

Mustansiriyah University
College of Engineering
Materials Engineering Department
B.Sc. Second Class

Engineering Fluid Mechanics

Lecture 2-4

Flow Rate and Velocity Measurement

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Flow Rate and Velocity Measurement

A major application area of fluid mechanics is the determination of the flow rate of fluids, and numerous devices have been developed.

Pitot and Pitot Static Probes

Pitot probe (also called Pitot tube) is just a tube with a pressure tap at the stagnation point that measures stagnation pressure (static + dynamic).

Pitot static probe has both a stagnation pressure tap and several circumferential static pressure taps and it measures both stagnation and static pressures.

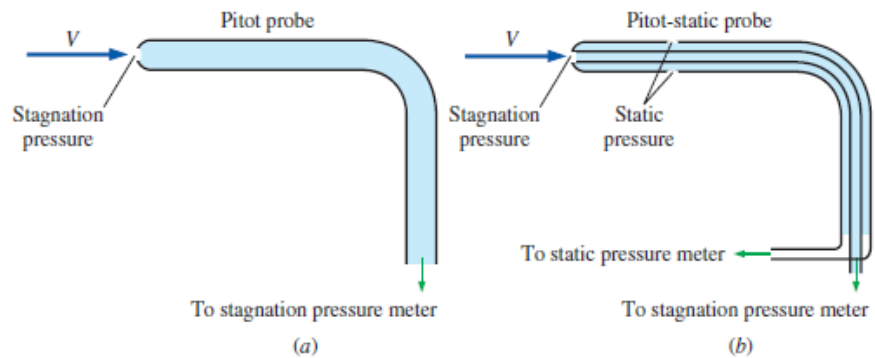


FIGURE 8-55

(a) A Pitot probe measures stagnation pressure at the nose of the probe, while (b) a Pitot-static probe measures both stagnation pressure and static pressure, from which the flow speed is calculated.

The Pitot static probe measures local velocity by measuring the pressure difference in conjunction with the Bernoulli equation.

For incompressible flow with sufficiently high velocities (so that the frictional effects between points 1 and 2 are negligible), the Bernoulli equation is applicable and is expressed as

$$\frac{P_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

Noting that $z_1 \cong z_2$ since the static pressure holes of the Pitot static probe are arranged circumferentially around the tube and $u_1 = 0$ because of the stagnation conditions, the flow velocity $u = u_2$ becomes

$$u = \sqrt{\frac{2(P_1 - P_2)}{\rho}} \quad \text{Pitot formula}$$

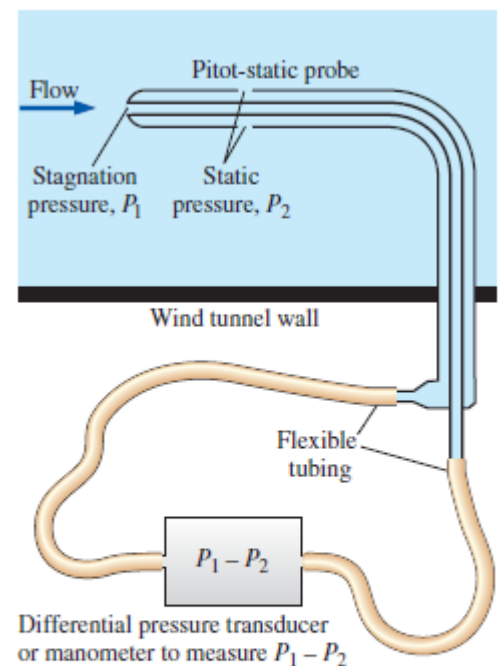


FIGURE 8-56

Measuring flow velocity with a Pitot-static probe. (A manometer may be used in place of the differential pressure transducer.)

Obstruction Flowmeters: Orifice, Venturi, and Nozzle Meters

Consider incompressible steady flow of a fluid in a horizontal pipe of diameter D that is constricted to a flow area of diameter d , as shown in the accompanying figure. The mass balance and the Bernoulli equations between a location before the constriction (point 1) and the location where constriction occurs (point 2) are written as

Mass balance $\dot{V} = u_1 A_1 = u_2 A_2$

$$u_1 = u_2 \left(\frac{A_2}{A_1} \right) = u_2 \left(\frac{d}{D} \right)^2 \text{ -----(1)}$$

Bernoulli equation ($z_1 = z_2$): $\frac{P_1}{\rho g} + \frac{u_1^2}{2g} = \frac{P_2}{\rho g} + \frac{u_2^2}{2g}$ -----(2)

Combining Eqs. (1) and (2) and solving for velocity u_2

gives $u_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}$ Obstruction with no loss -----(3)

Where $\beta = d/D$ is the diameter ratio. Once u_2 is known, the flow rate can be determined from

$$\dot{V} = u_2 A_2 = u_2 \left(\frac{\pi}{4} d^2 \right)$$

In reality, some pressure losses due to frictional effects are inevitable, and thus the actual velocity is less. Also, the fluid stream continues to contract past the obstruction, and the vena contracta area is less than the flow area of the obstruction. Both losses can be accounted for by incorporating a correction factor called the discharge coefficient C_d whose value (which is less than 1) is determined experimentally. Then the flow rate for obstruction flow meters is expressed as

$$\dot{V} = A_0 C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \text{ Obstruction flowmeters -----(4)}$$

Where $A_0 = A_2 = (\pi d^2/4)$ is the cross sectional area of the throat or orifice and $\beta = d/D$ is the ratio of the throat diameter to pipe diameter. The value of C_d depends on both β and the Reynolds number $Re = (\rho u_1 D / \mu)$, and charts and curve fit correlations for C_d are available for various types of obstruction meters.

For flows with high Reynolds numbers ($Re > 30,000$), the value of C_d can be taken to be 0.96 for flow nozzles and 0.61 for orifices. In the absence of specific data, we can take $C_d = 0.98$ for Venturi meters.

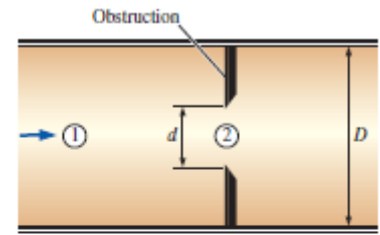
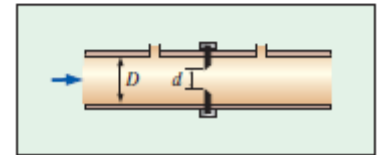
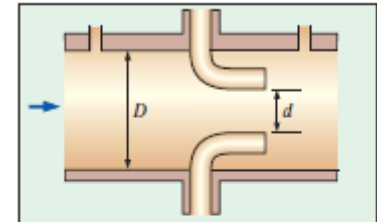


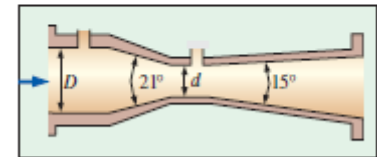
FIGURE 8-58
Flow through a constriction in a pipe.



(a) Orifice meter



(b) Flow nozzle

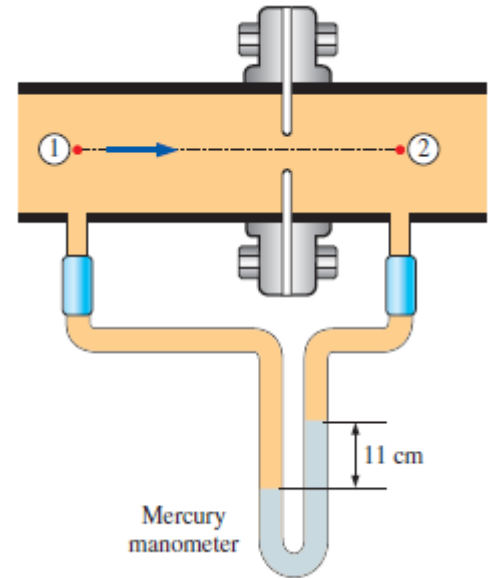


(c) Venturi meter

FIGURE 8-59
Common types of obstruction meters.

Example:

The flow rate of methanol at 20 °C ($\rho = 788.4 \text{ kg/m}^3$) through a 4 cm diameter pipe is to be measured with a 3 cm diameter orifice meter ($C_d = 0.6$) equipped with a mercury ($\rho = 13600 \text{ kg/m}^3$) manometer across the orifice plate, as shown in the accompanying figure. If the differential height of the manometer is 11 cm, determine the flow rate of methanol through the pipe and the average flow velocity.

**Solution:**

The flow rate relation for obstruction meters is

$$\dot{V} = A_0 C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}$$

The diameter ratio

$$\beta = \frac{d}{D} = \frac{3}{4} = 0.75$$

The throat area of the orifice plate is

$$A_0 = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.03 \text{ m})^2 = 7.069 \times 10^{-4} \text{ m}^2$$

The pressure drop across the orifice plate is

$$\Delta P = P_1 - P_2 = (\rho_{Hg} - \rho_{\text{methanol}})gh$$

$$\Delta P = (13600 - 788.4) \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{m}}{\text{s}^2} \times 0.11 \text{ m}$$

$$\Delta P = 13825 \frac{\text{N}}{\text{m}^2}$$

Substituting in the flow rate relation

$$\dot{V} = 7.069 \times 10^{-4} \text{ m}^2 \times 0.6 \times \sqrt{\frac{2 \times 13825 \frac{\text{N}}{\text{m}^2}}{788.4 \frac{\text{kg}}{\text{m}^3} \{1 - (0.75)^4\}}}$$

$$\dot{V} = 3.038 \times 10^{-3} \frac{\text{m}^3}{\text{s}}$$

The average flow velocity in the pipe is determined by dividing the flow rate by the cross sectional area of the pipe

$$u = \frac{\dot{V}}{A_{x,\text{pipe}}} = \frac{\dot{V}}{\frac{\pi}{4} D^2} = \frac{3.038 \times 10^{-3} \frac{\text{m}^3}{\text{s}}}{\frac{\pi}{4} (0.04 \text{ m})^2} = 2.418 \text{ m/s}$$

Variable Area Flowmeters (Rotameters)

A variable area flowmeter consists of a vertical tapered conical transparent tube made of glass or plastic with a float inside that is free to move. As fluid flows through the tapered tube, the float rises within the tube to a location where the float weight, drag force, and buoyancy force balance each other and the net force acting on the float is zero. The flow rate is determined by simply matching the position of the float against the graduated flow scale outside the tapered transparent tube. The float itself is typically either a sphere or a loose fitting piston like cylinder. Depending on the obstruction flowmeters Eq., the flow rate through the rotameter

$$\dot{V} = A_{annulus} C_d \sqrt{\frac{2V_{float} g (\rho_{float} - \rho_{fluid})}{A_{float} \rho_{fluid} \left[1 - \left(\frac{A_{annulus}}{A_{tube}} \right)^2 \right]}}$$

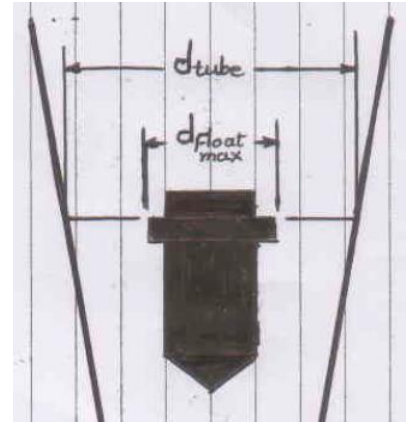
Where A_{float} is the maximum cross sectional area of the float, A_{tube} is the cross sectional area of the tube where the float is settled, and $A_{annulus}$ is the area of the annulus between the float and tube.



FIGURE 8-67

Two types of variable-area flowmeters: (a) an ordinary gravity-based meter and (b) a spring-opposed meter.

Example 1: A rotameter tube is 0.3 m long with an internal diameter of 25 mm at the top and 20 mm at the bottom. The diameter of the float is 20 mm, its specific gravity is 4.80 and its volume is 6.0 cm³. If the discharge coefficient is 0.7, what will be the flow rate of water ($\rho = 1000 \text{ kg/m}^3$) when the float is halfway up the tube?



Solution:

The flow rate relation for rotameters is:

$$\dot{V} = A_{\text{annulus}} C_d \sqrt{\frac{2V_{\text{float}} g (\rho_{\text{float}} - \rho_{\text{fluid}})}{A_{\text{float}} \rho_{\text{fluid}} \left[1 - \left(\frac{A_{\text{annulus}}}{A_{\text{tube}}} \right)^2 \right]}}$$

To determine A_{tube} and A_{annulus}

$$\tan \theta = \frac{x_2}{\text{tube height}} = \frac{x_1}{0.5 \times \text{tube height}} \rightarrow \frac{2.5 \text{ mm}}{300 \text{ mm}} = \frac{x_1}{150 \text{ mm}}$$

$$x_1 = 2.5 \text{ mm} \times \frac{150}{300} = 1.25 \text{ mm}$$

The diameter of the rotameter where the float stay at (be settled at) 15 cm height is $20 \text{ mm} + 2(1.25 \text{ mm}) = 22.5 \text{ mm}$

Area of tube at 15 cm height is:

$$A_{\text{tube}} = \frac{\pi}{4} (22.5 \times 10^{-3} \text{ m})^2 = 3.976 \times 10^{-4} \text{ m}^2$$

Annulus area equal to the area of tube where the float is settled minus area of the float

$$A_{\text{annulus}} = 3.976 \times 10^{-4} \text{ m}^2 - A_{\text{float}}$$

$$A_{\text{annulus}} = 3.976 \times 10^{-4} \text{ m}^2 - \frac{\pi}{4} (20 \times 10^{-3} \text{ m})^2$$

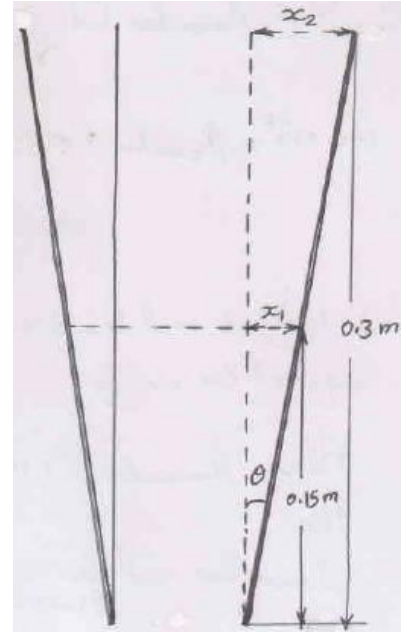
$$A_{\text{annulus}} = 3.976 \times 10^{-4} \text{ m}^2 - 3.1416 \times 10^{-4} \text{ m}^2 = 8.345 \times 10^{-5} \text{ m}^2$$

Substituting in the flow rate relation

$$\dot{V} = 8.345 \times 10^{-5} \times 0.7 \sqrt{\frac{2 \times 6 \times 10^{-6} \times 9.81 (4800 - 1000)}{3.1416 \times 10^{-4} \times 1000 \left[1 - \left(\frac{8.345 \times 10^{-5}}{3.976 \times 10^{-4}} \right)^2 \right]}}$$

$$\dot{V} = 5.8414 \times 10^{-5} \times \sqrt{1.48953} = 7.13 \times 10^{-5} \frac{\text{m}^3}{\text{s}}$$

$$\dot{V} = 7.13 \times 10^{-5} \frac{\text{m}^3}{\text{s}} = 7.13 \times 10^{-2} \frac{\text{L}}{\text{s}} = 7.13 \times 10^{-2} \frac{\text{kg}}{\text{s}}$$



Example 2: A rotameter has a tube 0.3 m long which has an internal diameter of 25 mm at the top and 20 mm at the bottom. The diameter of the float is 20 mm, its specific gravity is 4.80 and its volume is 6.6 cm³. If the discharge coefficient is 0.72, at what height will the float be when metering water ($\rho = 1000 \text{ kg/m}^3$) at 100 cm³/s?

Solution:

The flow rate relation for rotameters is: $\dot{V} = A_{\text{annulus}} C_d \sqrt{\frac{2V_{\text{float}} g(\rho_{\text{float}} - \rho_{\text{fluid}})}{A_{\text{float}} \rho_{\text{fluid}} \left[1 - \left(\frac{A_{\text{annulus}}}{A_{\text{tube}}}\right)^2\right]}}$

Initially it will be assumed that $\left[1 - \left(\frac{A_{\text{annulus}}}{A_{\text{tube}}}\right)^2\right]^{0.5}$ is approximately equal to unity.

$$\text{Then } 1 \times 10^{-4} = A_{\text{annulus}} \times 0.72 \sqrt{\frac{2 \times 6.6 \times 10^{-6} \times 9.81(4800 - 1000)}{\frac{\pi}{4}(0.02)^2 \times 1000}}$$

$$A_{\text{annulus}} = 1.10976 \times 10^{-4} \text{ m}^2$$

$$A_{\text{annulus}} = A_{\text{tube}} - A_{\text{float}}$$

$$1.10976 \times 10^{-4} \text{ m}^2 = A_{\text{tube}} - \frac{\pi}{4}(0.02 \text{ m})^2$$

$$A_{\text{tube}} = 4.25 \times 10^{-4} \text{ m}^2$$

$$A_{\text{tube}} = \frac{\pi}{4} D_{\text{tube}}^2 \rightarrow 4.25 \times 10^{-4} = \frac{\pi}{4} D_{\text{tube}}^2$$

$$D_{\text{tube}} = 0.023 \text{ m}$$

The height of the top of the float from the bottom of the tube can be determined as

$$\tan \theta = \frac{x_2}{\text{tube height}} = \frac{x_1}{\text{float height}}$$

$$\frac{2x_2}{\text{tube height}} = \frac{2x_1}{\text{float height}}$$

$$\frac{5 \times 10^{-3}}{0.3} = \frac{D_{\text{tube}} - D_{\text{float}}}{\text{float height}} \rightarrow \frac{5 \times 10^{-3}}{0.3} = \frac{0.023 - 0.02}{\text{float height}}$$

$$\text{float height} = 0.196 \text{ m}$$

The area can now be corrected to allow for the initial assumption that $\left[1 - \left(\frac{A_{\text{annulus}}}{A_{\text{tube}}}\right)^2\right]^{0.5}$ was equal to unity,

$$\left[1 - \left(\frac{A_{\text{annulus}}}{A_{\text{tube}}}\right)^2\right]^{0.5} = \left[1 - \left(\frac{1.10976 \times 10^{-4} \text{ m}^2}{4.25 \times 10^{-4} \text{ m}^2}\right)^2\right]^{0.5} = 0.965$$

$$\therefore \text{true annulus area} = 1.10976 \times 10^{-4} \text{ m}^2 \times 0.965 = 1.07 \times 10^{-4} \text{ m}^2$$