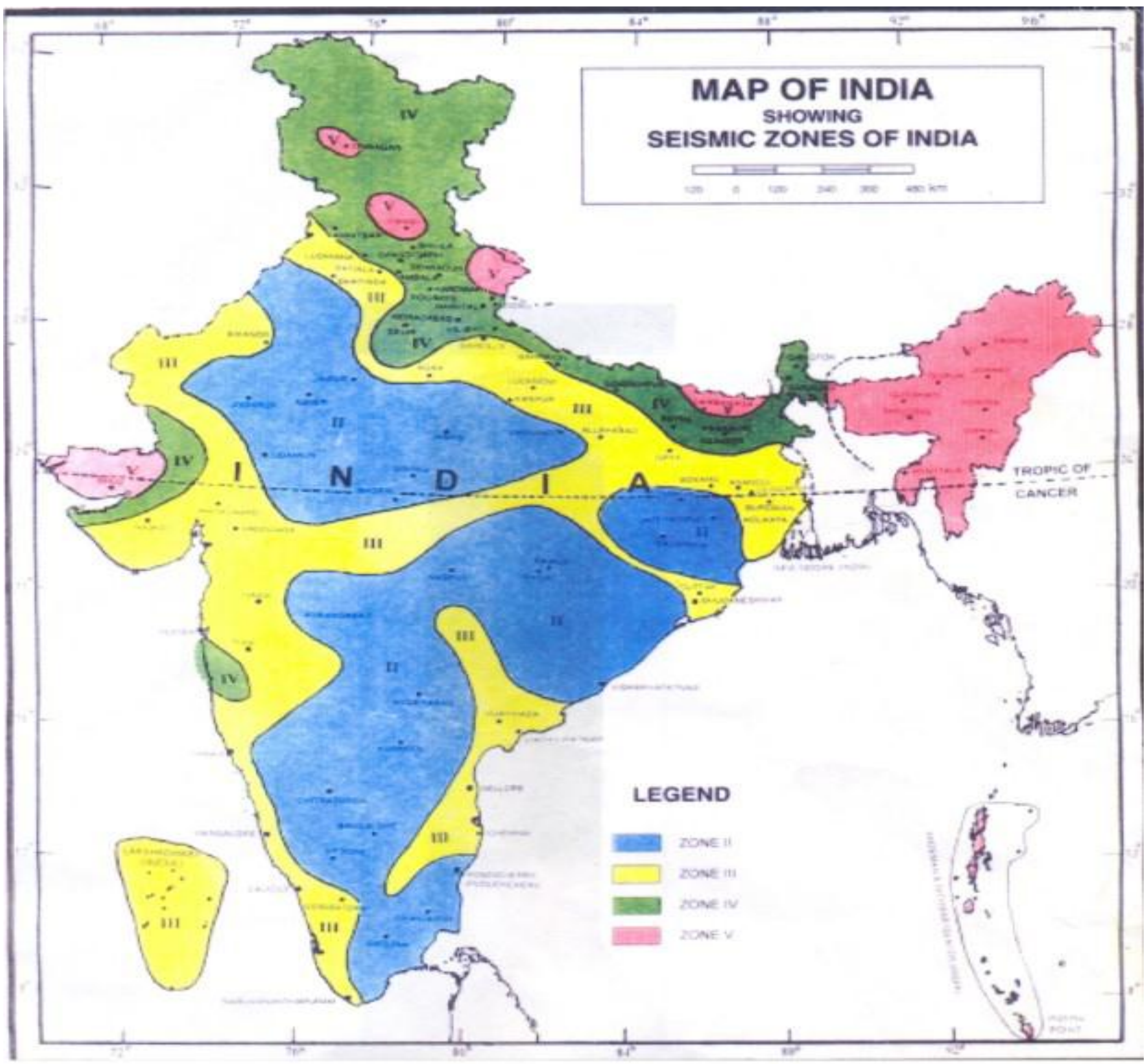



FIGURE 28. Seismic zones of India as per IS : 1893 - 2002 (Part 1)

As mentioned earlier, the earthquake forces cause both the dam structure as well as the water stored in the reservoir to vibrate. The force generated in the dam is called the Inertia Force and that in the water body, Hydrodynamic Force. Since the earthquake forces are generated due to the vibration of the earth itself, which may be shaking horizontally in the two directions as well as vibrating vertically. For design purpose, one has to consider the worst possible scenario, and hence the combination that is seen to be the least favorable to the stability of the dam has to be considered.

When the dam has been newly constructed, and the reservoir has not yet been filled, then the worst combination of vertical and horizontal inertia forces would have to be taken that causes the dam to topple backward as shown in Figure 30. The notations used in the figure are as follows:

$H_u$ : Horizontal earthquake force acting in the upstream direction

$H_D$: Horizontal earthquake force acting in the downstream direction

$V_u$: Vertical earthquake force acting upwards

$V_D$: Vertical earthquake force acting downwards

Under the reservoir full condition, the worst combination of the inertia forces is the one which tries to topple the dam forward, as shown in Figure 31.

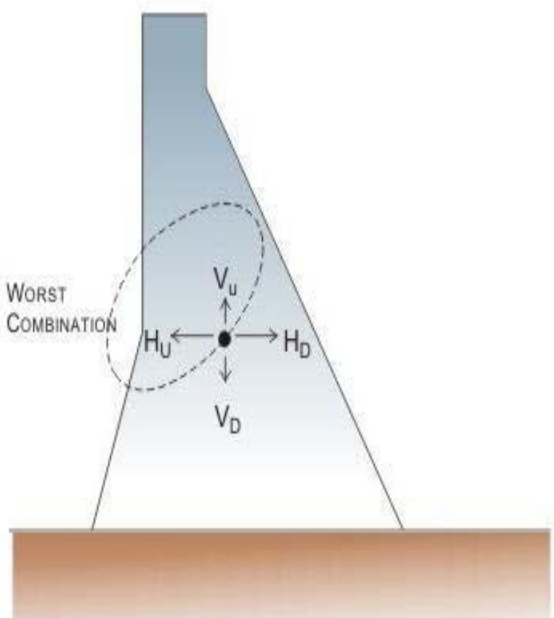


FIGURE 30. Worst combination of earthquake forces under reservoir empty condition

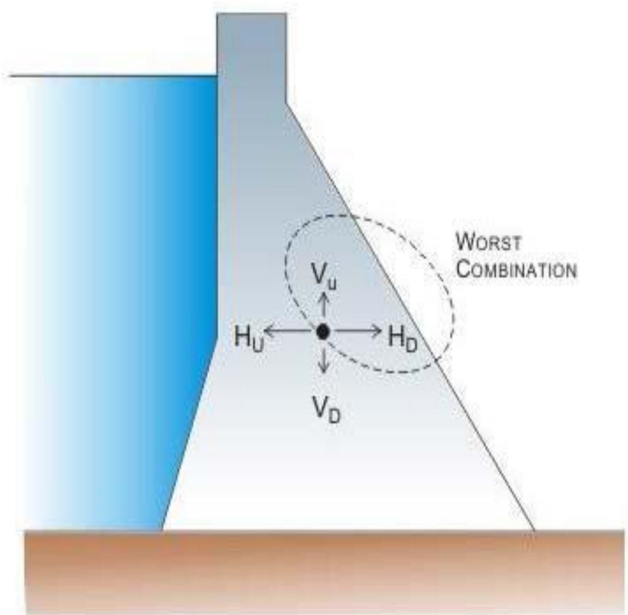


FIGURE 31. Worst combination of earthquake forces under reservoir full condition

### *5.7.1 Effect Earthquake pressure*

### *a- Effect in vertical direction*

Inertia force = Mass X acceleration due to earthquake

$$\boldsymbol{P_{ev} = M \times \alpha_v = \frac{W}{g} \times \alpha_v = W \times Kv}$$

Where $Kv = \frac{\alpha_v}{g} = \boldsymbol{\alpha}_\mathbf{v}$ = Coefficient of acceleration of the earthquake

$$\alpha_v = \boldsymbol{\alpha}_\mathbf{v}\,\mathbf{g}$$

***Note that:***

1. ***the position of $P_{ev}$ is at the Center of the mass M.***
2. ***the direction of $P_{ev}$ is downward if the earthquake direction is upward, while it is upward if the earthquake direction is downward.***

***So:*** ***<u>For upward direction</u>***

$$\boldsymbol{Net\ effective\ weight\ of\ W_i = W_i + W_i} \times Kv = \boldsymbol{W_i\,(1 +} Kv) = \boldsymbol{V_i\,\gamma\,(1 +} Kv) = \boldsymbol{V_i\,\gamma_{new}}$$

***<u>For downward direction</u>***

$$\boldsymbol{Net\ effective\ weight\ of\ W_i = W_i - W_i} \times Kv = \boldsymbol{W_i\,(1 -} Kv) = \boldsymbol{V_i\,\gamma\,(1 -} Kv) = \boldsymbol{V_i\,\gamma_{new}}$$

***b- Effect in horizontal direction***

1. Inertia force in body of the dam in the horizontal direction
2. Hydrodynamic pressure of the water

So:

**1- For the inertia force $P_{eh}$**

Inertia force = Mass X acceleration due to earthquake

$$\boldsymbol{P_{eh} = M \times \alpha_h = \frac{W}{g} \times \alpha_h = W \times Kh}$$

Where $Kh = \frac{\alpha_h}{g} = \boldsymbol{\alpha}_\mathbf{h}$ = Coefficient of acceleration of the earthquake

$$\alpha_h = \boldsymbol{\alpha}_\mathbf{h}\,\mathbf{g}$$

**2- Hydrodynamic pressure of the water**

$$P_e = C.k_h.\gamma.h$$

$$C = \frac{C_m}{2}\left[\frac{Z}{h}\left(2-\frac{Z}{h}\right)+\sqrt{\frac{Z}{h}\left(2-\frac{Z}{h}\right)}\right]$$

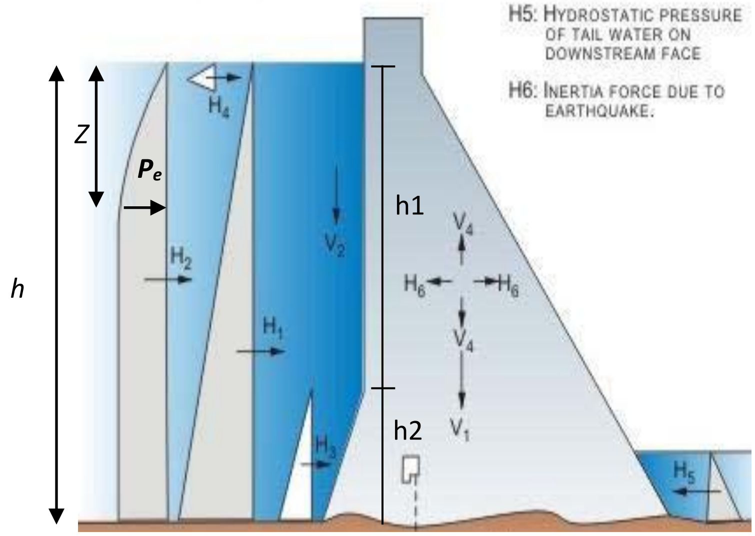
$$C_m = 0.73\ \frac{\beta}{90}$$

$P_e$ = Pressure intensity

$h$ = Maximum depth of the reservoir

z = Depth from the top of the reservoir

So:




$$Force = H_2 = 0.726\,P_e\,Z \quad at\ \frac{4\,h}{3\pi}$$

$$Moment = m_e = 0.3\,P_e\,Z^2 = 0.4132\,Z\,H_2$$

***Note that if h1 > (h1 + h2 = h)/2 , use β=90, else use β from the heel point to the point of water surface at the upstream face.***

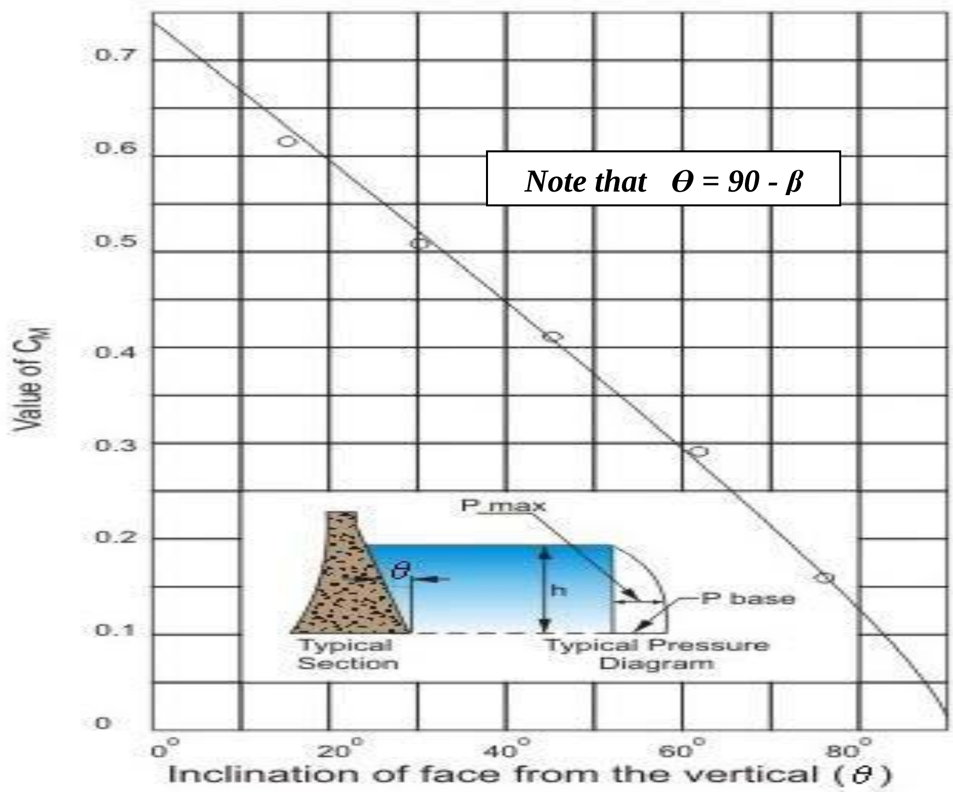



FIGURE.35: Maximum values of pressure coefficient ($C_M$) for dams with constant sloping faces

# 6. Stability analysis of gravity dams

The stability analysis of gravity dams may be carried out by various methods, where the gravity method is described here. In this method, the dam is considered to be made up of a number of vertical cantilevers which act independently for each other. The resultant of all horizontal and vertical forces including uplift should be balanced by an equal and opposite reaction at the foundation consisting of the total vertical reaction and the total horizontal shear and friction at the base and the resisting shear and friction of the passive wedge, if any. For the dam to be in static equilibrium, the location of this force is such that the summation of moments is equal to zero. The distribution of the vertical reaction is assumed as trapezoidal for convenience only. Otherwise, the problem of determining the actual stress distribution at the base of a dam is complicated by the horizontal reaction, internal stress relations, and other theoretical considerations. Moreover, variations of foundation materials with depth, cracks and fissures which affect the resistance of the foundation also make the problem more complex. The internal stresses and foundation pressures should be computed both with and without uplift to determine the worst condition. ***The stability analysis of a dam section is carried out to check the safety with regard to:***

1. *Rotation and overturning*
2. *Translation and sliding*
3. *Overstress and material failure*

## *6.1 Stability against overturning*

Before a gravity dam can overturn physically, there may be other types of failures, such as cracking of the upstream material due to tension, increase in uplift, crushing of the toe material and sliding. ***However, the check against overturning is made to be sure that the total stabilizing moments weigh out the de-stabilizing moments***. The factor of safety against overturning (F.S.O), may be taken as 1.5 to 2.5. As such, a gravity dam is considered safe also from the point of view of overturning if there is no tension on the upstream face.

$$F.S.O = \frac{Sum\ of\ Resisting\ moments}{Sum\ of\ Overturning\ moments} = \frac{\sum Mr}{\sum MO} \quad > 1.5\ to\ 2.5$$

## *6.2 Stability against sliding*

Many of the loads on the dam act horizontally, like water pressure, horizontal earthquake forces, etc. These forces have to be resisted by frictional or shearing forces along horizontal or nearly-horizontal seams in foundation. ***The stability of a dam against sliding is evaluated by comparing the minimum total available resistance along the critical path of sliding (that is, along that plane or combination of plans which mobilizes the least resistance to sliding) to the total magnitude of the forces tending to induce sliding.*** The factor of safety against sliding (F.S.S), may be taken as 4 to 5

$$F.S.S = \frac{Sum\ of\ Resisting\ or\ shear\ forces}{Sum\ of\ Moving\ Forces} = \frac{\mu \sum V + B\ .q}{\sum H} \quad > 4\ to\ 5$$

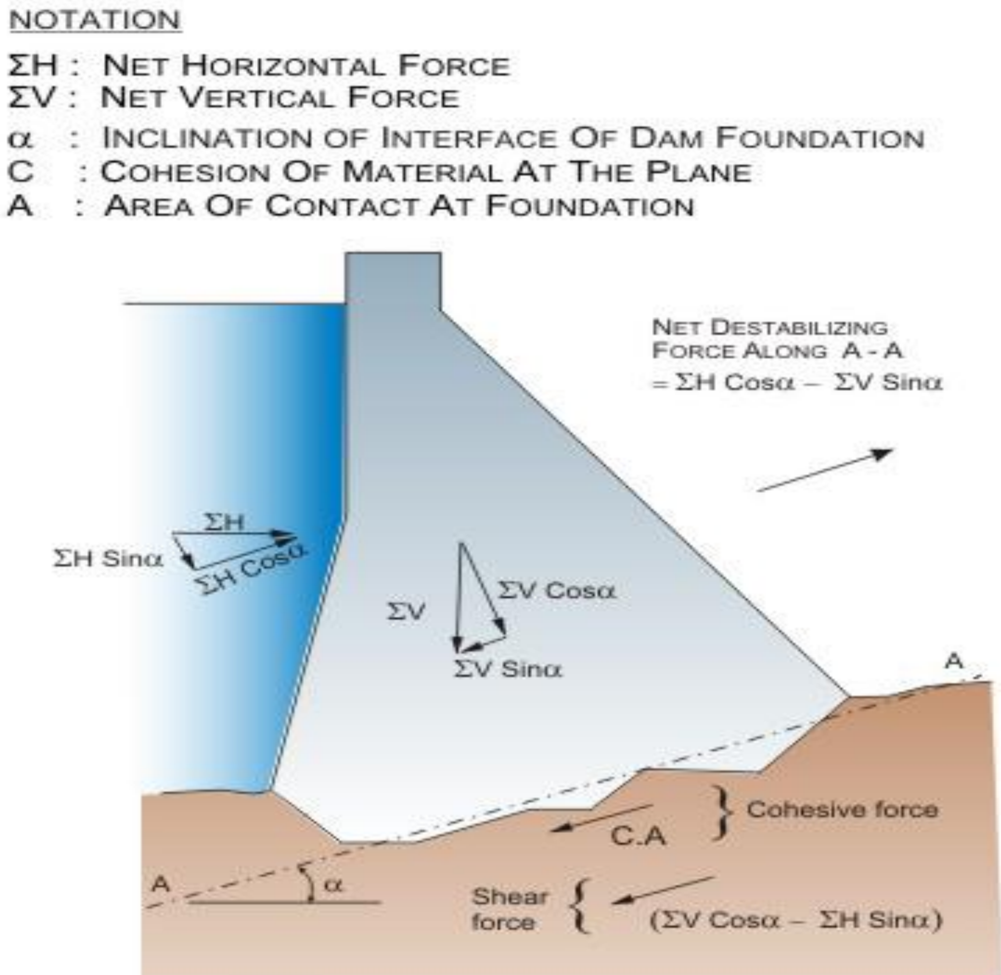



FIGURE. 37: Stability against sliding along concrete dam - rock base interface. Good rock is assumed to exist below

### *6.3 Failure against overstressing*

A dam may fail if any of its part is overstressed and hence the stresses in any part of the dam must not exceed the allowable working stress of concrete. In order to ensure the safety of a concrete gravity dam against this sort of failure, the strength of concrete shall be such that it is more than the stresses anticipated in the structure by a safe margin. The maximum compressive stresses occur at heel (mostly during reservoir empty condition) or at toe (at reservoir full condition) and on planes normal to the face of the dam. The strength of concrete and masonary varies with age, the kind of cement and other ingredients and their proportions in the work can be determined only by experiment.

The calculation of the stresses in the body of a gravity dam follows from the basics of elastic theory, which is applied in a two-dimensional vertical plane, and assuming the block of the dam to be a cantilever in the vertical plane attached to the foundation. Although in such an analysis, it is assumed that the vertical stresses on horizontal planes vary uniformly and horizontal shear stresses vary parabolically, they are not strictly correct. Stress concentrations develop near heel and toe, and modest tensile stresses may develop at heel. The basic stresses that are required to be determined in a gravity dam analysis are discussed below:

### *6.4 Normal stresses on horizontal planes*

On any horizontal plane, the vertical normal stress ($\sigma_z$) may be determined as:

$$\sigma_Z = \frac{\sum V}{T} + \frac{12 \sum V \times e}{T^3} y$$

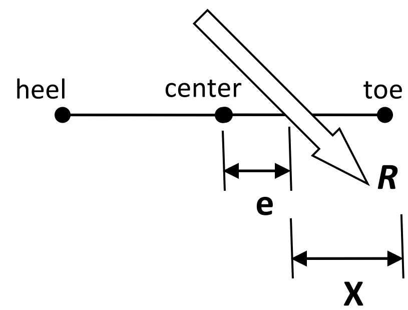
***Where***

$\Sigma V$ = Resultant vertical load above the plane considered
T = Thickness of the dam block, that is, the length measured from heel to toe
e = Eccentricity of the resultant load
y = Distance from the neutral axis of the plane to the point where ($\sigma_Z$) is being determined

At the heel or toe point, y = T/2. Thus, at these points, the normal stresses will be a maximum or minimum stress according to the position of the result from the center, (The eccentricity e).

$$\sigma_{Z\,max} = \frac{\sum V}{T} + \frac{6 \sum V \times e}{T^2} = \frac{\sum V}{T}\left(1 + \frac{6\,e}{T}\right)$$

$$\sigma_{Z\,min} = \frac{\sum V}{T} - \frac{6 \sum V \times e}{T^2} = \frac{\sum V}{T}\left(1 - \frac{6\,e}{T}\right)$$




Where $\boldsymbol{X} = \frac{\textbf{Net Moment}}{\textbf{Net Vertical Force}} = \frac{\sum \boldsymbol{M}}{\sum \boldsymbol{V}}$

$\boldsymbol{e} = \frac{\boldsymbol{T}}{\boldsymbol{2}} - \boldsymbol{X}$ and $\boldsymbol{R} = \sqrt{(\sum \boldsymbol{V})^2 + (\sum \boldsymbol{h})^2}$

Naturally, there would be tension on the upstream face if the overturning moments under the reservoir full condition increase such that ***e*** becomes greater than **T/6**. The total vertical stresses at the upstream and downstream faces are obtained by addition of external hydrostatic pressures.

### *6.5 Shear stresses on horizontal planes*

Nearly equal and complimentary horizontal stress ($\tau_{zy}$) and shear stresses ($\tau_{yz}$) are developed at any point as a result of the variation in vertical normal stress over a horizontal plane (Figure 39).

$\tau_{yz\,D} = (\sigma_{zD} - p_D)\tan\varphi_{D,}$ the shear stress at downstream face

$\tau_{yz\,U} = -(\sigma_{zU} - p_U)\tan\varphi_{U,}$ the shear stress at upstream face

The shear stress is seen to vary parabolically from $\tau_{yz\,U}$ at the upstream face up to $\tau_{yz\,D}$ at the downstream face.

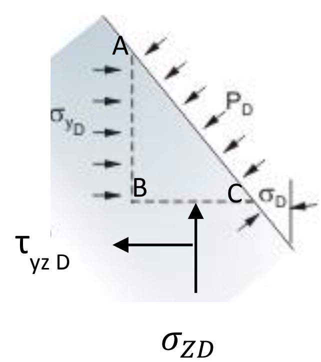
### *6.6 Normal stresses on vertical planes*

These stresses, $\boldsymbol{\sigma_y}$ can be determined by consideration of the equilibrium of the horizontal shear forces operating above and below a hypothetical element within the dam (Figure 39). The difference in shear forces is balanced by the normal stresses on vertical planes. Boundary values of $\boldsymbol{\sigma_y}$ at upstream and downstream faces are given by the following relations:

$$\sum F_x = 0$$

$$\sigma_{yD}\ AB = P_D\ AC\ \cos\emptyset D + \tau_{yZD}\ BC$$

$$\sigma_{yD} = P_D\,\frac{AC}{AB}\cos\emptyset D + (\sigma_{zD} - P_D)\tan\emptyset_D\,\frac{BC}{AB}$$




$$\sigma_{yD} = P_D\,\frac{1}{\cos\emptyset D}\cos\emptyset D + (\sigma_{zD} - P_D)\tan\emptyset_D\,\tan\emptyset_D$$

$$\boldsymbol{\sigma_{yD} = P_D + \left(\sigma_{zD} - P_D\right)tan^2\emptyset D}$$

***Simillary, for the upstream it could be seen that:***

$$\boldsymbol{\sigma_{yu} = P_u + \left(\sigma_{zu} - P_u\right)tan^2\emptyset u}$$

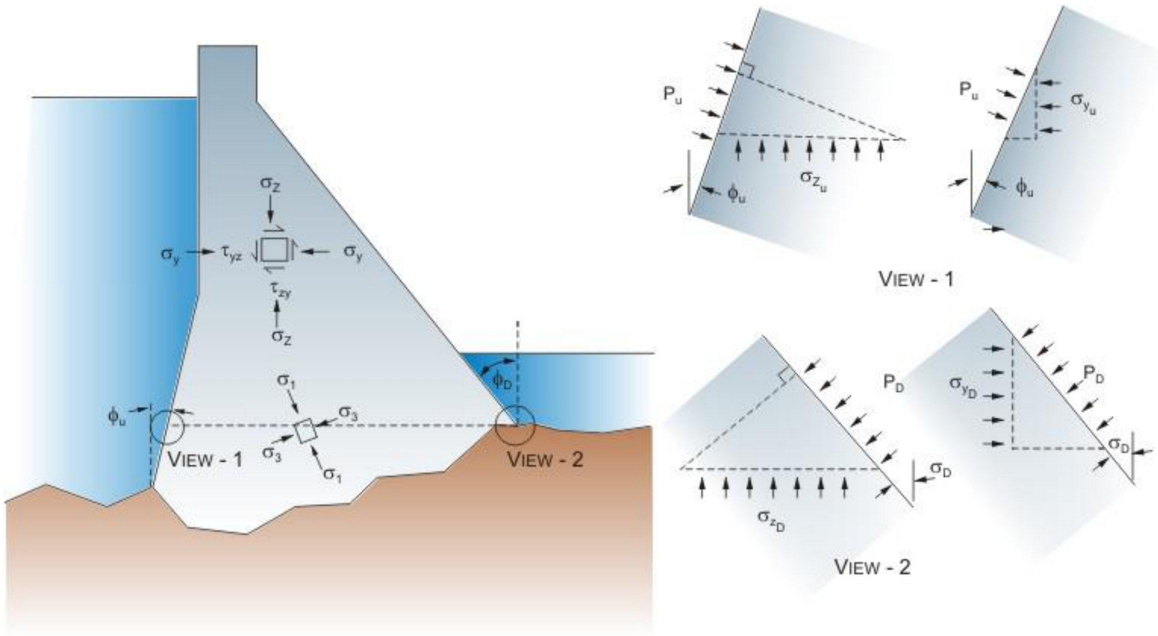



FIGURE.39: State of stress in a concrete gravity dam

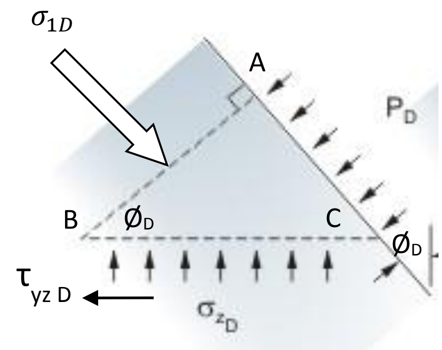
### *6.7 Principal stresses*

These are the maximum and minimum stresses that may be developed at any point within the dam. Usually, these are denoted as $\sigma_1$ and $\sigma_3$ respectively, and are oriented at a certain angle to the reference horizontal or vertical lines. The magnitude of $\sigma_1$ and $\sigma_3$ may be determined from the state of stress $\sigma_z$, $\sigma_y$ and $\tau_{yz}$ at any point by the following formula:

$$\sigma_{1,3} = \frac{\sigma_z + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_z - \sigma_y}{2}\right)^2 + \tau^2{}_{zy}}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_z - \sigma_y}{2}\right)^2 + \tau^2{}_{zy}}$$

The upstream and downstream faces are each planes of zero shear, and therefore, are planes of principal stresses. The principal stresses at these faces are given by the following expressions:




$\sum F_y = 0$

$$\sigma_{1D}\ AB \cos \emptyset D = \sigma_{zD}\ \ BC\ - \mathrm{P_D} AC\ \sin \emptyset D$$

$$\sigma_{1D}\ = \sigma_{zD}\ \frac{BC}{AB} \sec \emptyset D - \mathrm{P_D} \frac{AC}{AB} \tan \emptyset_D$$

$$\boldsymbol{\sigma_{1D}\ = \sigma_{zD}\ \ sec^2 \emptyset D - P_D tan^2 \emptyset_D}$$

***Also, for***

$\sum F_y = 0$

$$\boldsymbol{\sigma_{1D}\ AB \sin \emptyset D = \tau_{yzD}\ \ BC\ + \mathrm{P_D} AC\ \cos \emptyset D}$$

***Or***

$$\boldsymbol{\tau_{yzD}\ = (\sigma_{zD}\ \ - P_D)\ tan \emptyset D}$$

***Similarly we can obtain expression for upstream end as below:***

$$\boldsymbol{\sigma_{1u}\ = \sigma_{zu}\ \ \sec^2 \emptyset u - P_u \tan^2 \emptyset_u}$$

$$\boldsymbol{\tau_{yzu}\ = (\sigma_{zu}\ \ - P_u)\ \tan \emptyset u}$$

Where $P_D$ or Pu are the combined normal stress due to tail or upstream water and earthquake ***(if any; proper sign to be used for earthquake stress) i.e.***

$$\boldsymbol{PD\ Or\ Pu\ = \ w\ h\ \pm\ P_e}$$

### *6.8 Permissible stresses in concrete*

According to IS: 6512-1984, the following have to be followed for allowable compressive and tensile stresses in concrete:

Compressive strength of concrete is determined by testing 150mm cubes. The strength of concrete should satisfy early load and construction requirements and at the age of one year, ***it should be four times the maximum computed stress in the dam or 14N/mm$^2$, whichever is more.*** The allowable working stress in any part of the structure shall also not exceed 7N/mm$^2$.

No tensile stress is permitted on the upstream face of the dam for load combination B. Nominal tensile stresses are permitted for other load combinations and their permissible values should not exceed the values given in the following table:

| ***Version 2 CE IIT, Kharagpur* Load combination** | ***Permissible tensile stress*** |
|---|---|
| C | $0.01f_c$ |
| E | $0.02f_c$ |
| F | $0.02f_c$ |
| G | $0.04f_c$ |

***Where $f_c$ is the cube compressive strength of concrete.***

Small values of tension on the downstream face is permitted since it is improbable that a fully constructed dam is kept empty and downstream cracks which are not extensive and for limited depths from the surface may not be detrimental to the safety of the structure.

## ***Stability Analysis Steps***

Stability analysis by analytical method is done as the following steps:

1- Consider a unit length of the dam.
2- Find out the algebraic sum of all vertical forces acting on the dam, $\sum V$.
3- Find out the algebraic sum of all horizontal forces acting on the dam, $\sum H$.
4- Determine the obverting moment $\sum mo$ and the resisting moment $\sum mr$ , about the toe of the dam and find out the net moment as:

$$\sum m = \sum mr - \sum mo$$

5- Determine the position of the results R from the toe as:

$$\boldsymbol{X} = \frac{\textbf{Net Moment}}{\textbf{Net Vertical Force}} = \frac{\sum M}{\sum V}$$

6- Determine the eccentricity**, e ,** of the result ***R*** from the toe as

$$\boldsymbol{e} = \frac{T}{2} - \boldsymbol{X}$$

7- Find out the factor of safety against overtaking and sliding as:

$$F.S.O = \frac{Sum\ of\ Resisting\ moments}{Sum\ of\ Overturning\ moments} = \frac{\sum Mr}{\sum MO} \quad > 1.5\ to\ 2.5$$

$$F.S.S = \frac{Sum\ of\ Resisting\ or\ shear\ forces}{Sum\ of\ Moving\ Forces} = \frac{\mu \sum V + B\,.q}{\sum H} \quad > 4\ to\ 5$$

8- Find out the normal stresses at the toe and heel points as:

$$\sigma_{Z\,max} = \frac{\sum V}{T}\left(1 + \frac{6\,e}{T}\right)$$

$$\sigma_{Z\,min} = \frac{\sum V}{T}\left(1 - \frac{6\,e}{T}\right)$$

9- Determine the principle stresses at the toe and heal points as:

$$\sigma_{1D} = \sigma_{zD}\ sec^2 \emptyset D - P_D tan^2 \emptyset_D$$

$$\sigma_{1u} = \sigma_{zu}\ sec^2 \emptyset u - P_u tan^2 \emptyset_u$$

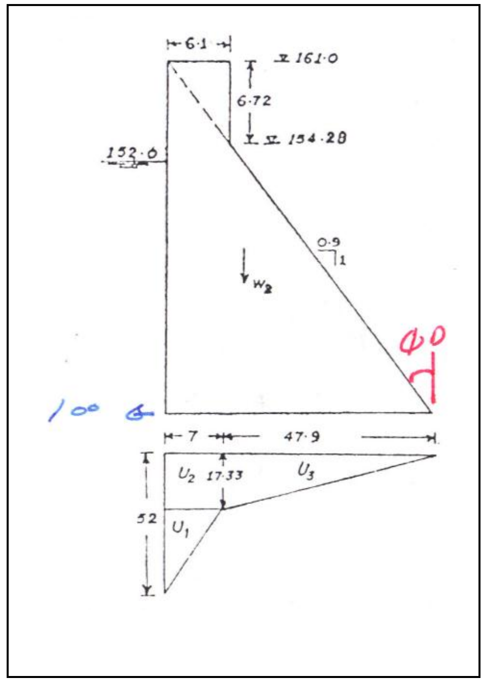
$$\tau_{yzD} = (\sigma_{zD} - P_D)\ tan \emptyset D$$

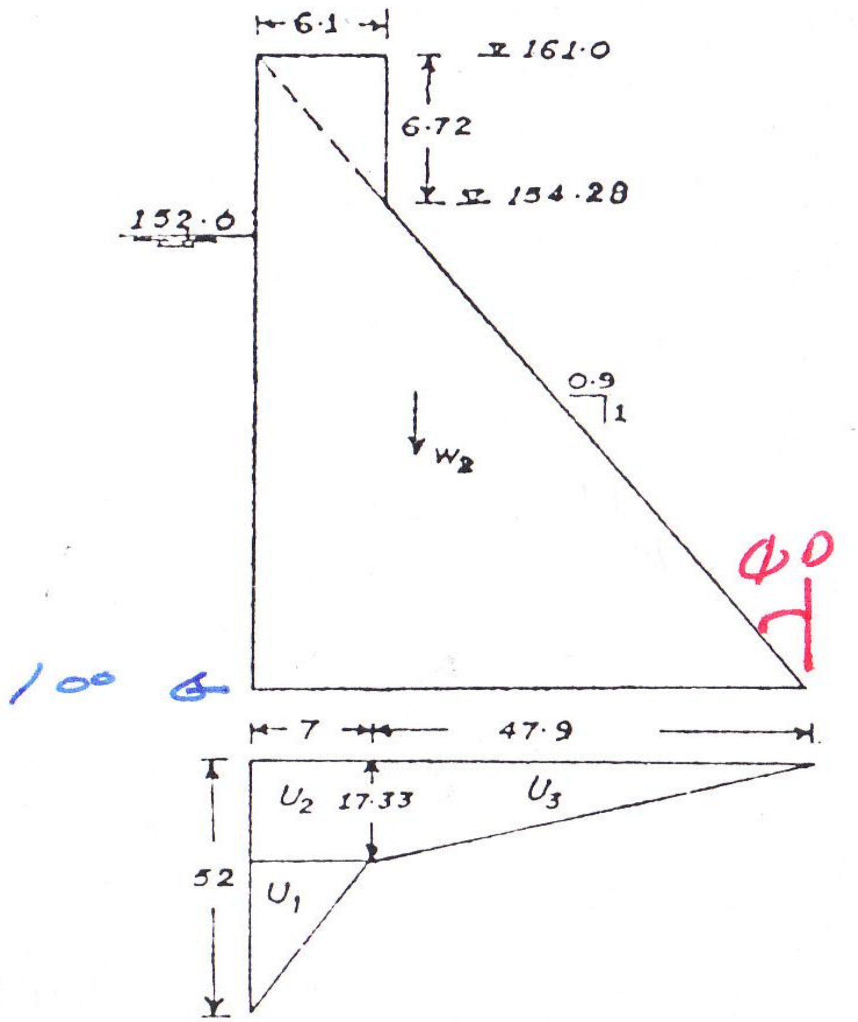
$$\tau_{yzu} = (\sigma_{zu} - P_u)\ tan \emptyset u$$

## Analysis Example

Check the non-overflow section of a gravity dam with the following data:

| | | |
|---|---|---|
| 1 | R. L. of deepest foundation level | 100 m |
| 2 | R. L. of roadway at the top of dam | 161 m |
| 3 | Maximum pond level | 152 m |
| 4 | Maximum tail water level | 123.9 m |
| 5 | Location of center of drainage gallery form U.S. face of dam | 7 m |
| 6 | Roadway width at the top | 6.1 m |
| 7 | Downstream vertical face up to El. | 154.28 m |
| 8 | Upstream face of dam | vertical |
| 9 | Downstream face of dam ( H:V) | 0.9 : 1 |
| 10 | Density of silt laden water | 1.36 t/m$^3$ |
| 11 | Weight of concrete | 2.4 t/m$^3$ |
| 12 | Safe bearing capacity | 1500 t/m$^2$ |
| 13 | Average shearing resistance | 210 t/m$^2$ |
| 14 | Maximum allowable shearing friction factor under: | |
| | Normal loading | 5 |
| | Abnormal loading | 4 |
| 15 | Maximum coefficient of sliding under normal loading | 0.75 |
| 16 | Horizontal Seismic coefficient | 0.2 |
| 17 | Vertical Seismic coefficient | 0.1 |




6.1
161.0
6.72
154.28
152.0
0.9
1
W2
7
47.9
U2
17.33
U3
52
U1