Circle:

$$A = \pi R^2 = \pi (20)^2 = 1257 \ mm^2$$

$$\bar{I}_x = \frac{\pi R^4}{4} = \frac{\pi (20)^4}{4} = 0.1257 * 10^6 \ mm^4$$

$$I_x = \bar{I}_x + A\bar{y}^2 = (0.1257 * 10^6) + (1257)(100)^2 = 12.70 * 10^6 \ mm^4$$

$$\bar{I}_y = \frac{\pi R^4}{4} = \frac{\pi (20)^4}{4} = 0.1257 * 10^6 \ mm^4.$$

$$I_y = \bar{I}_y + A\bar{x}^2 = (0.1257 * 10^6) + (1257)(45)^2 = 2.67 * 10^6 \ mm^4.$$

For the Composite Area

$$A = \Sigma A = 4500 + 3181 - 1257 = 6424 \ mm^2$$

$$I_x = \Sigma I_x = (22.52 + 45.57 - 12.70) * 10^6 = 55.39 * 10^6 \ mm^4$$

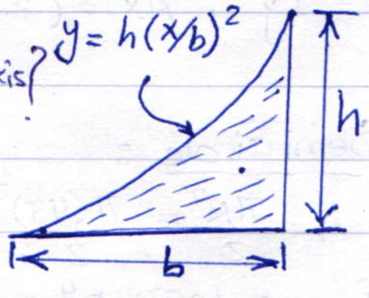
$$I_y = \Sigma I_y = (18.23 + 8.05 - 2.67) * 10^6 = 23.61 * 10^6 \ mm^4$$

Therefore;

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{55.39 * 10^6}{6424}} = 92.9 \ mm$$

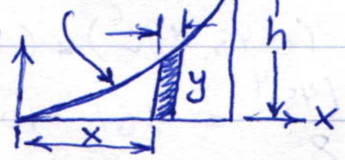
$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{23.61 * 10^6}{6424}} = 60.6 \ mm$$

Ex 3: Determine the moment of inertia about y-axis, and about x-axis?



Solu: ① Applying vertical element.

$$dA = y\,dx = \frac{h}{b^2} x^2 dx$$

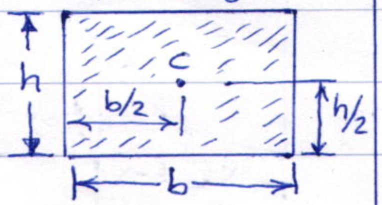


$$\therefore I_y = \int_A x^2 dA$$

$$\therefore I_y = \int_0^b x^2 \left(\frac{h}{b^2}\right) x^2 dx = \frac{h}{b^2} \int_0^b x^4 \cdot dx = \frac{h}{b^2} \frac{b^5}{5} = \frac{b^3 h}{5}$$

Ans.

Rectangle

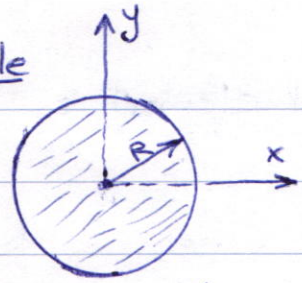


$\bar{I}_x = \frac{bh^3}{12}$; $\bar{I}_y = \frac{b^3h}{12}$

$I_x = \frac{bh^3}{3}$; $I_y = \frac{b^3h}{3}$

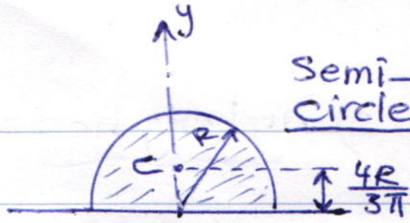
$I_{xy} = \frac{b^2h^2}{4}$

Circle



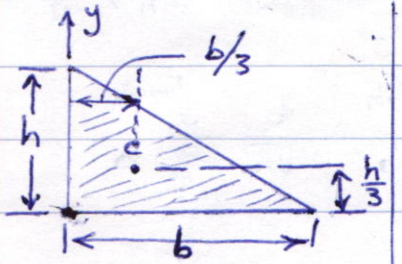
$I_x = I_y = \frac{\pi R^4}{4}$

Semi-circle



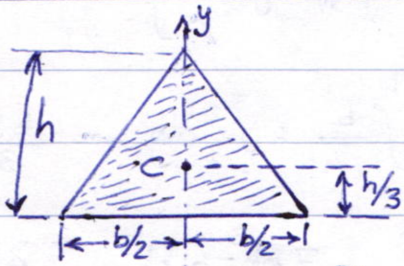
$\bar{I}_x = 0.1098\ R^4$

$I_x = I_y = \frac{\pi R^4}{8}$



$\bar{I}_x = \frac{bh^3}{36}$; $\bar{I}_y = \frac{b^3h}{36}$

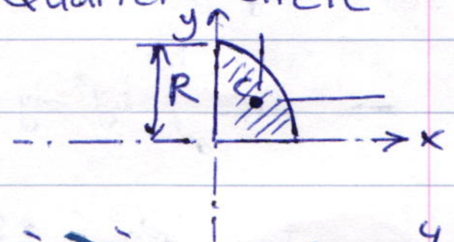
$I_x = \frac{bh^3}{12}$; $I_y = \frac{b^3h}{12}$



$\bar{I}_x = \frac{bh^3}{36}$; $\bar{I}_y = \frac{b^3h}{48}$

$I_x = \frac{bh^3}{12}$

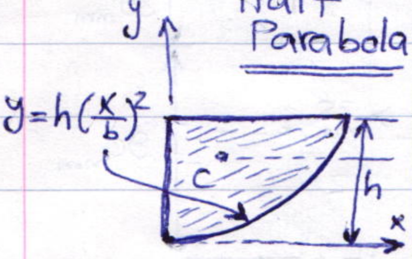
Quarter Circle



$\bar{I}_x = \cancel{\bar{I}_y} = 0.05488\,R^4$

$I_x = I_y = \frac{\pi R^4}{16}$

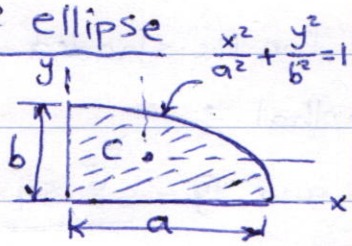
Half Parabola



$\bar{I}_x = \frac{8bh^3}{175}$, $\bar{I}_y = \frac{19b^3h}{480}$

$I_x = \frac{2bh^3}{7}$; $I_y = \frac{2b^3h}{15}$

Half ellipse



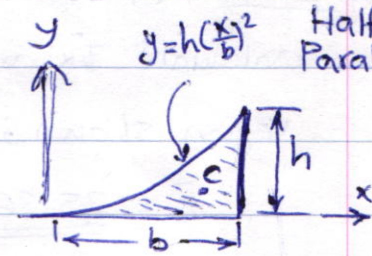
$\bar{I}_x = 0.05488\ ab^3$

$\bar{I}_y = 0.05488\ a^3b$

$I_x = \frac{\pi ab^3}{16}$

$I_y = \frac{\pi a^3b}{16}$

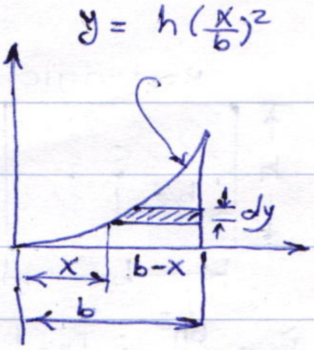
Half Parabolic



$\bar{I}_x = \frac{37\,bh^3}{2100}$, $I_x = \frac{bh^3}{21}$

$\bar{I}_y = \frac{b^3h}{80}$; $I_y = \frac{b^3h}{5}$

2) Applying horizontal element :



$$I_x = \int_A y^2 \cdot dA$$

$$\Rightarrow I_x = \int_0^h y^2 \cdot \left(b - b\sqrt{\frac{y}{h}}\right) dy$$

$dA = (b-x)\, dy$

$x = b\sqrt{\frac{y}{h}}$ from $y = h(\frac{x}{b})^2$

$\therefore dA = \left(b - b\sqrt{\frac{y}{h}}\right) dy$

$$\Rightarrow I_x = \int_0^h \left[b y^2 dy - b\sqrt{\frac{y}{h}} \cdot y^2 dy \right]$$

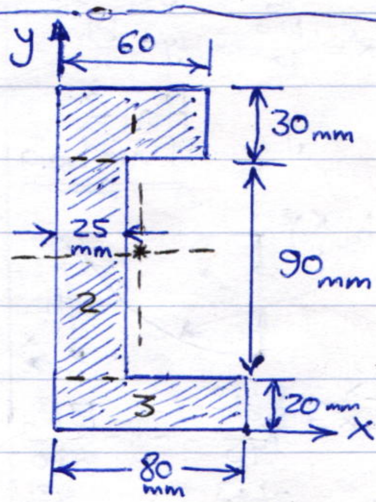
$$\Rightarrow I_x = \int_0^h b y^2 dy - \int_0^h \frac{b}{\sqrt{h}} \cdot y^{5/2} dy = \frac{b y^3}{3}\Big|_0^h - \frac{b}{\sqrt{h}} \cdot \frac{y^{7/2}}{7/2}\Big|_0^h$$

$$\Rightarrow I_x = \frac{bh^3}{3} - \frac{2bh^3}{7} = \frac{7bh^3 - 6bh^3}{21} = \frac{bh^3}{21} \quad \text{Ans}$$

Ex 4 : (9.20 and 9.21 ; PP. 489)

calculate $\bar{I}_x$ and $\bar{I}_y$ for the shaded region shown ; givin that :

$\bar{x} = 25.86$ mm and $\bar{y} = 68.54$ mm



Solu :

	A	x̀	x̀A	ỳ	ỳA
1	1800	30	54000	125	225000
2	2250	12.5	28125	65	146250
3	1600	40	64000	10	16000

$\Sigma A = 5650$ $\quad \Sigma \grave{x}A = 146{,}125$ $\quad \Sigma \grave{y}A = 387250$

$$\grave{x} = \frac{\Sigma \grave{x}A}{\Sigma A} = \frac{146125}{5650} = 25.86 \text{ mm}$$

$$\grave{y} = \frac{\Sigma \grave{y}A}{\Sigma A} = \frac{387250}{5650} = 68.54 \text{ mm}$$

$$\bar{I}_x = I_x + Ad^2$$