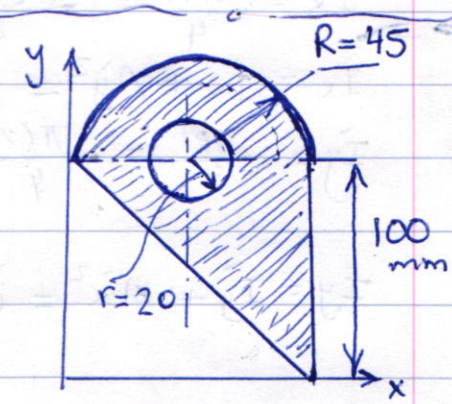
or $\bar{J_c} = J_o - Ad^2 = \frac{bh}{3}(h^2+b^2) - bh\left(\frac{b^2}{4}+\frac{h^2}{4}\right)$

$\Rightarrow \bar{J_c} = \frac{bh}{12}(h^2+b^2)$.

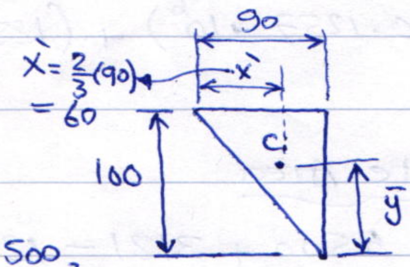
Ex 2:



Calculate the radii of gyration about the x- and y-axes.

Solu:

Triangle:



$$A = \frac{bh}{2} = \frac{90 \times 100}{2} = 4500 \text{ mm}^2$$

$$\bar{I}_x = \frac{bh^3}{36} = \frac{90(100)^3}{36} = 2.50 \times 10^6 \text{ mm}^4$$

$$I_x = \bar{I}_x + A\bar{y}^2 = (2.5 \times 10^6) + (4500)(66.7)^2 = 22.52 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = \frac{hb^3}{36} = \frac{100(90)^3}{36} = 2.025 \times 10^6 \text{ mm}^4$$

$$I_y = \bar{I}_y + A\bar{x}^2 = (2.025 \times 10^6) + (4500)(60)^2 = 18.23 \times 10^6 \text{ mm}^4.$$

Semicircle:

$$A = \frac{\pi R^2}{2} = \frac{\pi(45)^2}{2} = 3181 \text{ mm}^2$$

$$\bar{I}_x = 0.1098\, R^4 = 0.1098\,(45)^4 = 0.45 \times 10^6 \text{ mm}^4$$

$$I_x = \bar{I}_x + A\bar{y}^2 = (0.45 \times 10^6) + (3181)(119.1)^2 = 45.57 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = \frac{\pi R^4}{8} = \frac{\pi(45)^4}{8} = 1.61 \times 10^6 \text{ mm}^4$$

$$I_y = \bar{I}_y + A\bar{x}^2 = (1.61 \times 10^6) + (3181)(45)^2 = 8.05 \times 10^6 \text{ mm}^4.$$

<u>Circle:</u>

$A = \pi R^2 = \pi (20)^2 = 1257 \text{ mm}^2$

$\grave{I}_x = \frac{\pi R^4}{4} = \frac{\pi (20)^4}{4} = 0.1257 * 10^6 \text{ mm}^4$

$I_x = \grave{I}_x + A\grave{y}^2 = (0.1257 * 10^6) + (1257)(100)^2 = 12.70 * 10^6 \text{ mm}^4$

$\grave{I}_y = \frac{\pi R^4}{4} = \frac{\pi (20)^4}{4} = 0.1257 * 10^6 \text{ mm}^4.$

$I_y = \grave{I}_y + A\grave{x}^2 = (0.1257 * 10^6) + (1257)(45)^2 = 2.67 * 10^6 \text{ mm}^4.$

<u>For the Composite Area</u>

$A = \Sigma A = 4500 + 3181 - 1257 = 6424 \text{ mm}^2$

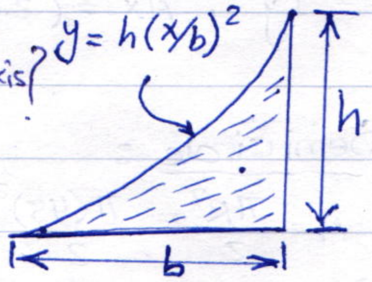
$I_x = \Sigma I_x = (22.52 + 45.57 - 12.70) * 10^6 = 55.39 * 10^6 \text{ mm}^4$

$I_y = \Sigma I_y = (18.23 + 8.05 - 2.67) * 10^6 = 23.61 * 10^6 \text{ mm}^4$

<u>Therefore;</u> $K_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{55.39 * 10^6}{6424}} = 92.9 \text{ mm}$

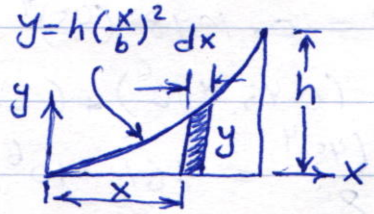
$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{23.61 * 10^6}{6424}} = 60.6 \text{ mm}$

<u>Ex 3:</u> Determine the moment of inertia about y-axis, and about x-axis?



<u>Solu:</u> ① <u>Applying vertical element.</u>

$dA = y\, dx = \frac{h}{b^2} x^2\, dx$



$\therefore I_y = \int_A x^2\, dA$

$\therefore I_y = \int_0^b x^2 \left(\frac{h}{b^2}\right) x^2\, dx = \frac{h}{b^2} \int_0^b x^4 \cdot dx = \frac{h}{b^2} \frac{b^5}{5} = \frac{b^3 h}{5}$ Ans.