

محاضرة / -2-
التاريخ /



الكورس الاول
السعر /

Engineering Mechanics

الميكانيك الهندسي

لطلبة الدراسات الاولى

المرحلة الاولى

قسم الهندسة الموارد المائية

Dr. Khitam Abdulhusein

النسخة الأصلية

في مكتب الغدير داخل كلية الهندسة / الفرع الاول

مكتب الغدير 2 مقابل كلية الهندسة / الفرع الثاني

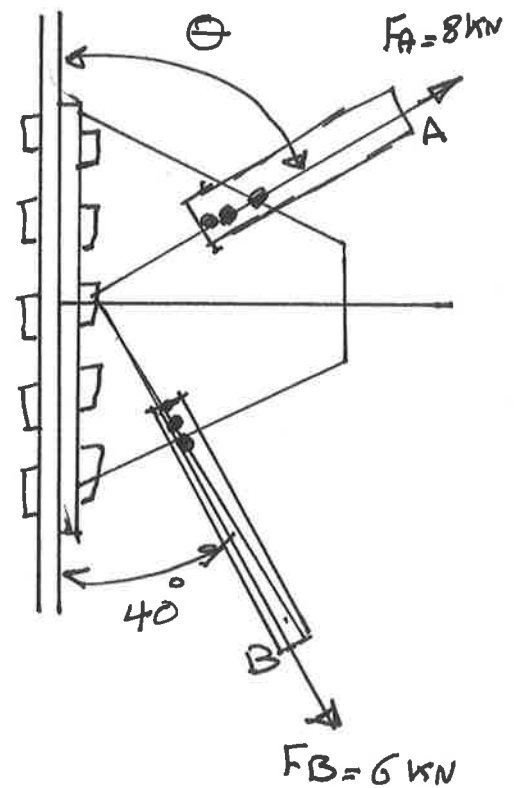
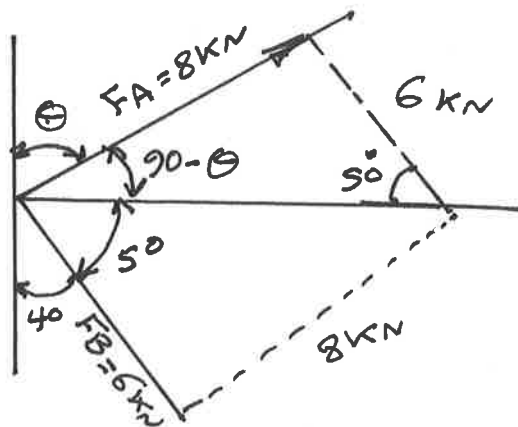
بإدارة / عادل الكناني

2018 - 2019

Ex^o1 Determine the angle θ so that the resultant force directed horizontally to the right. Also find magnitude of the resultant

Solution

Parallelogram Law: as shown in fig. below:



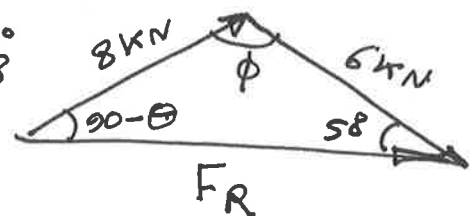
by using law of sines

$$\frac{\sin(90^\circ - \theta)}{6} = \frac{\sin 50^\circ}{8} \Rightarrow \theta = 54.93^\circ = 54.9^\circ$$

From the triangle, \Rightarrow

$$\phi = 180^\circ - (90^\circ - 54.9^\circ) - 50^\circ = 94.93^\circ$$

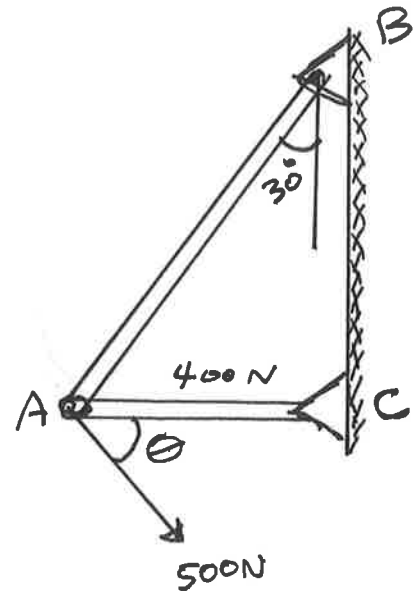
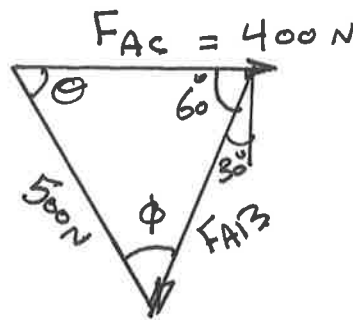
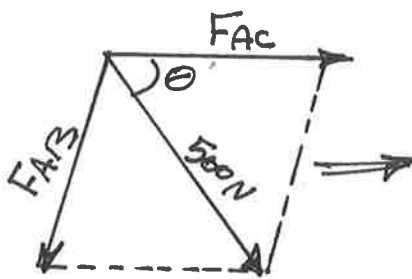
by law of cosines, it can be found the magnitude of F_R as



$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93^\circ} = \boxed{10.4 \text{ kN}}$$

Ex:2 For the frame shown, determine the angle θ so that the horizontal component F_{AC} has a magnitude of 400 N, also find F_{AB}

Solution :-



$$\frac{400}{\sin \phi} = \frac{500}{\sin 60^\circ}$$

$$\sin \phi = 0.6928$$

$$\phi = 43.85^\circ$$

$$\theta = 180 - (60 + 43.85) = 76.15^\circ$$

$$\therefore \frac{F_{AB}}{\sin 76.15} = \frac{500}{\sin 60} \implies F_{AB} = 560.56 \text{ N}$$

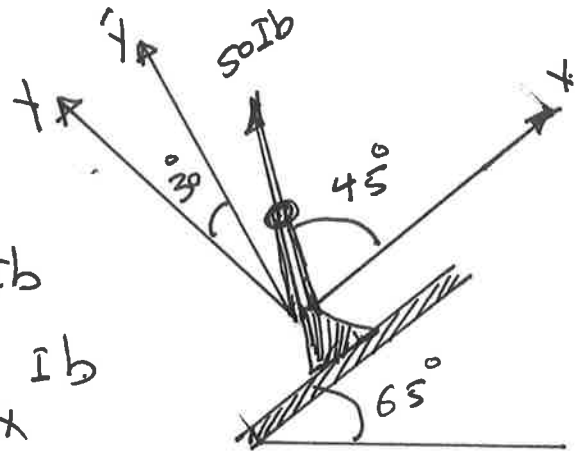
OR

$$F_{AB} = \sqrt{500^2 + 400^2 - 2(500)(400) \cos 76.15}$$

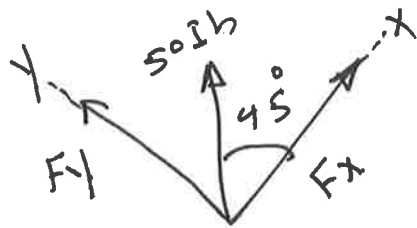
$$F_{AB} = 560.56 \text{ N}$$

Ex:3 Resolve the 50 lb force into components acting along (a) the x and y axes, and (b) the x and \hat{y} axes.

Solution :-

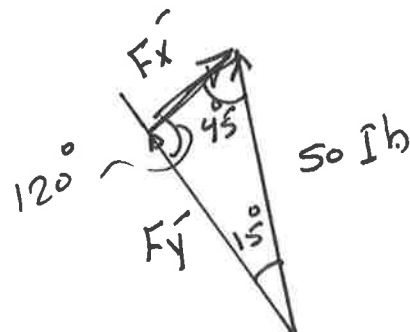


(a) $F_x = 50 \cos 45^\circ = 35.4 \text{ lb}$
 $F_y = 50 \sin 45^\circ = 35.4 \text{ lb}$



(b) $\frac{F_x}{\sin 15^\circ} = \frac{50}{\sin 120^\circ} \Rightarrow F_x = 14.9 \text{ lb}$

$\frac{F_{\hat{y}}}{\sin 45^\circ} = \frac{50}{\sin 120^\circ} \Rightarrow F_{\hat{y}} = 40.8 \text{ lb}$



Ex 84 The screw eye shown is subjected to two forces F_1 and F_2 . Determine the magnitude and direction of the resultant force.

Solution:-

$$R = \sqrt{(100\text{N})^2 + (150\text{N})^2 - 2(100\text{N})(150\text{N}) \cos 115^\circ}$$

$$R = \sqrt{10000 + 22500 - 30000(-0.4226)}$$

$$R = 213\text{N}$$

Applying the law of sines to determine θ

$$\frac{150\text{N}}{\sin \theta} = \frac{212.6\text{N}}{\sin 115^\circ}$$

$$\sin \theta = \frac{150\text{N}}{212.6\text{N}} (\sin 115^\circ)$$

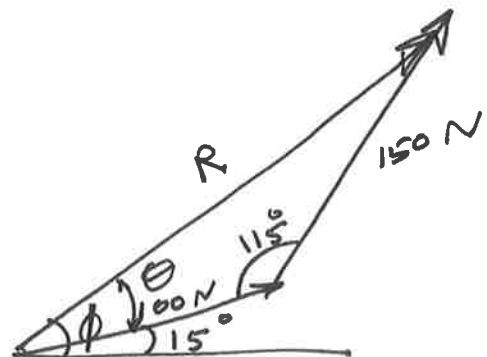
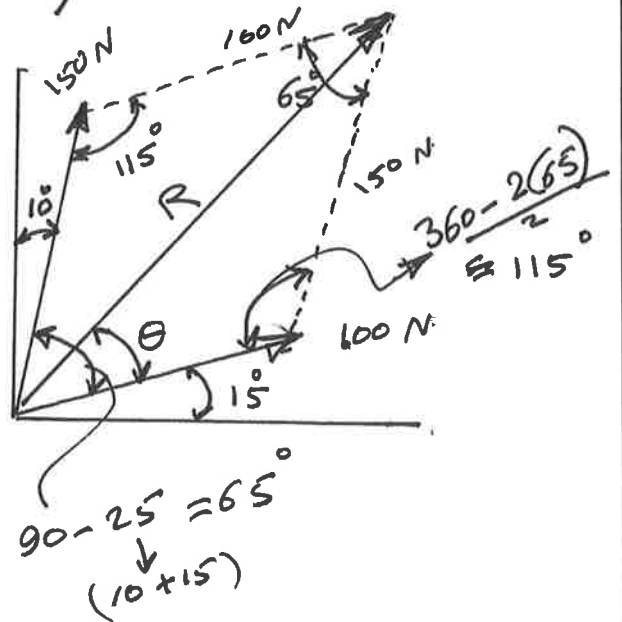
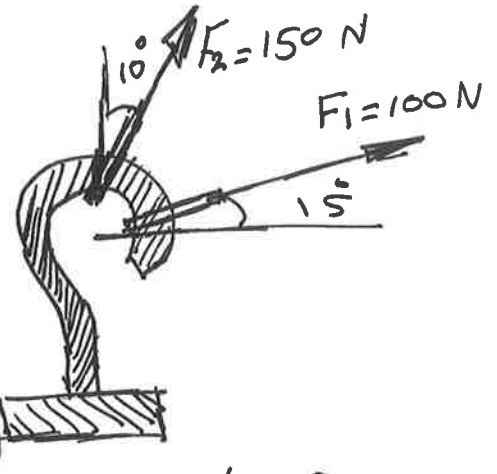
$$\theta = 39.8^\circ$$

Thus, the direction (ϕ)

of R measured from the horizontal is

$$\phi = 39.8^\circ + 15.0^\circ = 54.8^\circ$$

(21)

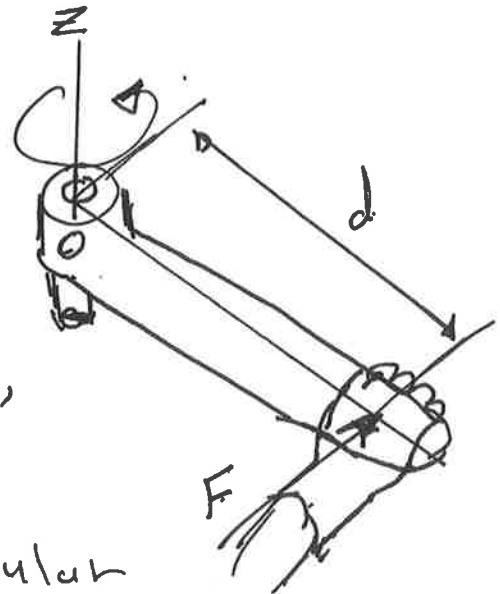


Moment and Couples

1. Moment of a force

- The moment of a force about a point or axis provides a measure of the tendency of the force to cause a body to rotate about the point or axis.

- The force F and the point O lie in a plane. The moment about the point O , or about an axis passing through O and perpendicular to the plane is a vector quantity.

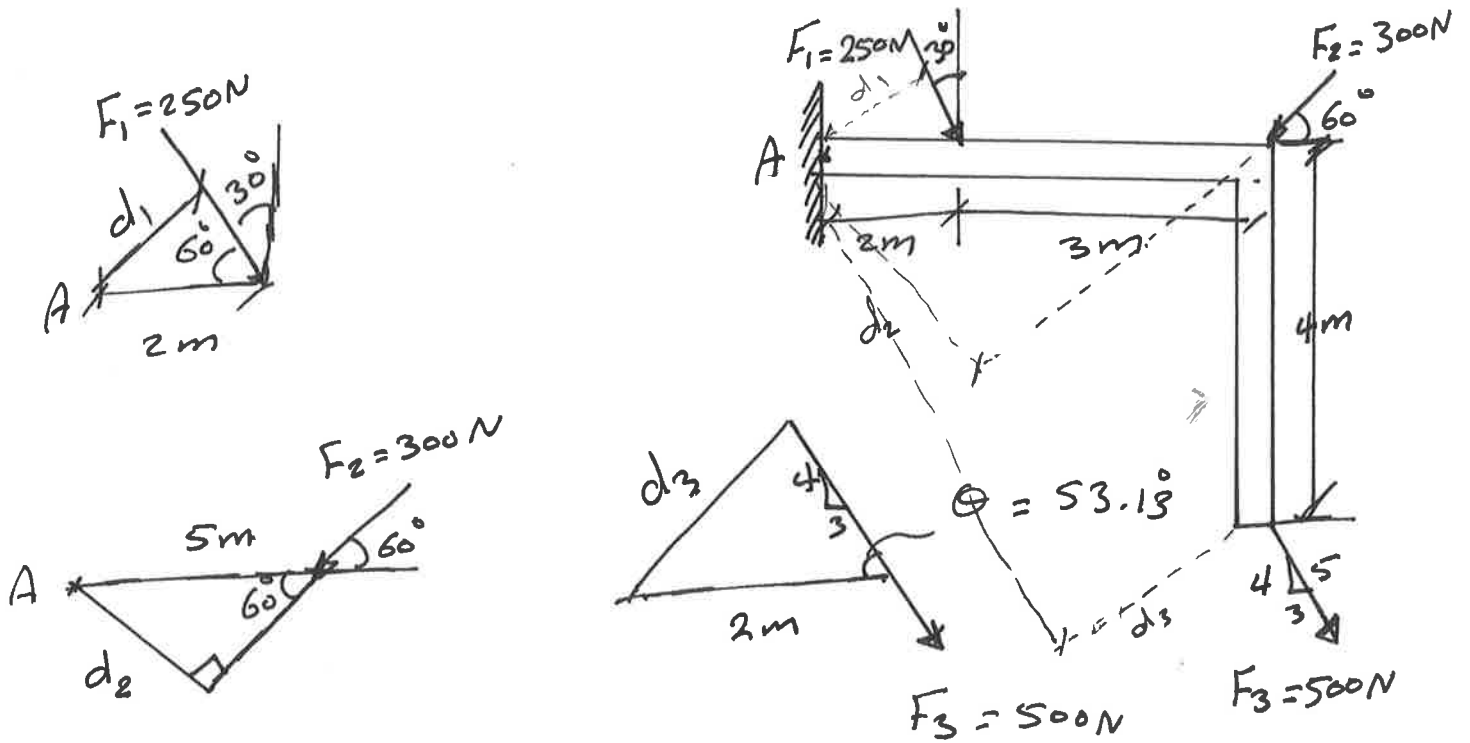


The magnitude of M is :

$$M = F \cdot d$$

where d is the moment arm or perpendicular distance from point O to the line of action of the force F . Units of moment consist of force times distance, e.g. $N \cdot m$ or $lb \cdot ft$.

Ex:- Determine the moment of each of the three forces about point A



The moment arm measured perpendicular to each force from point A is

$$d_1 = 2 \sin 60 = 1.732\text{ m} \quad \rightarrow \sin 60 = \frac{d_1}{2}$$

$$d_2 = 5 \sin 60 = 4.330\text{ m} \quad \rightarrow \sin 60 = \frac{d_2}{5}$$

$$d_3 = 2 \sin 53.13 = 1.60\text{ m} \quad \rightarrow \sin 53.13 = \frac{d_3}{2}$$

Using each force where $M_A = Fd$ we have

$$\begin{aligned} \sum (M_{F_i})_A &= -250(1.732) \\ &= -433\text{ N}\cdot\text{m} = 433\text{ N}\cdot\text{m} \text{ (clockwise)} \end{aligned}$$

$$\begin{aligned} \curvearrowright (M_{F_2})_A &= -300(4.33) = -1299 \text{ N.m} = 1.3 \text{ kN.m} \\ &\quad \text{(clockwise)} \end{aligned}$$

$$\begin{aligned} \curvearrowright (M_{F_3})_A &= -500(1.60) = -800 \text{ N.m} = 800 \text{ N.m} \\ &\quad \text{(clockwise)} \end{aligned}$$

2. Resultant moment of a system of Coplanar Forces:

Resultant moment M_R of the system can be determined by adding the moments of all forces algebraically:

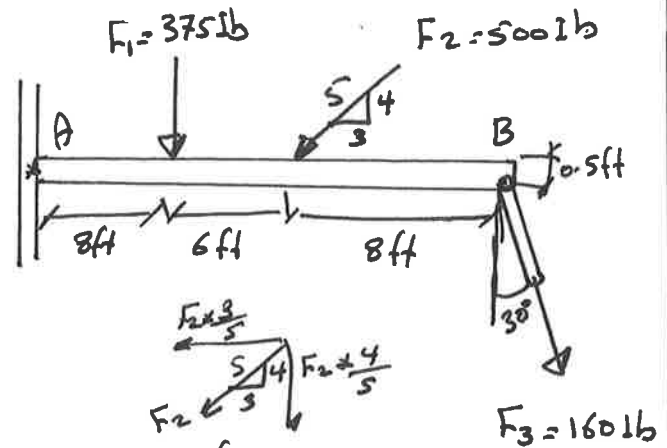
$$\text{or } \curvearrowright \quad \curvearrowleft \quad M_R = \sum Fd$$

The moment of any force will be positive if it rotates the body clockwise, whereas a negative moment rotates the body counterclockwise.

Exo- Determine the moment about point B of each of the three forces acting on the beam.

Solution:

$$\begin{aligned} \overset{+}{\curvearrowright} (M_{F_1})_B &= 375 \times 14 \\ &= 5250 \text{ lb.ft} \\ &\text{(Counterclockwise)} \end{aligned}$$



$$\begin{aligned} \overset{+}{\curvearrowright} (M_{F_2})_B &= 500 \left(\frac{4}{5} \right) \times 8 = 3200 \text{ lb.ft} \\ &\text{(Counterclockwise)} \end{aligned}$$

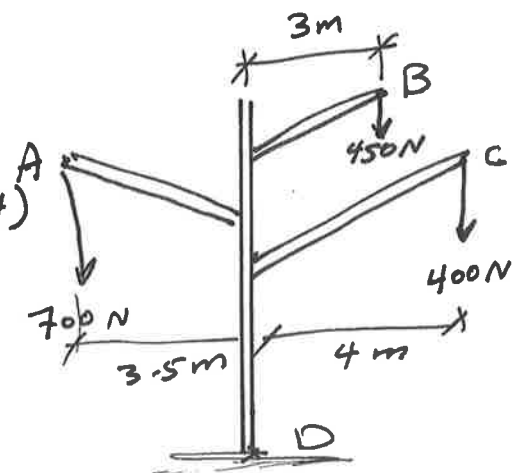
$$\begin{aligned} \overset{+}{\curvearrowright} (M_{F_3})_B &= 160 \sin 30^\circ (0.5) - 160 \cos 30^\circ (0) \\ &= 40 \text{ lb.ft} \text{ (counterclockwise)} \end{aligned}$$

Exo- For the power pole shown, determine the resultant moment about base D. Then determine the resultant moment if line A removed.

Solution

$$\overset{+}{\curvearrowright} M_{R_D} = \sum Fd$$

$$\begin{aligned} M_{R_D} &= 700(3.5) - 450(3) - 400(4) \\ &= -500 \text{ N.m} = 500 \text{ N.m} \\ &\text{(clockwise)} \end{aligned}$$



When the cable A is removed it will create the greatest moment at point D

$$\begin{aligned} \sum (M_{R_D}) &= \sum Fd \\ &= -450(3) - 400(4) \\ &= -2950 \text{ N.m (clockwise).} \end{aligned}$$

Exo:- Determine the moment of force F with respect to the vertical line **ed**

Solution

the length of AB is

$$\sqrt{4^2 + 5^2 + 7^2} = 9.49 \text{ cm}$$

$$\therefore \text{the scale for } F = \frac{400}{9.49}$$

$$F = 42.2 \text{ N/cm}$$

since $F_y \parallel ed$,
 therefore it has no moment with respect to **ed**.

$$F_x = 42.2(7) = 295 \text{ N} \leftarrow$$

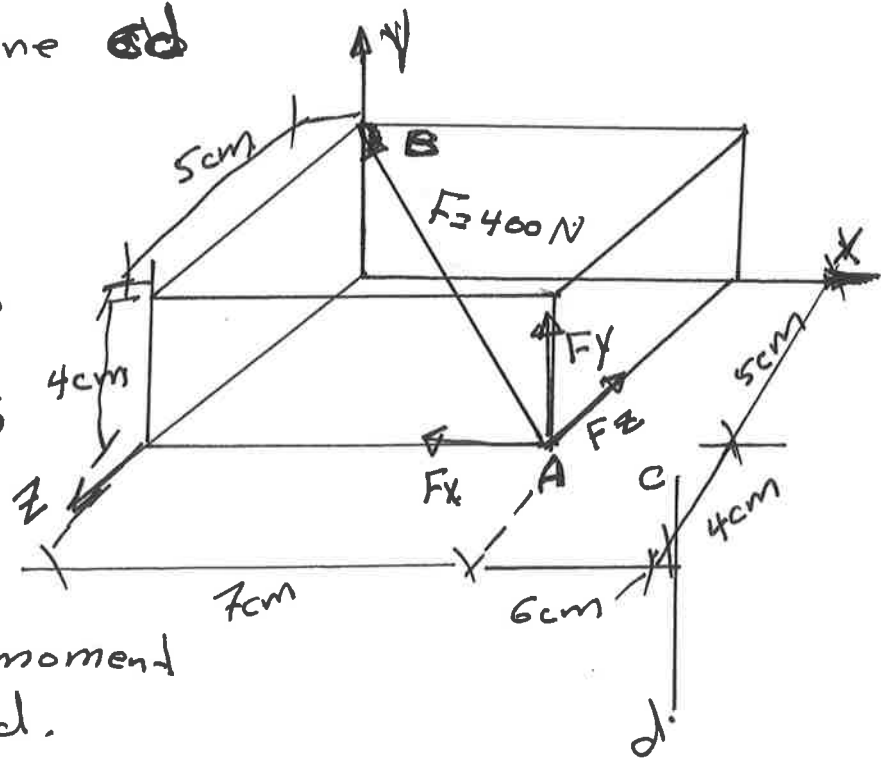
$$F_x = F \cos \theta_x \Rightarrow 400 \times \frac{7}{9.49}$$

$$F_z = 42.2(5) = 211 \nearrow$$

$$\begin{aligned} \sum M_{ed} &= -295(4) + 211(6) \\ &= +86 \end{aligned}$$

$$\therefore M_{ed} = 86 \text{ N.c}$$

(36)



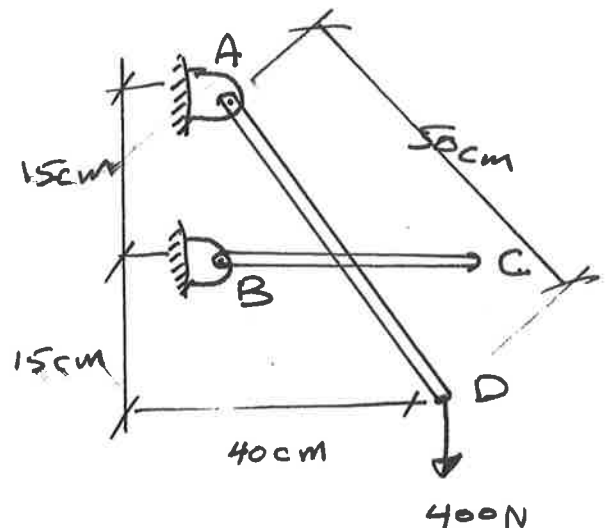
Problems

1-25 Determine the moment of the force with respect to the point A for each system shown in fig.

Solution

a)

$$\begin{aligned}
 \downarrow + \quad M_A &= 400 \times 40 \\
 &= 16000 \text{ N.cm}
 \end{aligned}$$



b)

$$F_x = 1000 \times \frac{4}{5} = 800 \text{ N} \rightarrow$$

$$F_y = 1000 \times \frac{3}{5} = 600 \text{ N} \uparrow$$

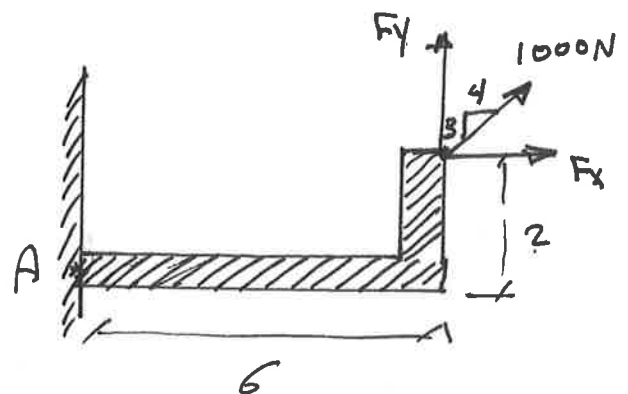
$\downarrow +$

$$\Sigma M_A = 800 \times 2 - 600 \times 6$$

$$= 1600 - 3600$$

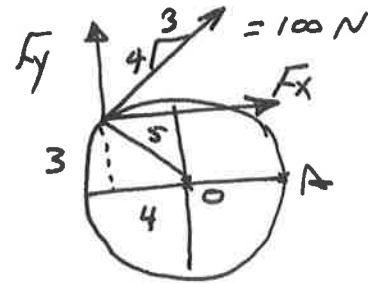
$$= -2000$$

$$= + 2000 \text{ N.cm}$$



(c) $F_x = 100 \times \frac{3}{5} = 60 \text{ N} \rightarrow$

$F_y = 100 \times \frac{4}{5} = 80 \text{ N} \uparrow$

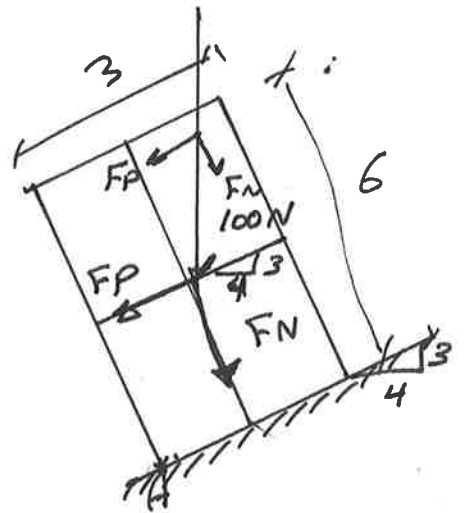


$\sum M_A = 60 \times 3 + 80(4+5)$
 $= 180 + 720 = 900 \text{ N.cm}$

1-26 Determine the moment of the force with respect to point A

(a) $F_P = 100 \times \frac{3}{5} = 60 \text{ N}$

$F_N = 100 \times \frac{4}{5} = 80 \text{ N}$



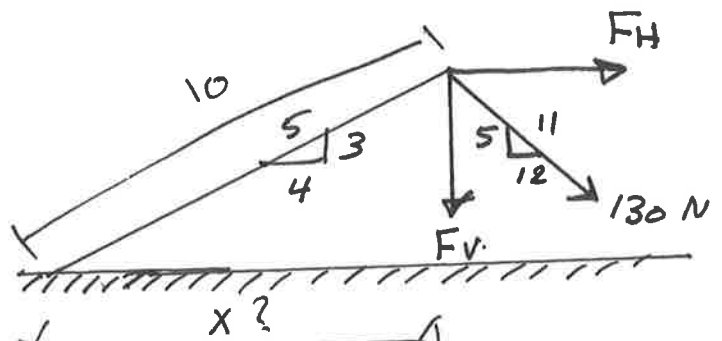
$\sum M_A = 80(1.5) - 60(3)$
 $= 120 - 180 = -60$
 $= 60 \text{ N.cm}$

(b) $F_H = 130 \times \frac{12}{13}$

$= 120 \text{ N}$

$F_V = 130 \times \frac{5}{13}$

$= 50 \text{ N}$



$\sum M_A = 50(8) + 120(6)$
 $= 1120 \text{ N.cm}$

$\frac{4}{5} = \frac{x}{10} \Rightarrow x = 8$

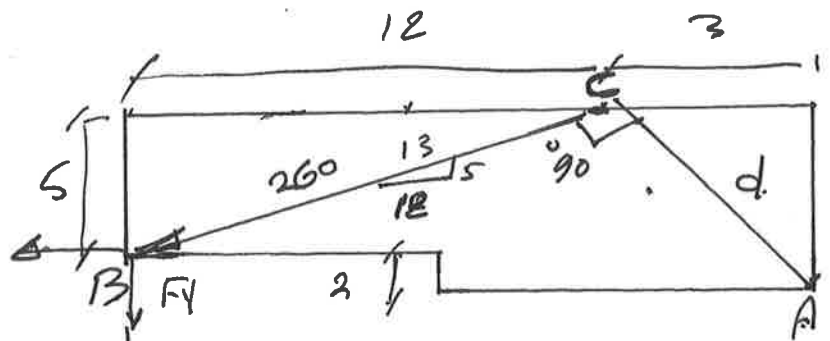
1-28 (a) Determine the moment of the 260 N force at point A - (1) if resolved at B (2) if resolved at C

(b) Determine the \perp distance from (f) to point A By using the principle of moments.

(a)

(1) $F_x = 260 \times \frac{12}{13}$
 $= 240 \text{ N} \leftarrow$

$F_y = 260 \times \frac{5}{13} = 100 \text{ N} \downarrow$



(2) $\sum M_A = 240(2) + 100(15)$
 $= 480 + 1500 = 1980 \text{ N}\cdot\text{cm}$

(2) $\sum M_A = 240(7) + 100(3)$
 $= 1680 + 300 = 1980$

(b) The moment of force = \sum moment of its components

$260(d) = 1980$

$\therefore d = \frac{1980}{260} = 7.61 \text{ cm}$

1-34 (a) Determine the moment of the forces with respect to a-axis.

(b) Determine the \perp distance from the 130 N force to the a-axis

$$L = \sqrt{5^2 + 12^2 + 4^2}$$

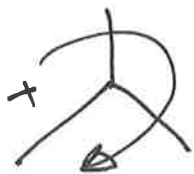
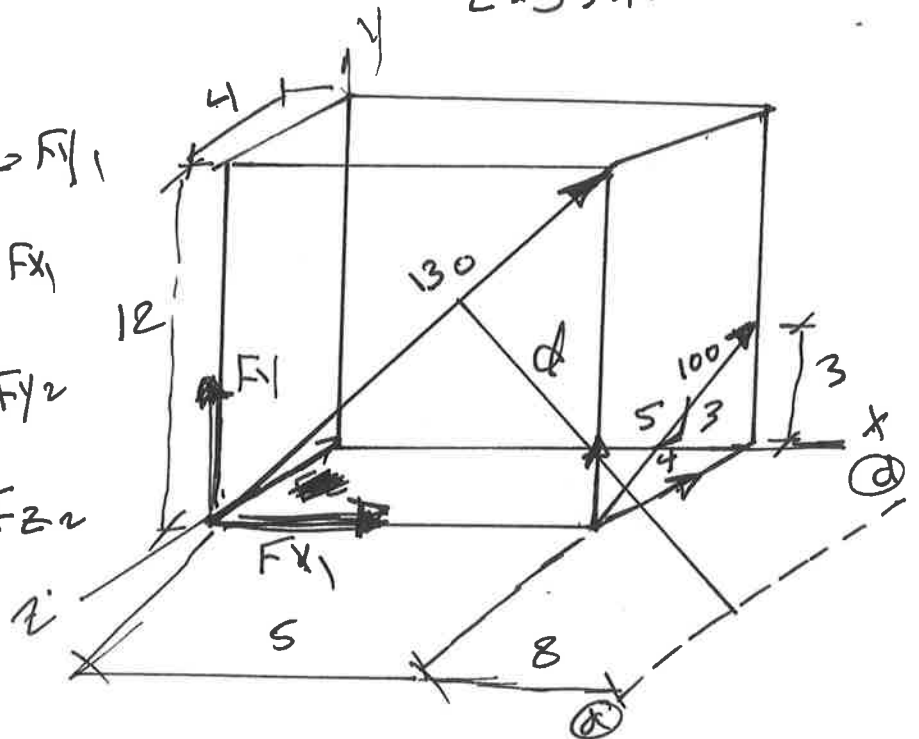
(a)

$$130 \times \frac{12}{13} = 120 \text{ N} = F_{y1}$$

$$130 \times \frac{5}{13} = 50 \text{ N} = F_{x1}$$

$$100 \times \frac{3}{5} = 60 \text{ N} = F_{y2}$$

$$100 \times \frac{4}{5} = 80 \text{ N} = F_{z2}$$



$$\begin{aligned} \sum M_{a-a} &= 120(13) + 60(8) \\ &= 1560 + 480 \\ &= 2040 \text{ N}\cdot\text{cm} \end{aligned}$$

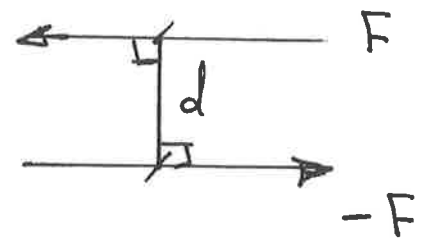
(b) mom. of a force = $\sum M$ of it component
 $130(d) = 1560$

$$\therefore d = \frac{1560}{130} = 12 \text{ cm}$$

Couples

A couple is defined as two parallel forces that may have the same magnitude, and opposite directions, and are separated by a perpendicular distance e

- since the resultant of the two forces is zero, the only effect of a couple is to produce a rotation.



- The moment produced by a couple ~~moment~~ called a couple moment and is

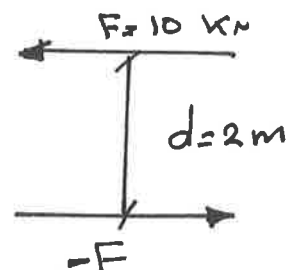
$$M_c = F \cdot d$$

where F is a magnitude of one of the forces and, d is the perpendicular distance between the two forces.

Ex:1 Determine the magnitude of the couple

Solu:

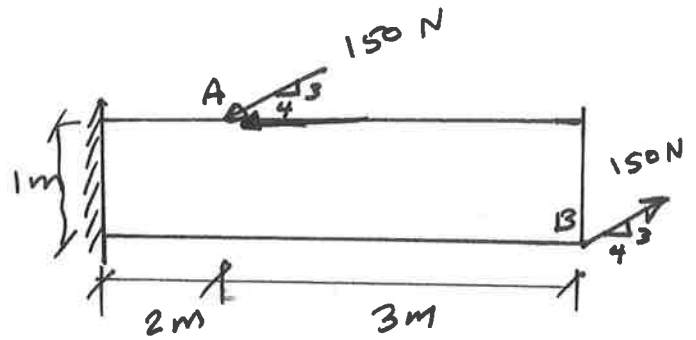
$$\begin{aligned} M_c &= F \cdot d \\ &= 10 * 2 = 20 \text{ kN}\cdot\text{m} \end{aligned}$$



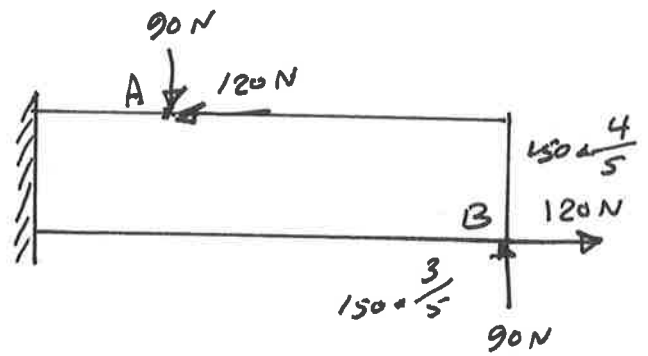
Exs 2 Determine the magnitude of the couple.

Solu:

It is difficult to find the perpendicular distance between the forces.



Thus, we can resolve each force into components.



$$F_x = 150 \left(\frac{4}{5}\right) = 120 \text{ N}$$

$$F_y = 150 \left(\frac{3}{5}\right) = 90 \text{ N}$$

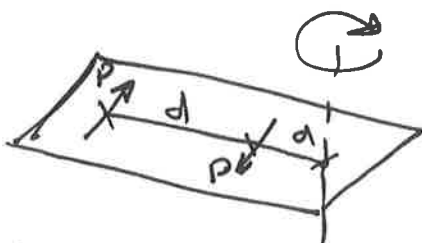
There are two couples,

$$M_1 = 120(1) = 120 \text{ N}\cdot\text{m}$$

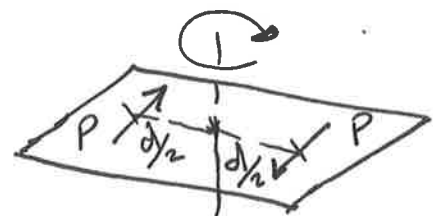
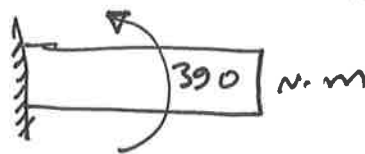
$$M_2 = 90(3) = 270 \text{ N}\cdot\text{m}$$

$$M_c = 120 + 270 = 390 \text{ N}\cdot\text{m}$$

- Notice that the couple moment can be act at any point of the member since the M_c is free vector



$$\begin{aligned} M &= P(d+a) - P(a) \\ &= Pd + Pa - Pa \\ &= Pd \end{aligned}$$



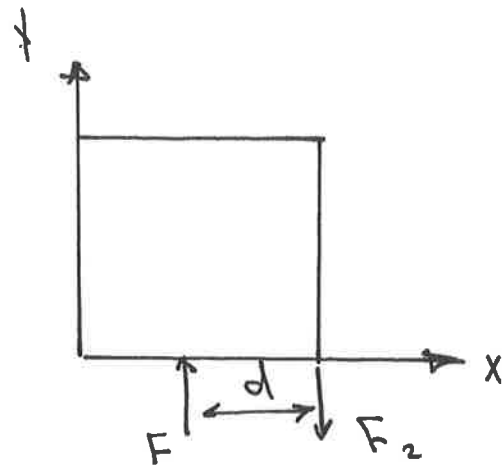
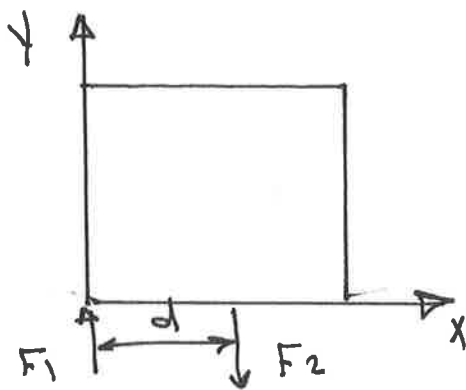
$$\begin{aligned} M &= P\left(\frac{d}{2}\right) + P\left(\frac{d}{2}\right) \\ &= Pd \end{aligned}$$

Engineering Mechanics

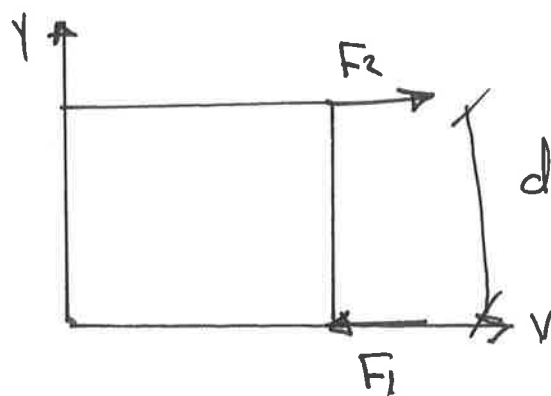
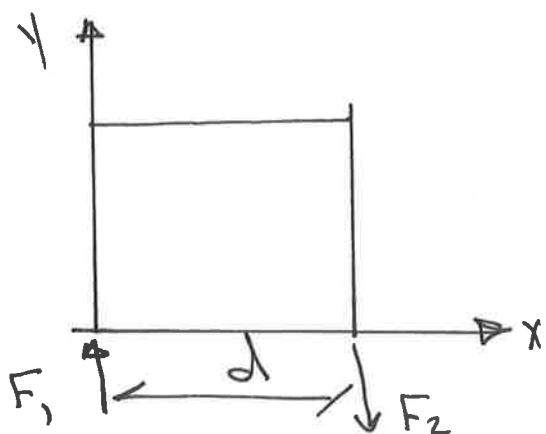
Transformations of A couple:

Transformations of a couple are operations on the couple that do not change any of its characteristics.

- 1- The couple is moved to a parallel position in its plan.



- 2- The couple is rotated in its plane.



Problems:

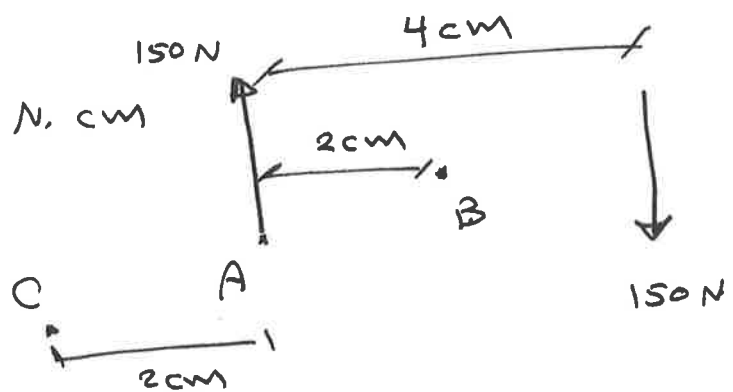
1-39: Determine the moment of the couple in fig. with respect to (a) point A (b) point B (c) point C

Solu:

(a) $M_A = 150 \times 4 = 600 \text{ N}\cdot\text{cm}$

(b) $M_B = 150 \times 2 + 150 \times 2 = 600 \text{ N}\cdot\text{cm}$

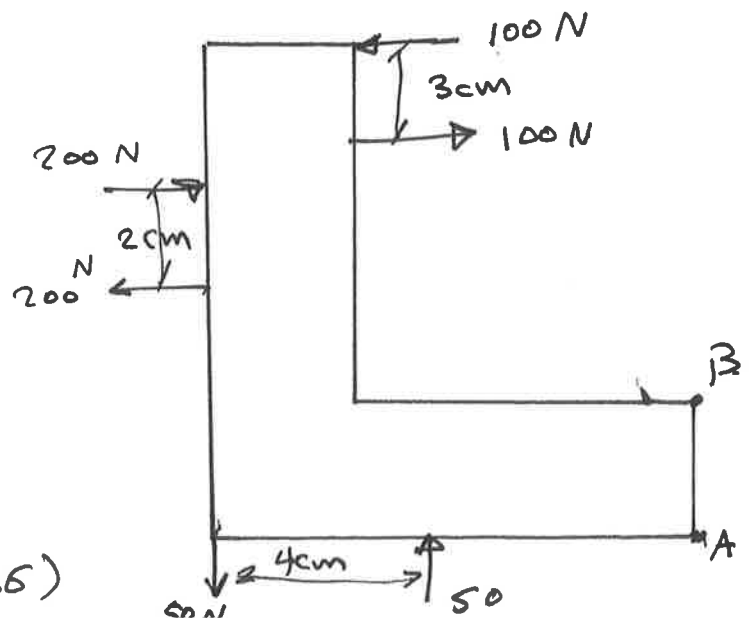
(c) $M_C = 150 \times 6 - 150 \times 2 = 600 \text{ N}\cdot\text{cm}$

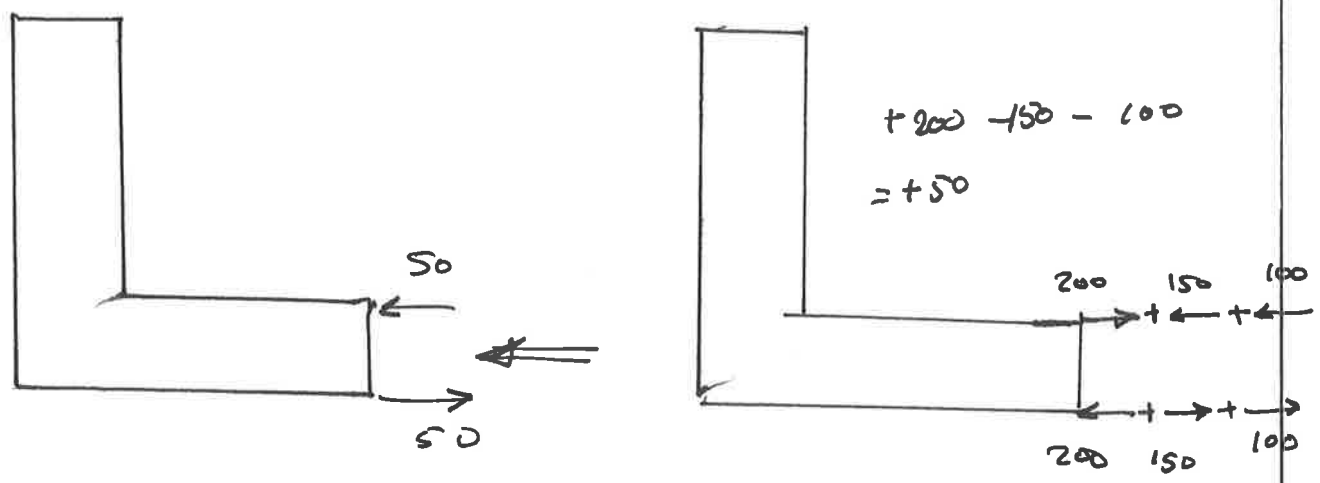


1-40: By using the transformations of a couple, replace the three couple of fig. by one couple with the forces acting horizontally at A and B

Solu:

$M = 200 \times 2 = 400$
 $M = F \cdot d$
 $100 \times 3 = 300$
 $\frac{300}{2} = 150$
 $50 \times 4 = 200$
 $\frac{200}{2} = 100$

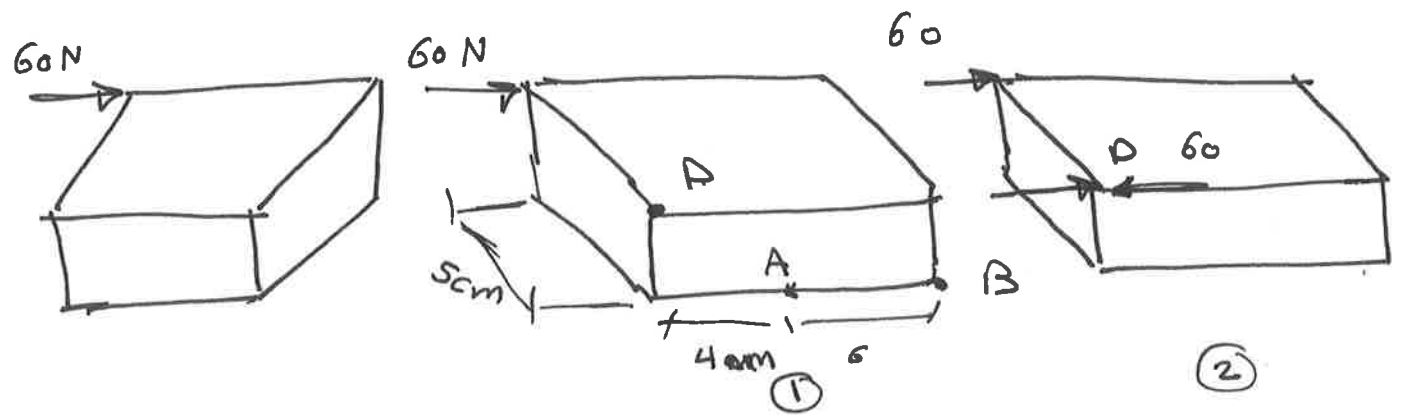


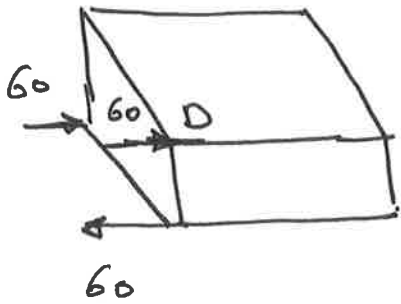


Resolution of a force into a force and couple

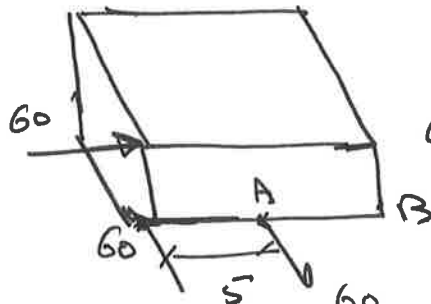
A force may be replaced by an equal parallel force through any other point of the body and a couple. This can be done by adding two equal collinear forces of opposite sense to a force system on a rigid body.

Ex: - Replace the 60 N force shown in the fig. by a force through D and a couple where forces act horizontally through A and B

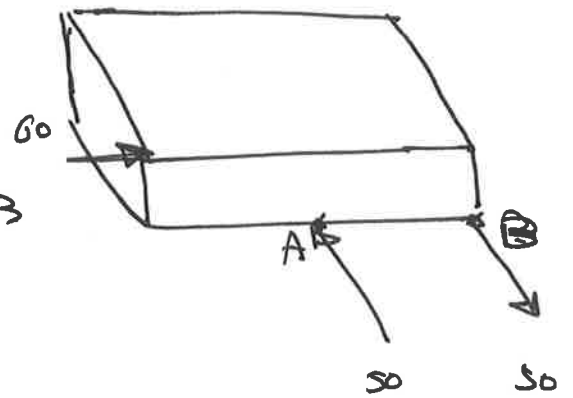




(3)

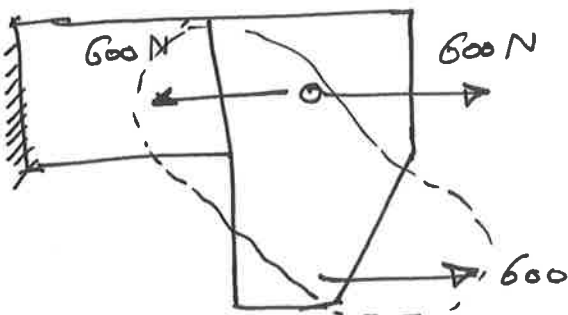


(4)

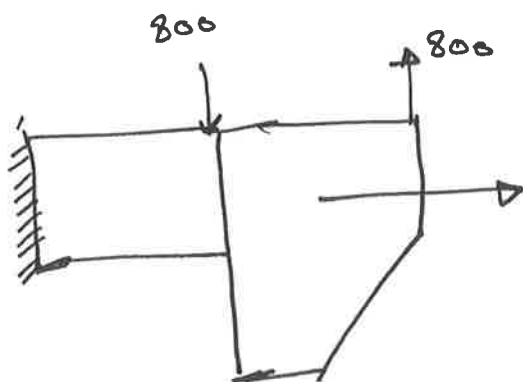
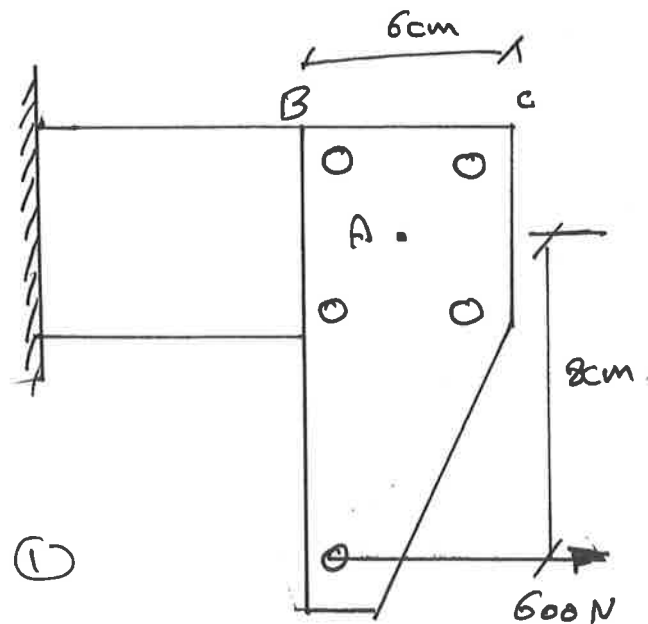


1.44 Replace the 600 N force by a force through A and a couple whose forces act vertically through points B and c.

Solu:

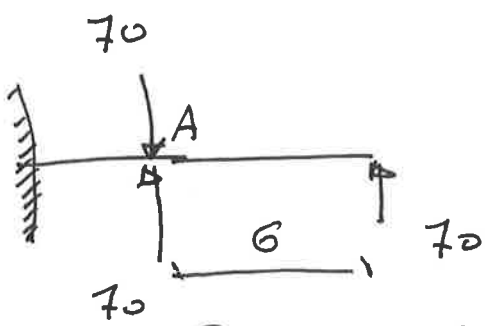


$600 \times 8 = 4800 \quad \text{--- (1)}$

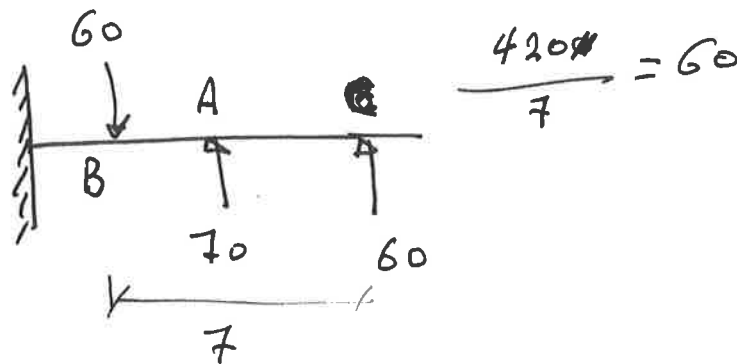
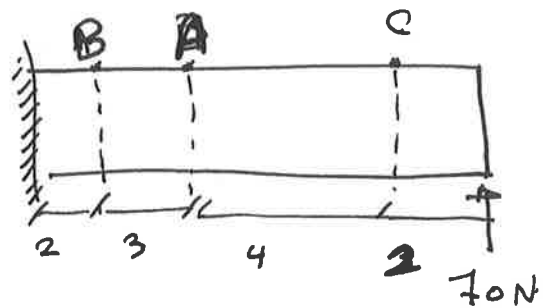


$\frac{4800}{6} = 800 \text{ N} \quad \text{--- (2)}$

1.45 By transformation of a couple replace the 70 N force by a force through A and a couple whose forces act vertically through B and C.



① $70 \times 6 = 420$

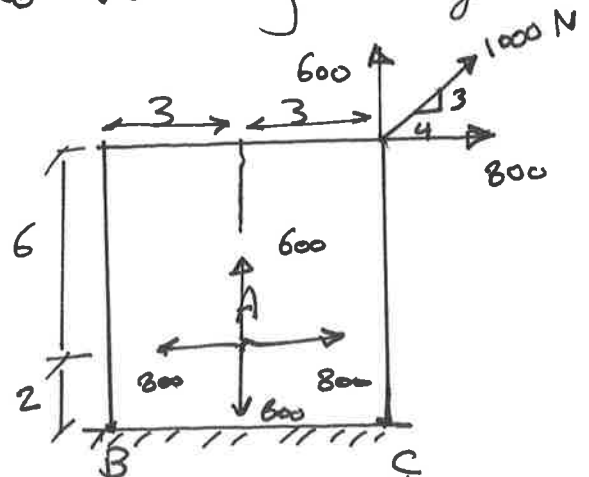


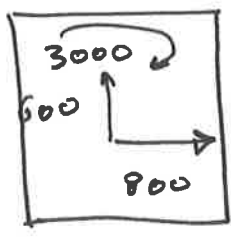
1.48 Replace the 1000 N force by a force through A and couple whose forces act vertically through B and C.

Solu

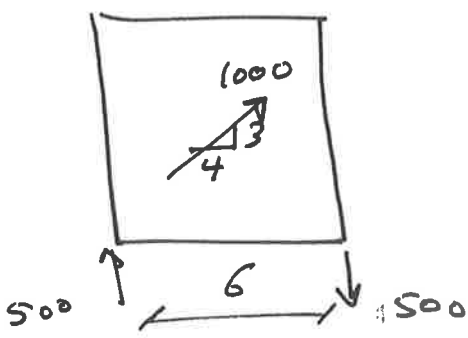
$$\begin{aligned} \sum \text{Couples} &= 800 \times 6 - 600 \times 3 \\ &= 4800 - 1800 \\ &= 3000 \text{ N.cm} \end{aligned}$$

$$\frac{3000}{6} = 500$$



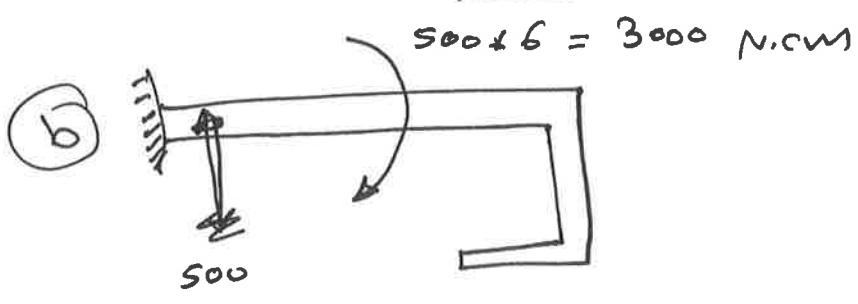
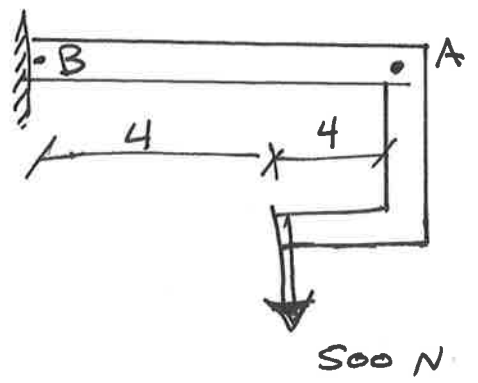
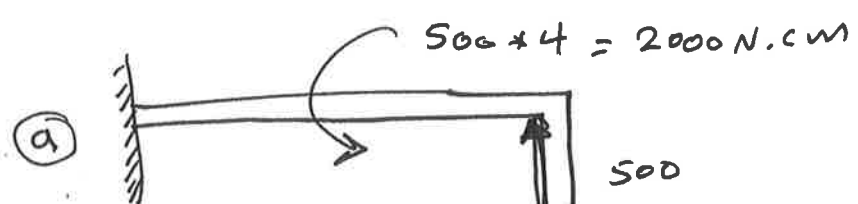


①



②

1-51 Replace the 500 N force by (a) a force through A and a couple (b) a force through B and a couple.



1.53 Replace the two 100 N forces with a single force acting through G and a couple whose forces act vertically at A and B

Soln:

$$L = \sqrt{1^2 + 1^2}$$

$$= \sqrt{2}$$

$$100 = R \cdot \frac{1}{\sqrt{2}}$$

$$R = \frac{100}{\frac{1}{\sqrt{2}}}$$

$$= 141.42 \text{ N}$$

$$141.42 \cdot 3 = 424.26$$

$$424.26 / 2 = 212.13$$

