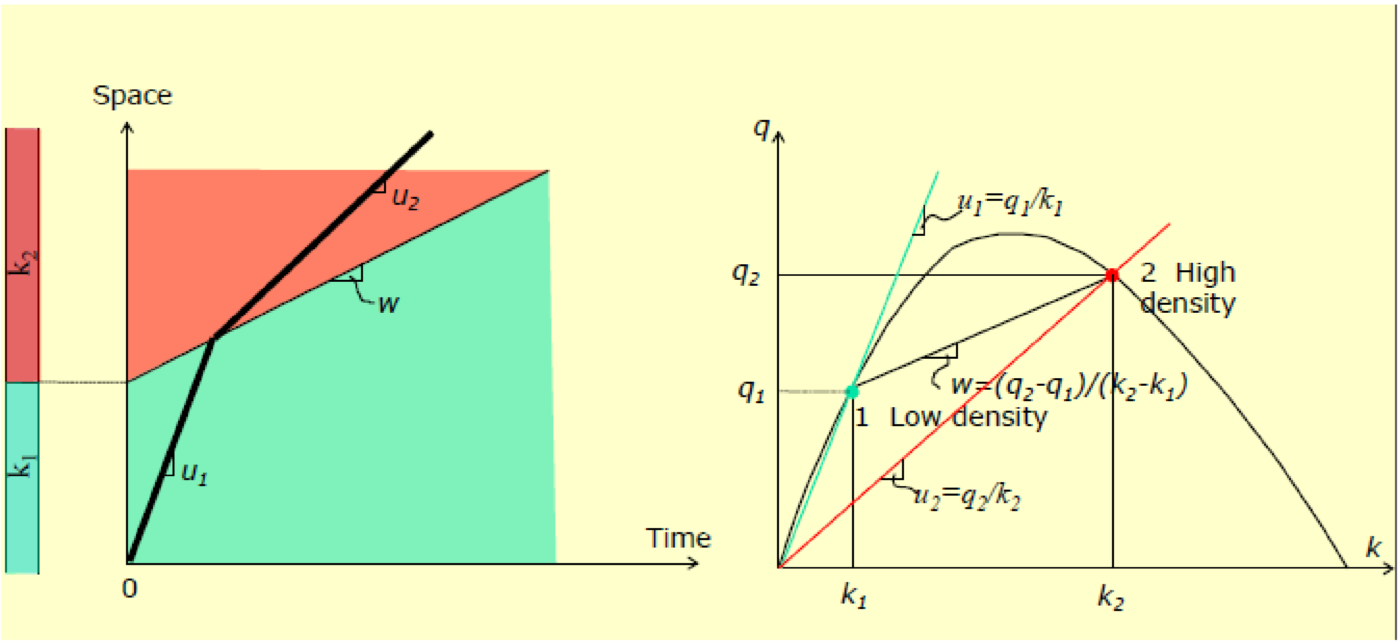
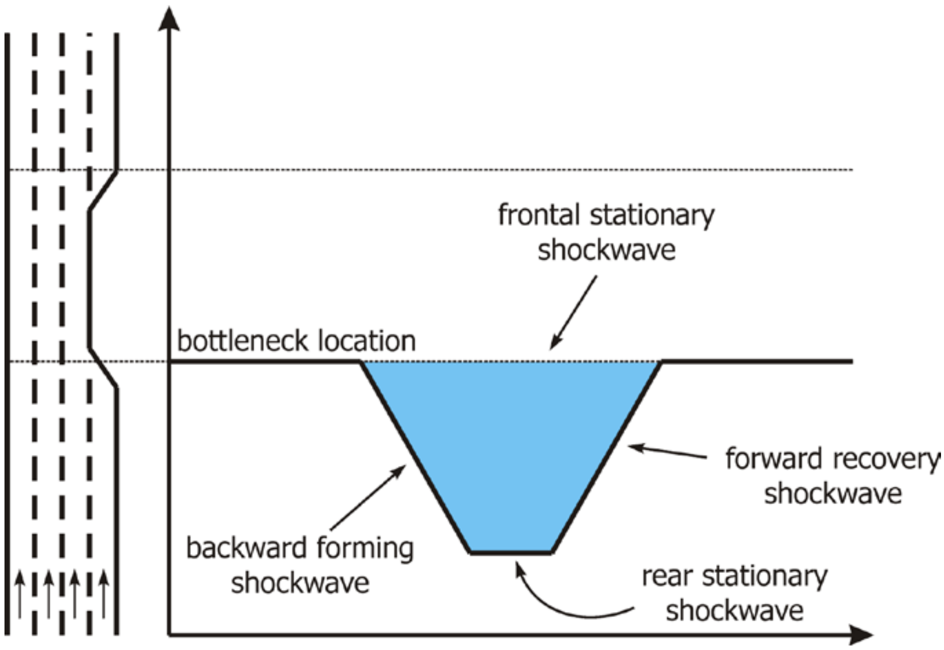
Shockwave Theory

Definitions:

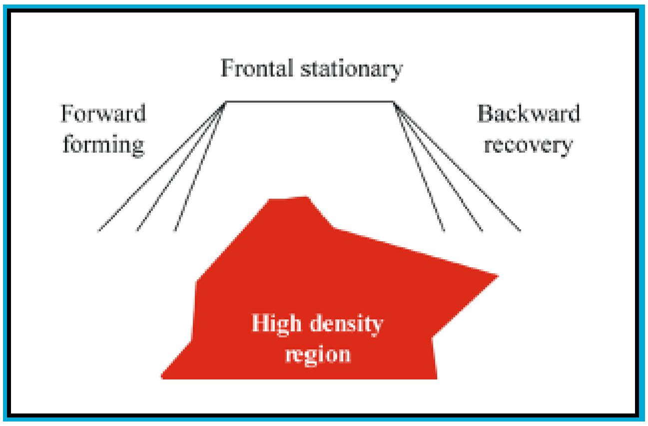
A shockwave describes the boundary between two traffic states that are characterized by different densities, speeds and/or flow rates. Shockwave theory describes the dynamics of shockwaves, in other words how the boundary between two traffic states moves in time and space.



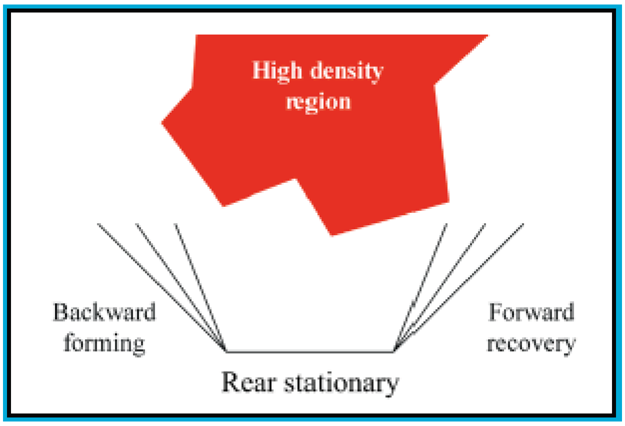
- Forward,
- Backward,
- And stationary shock waves.



Shockwave classification

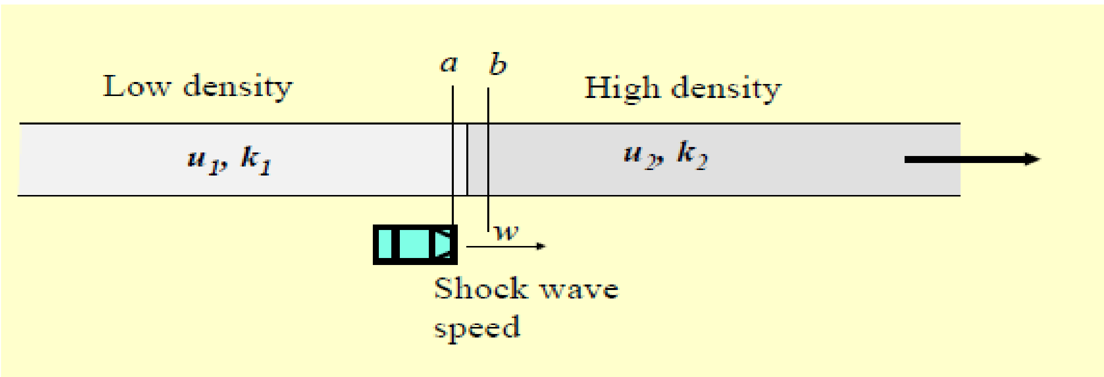


1. **Frontal stationary**: head of a queue in case of stationary / temporary bottleneck.
2. **Forward forming**: moving bottleneck (slow vehicle moving in direction of the flow given limited passing opportunities).
3. **Backward recovery**: dissolving queue in case of stationary or temporary bottleneck (demand greater than supply); forming or dissolving queue for moving bottleneck.



1. **Forward recovery**: removal of temporary bottleneck (e.g. clearance of incident, opening of bridge, signalized intersection).
2. **Backward forming**: forming queue in case of stationary, temporary, or moving bottleneck* (demand greater than supply);
3. **Rear stationary**: tail of queue in case recurrent congestion when demand is approximately equal to the supply.

Suppose that we have two traffic states: states 1 and 2. Let S denote the wave that separates these states. The speed of this shockwave S can be computed by:



$$\omega_{12} = \frac{q_2 - q_1}{k_2 - k_1}$$

In other words, the speed of the shockwave equals the jump in the flow over the wave divided by the jump in the density. This yields a nice graphical interpretation (Figure 1): if we consider the line that connects the two traffic states 1 and 2 in the fundamental diagram, then the slope of this line is exactly the same as the speed of the shock in the time–space plane.

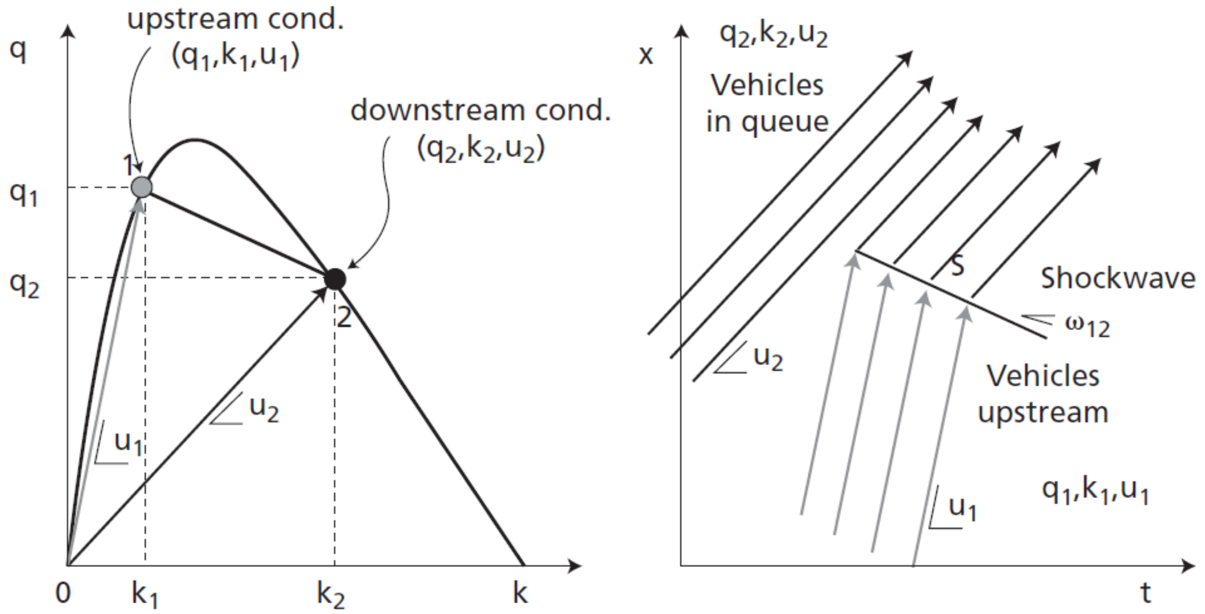
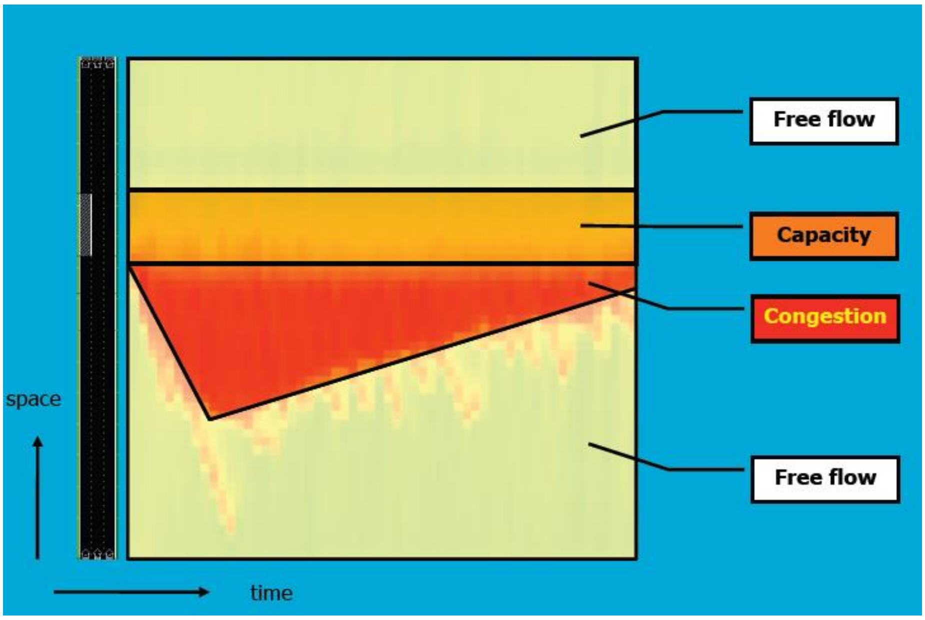


Figure 1. Graphical Interpretation of Shockwave Speed.

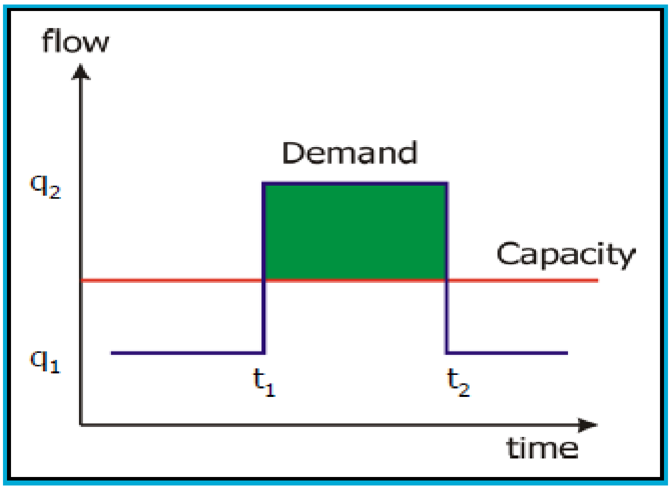
Shockwave theory provides a simple means to predict traffic conditions in time and space. These predictions are largely in line with what can be observed in practice, but they have their limitations:

1. Traffic driving away from congestion does not accelerate smoothly towards the free speed but continues driving at the critical speed.
2. Transition from one state to the other always occurs in jumps, not taking into account the bound acceleration characteristics of real traffic.
3. There is no consideration of hysteresis.
4. There are no spontaneous transitions from one state to the other.
5. Location of congestion occurrence is not in line with reality.



Application of Shockwave

- Temporary over-saturation



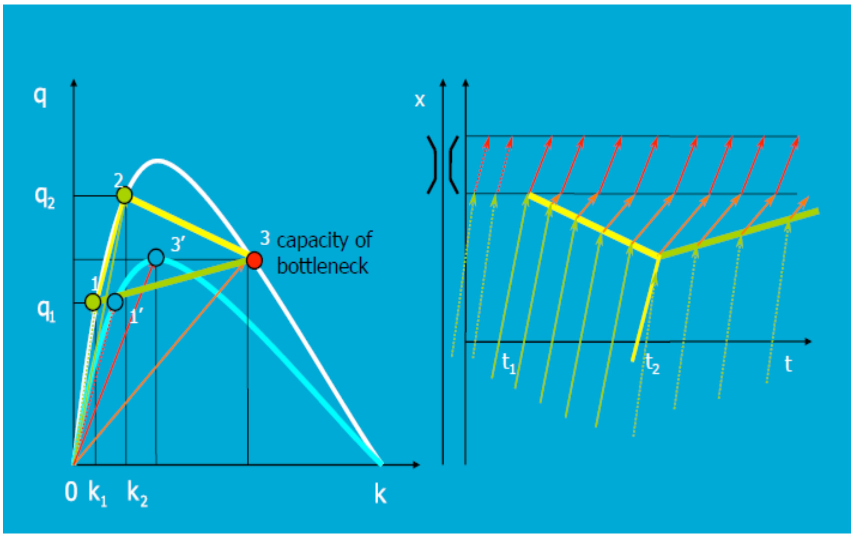
$$q(t) = \begin{cases} q_1 & t < t_1 \text{ or } t \geq t_2 \\ q_2 & t_1 \leq t < t_2 \end{cases}$$

Three simple steps to applying shockwave theory:

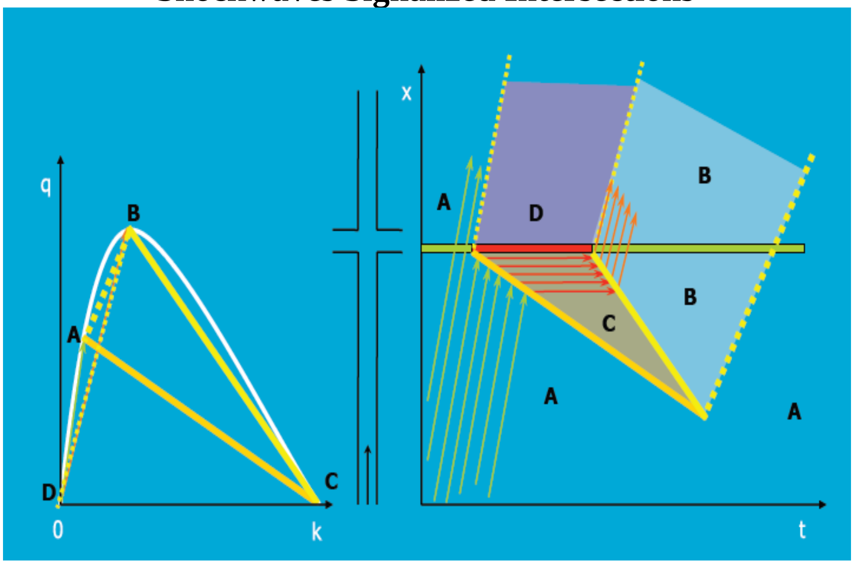
- Determine the Q (k) curve for all locations x.
- Determine the following ‘external conditions’:
  - Initial states (t = t0).
  - ‘Boundary’ states (inflow, outflow restrictions, moving bottleneck).

- Present in the x-t plane and the q-k plane
- Determine the boundaries between the states (=shockwaves) and determine their dynamics.
- Check for any omissions you may have made (are regions with different states separated by a shockwave?).

Shockwaves at Bottleneck



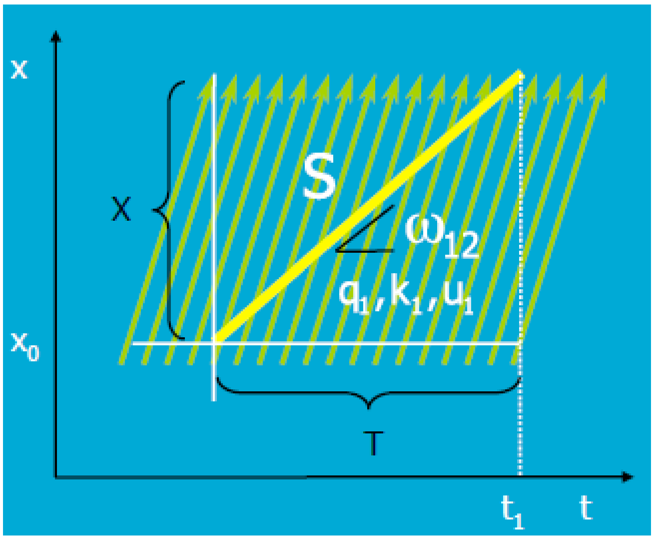
Shockwaves Signalized Intersections



Flow into shockwave

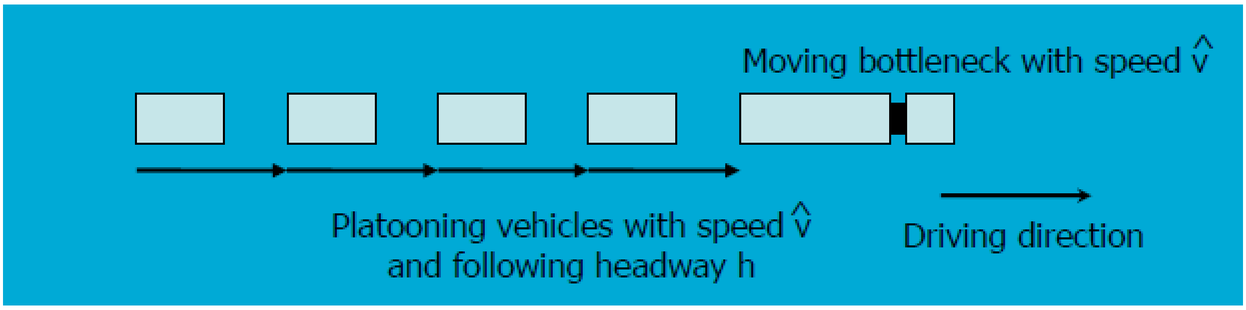
- Consider a shockwave moving with speed, ω_{12}
- Flow into the shockwave = flow observed by moving observer Travelling with speed of shockwave
- Number of vehicles observed on
  S = + Vehicles passing x0 during T - Vehicles on X at t_1

$$q_{in}^{S}T = q_1 T - k_1 X, \quad X = \omega_{12} T$$
$$q_{in}^{S}T = (k_1 u_1 - k_1 \omega_{12})T$$
$$q_{in}^{S} = k_1 (u_1 - \omega_{12})$$

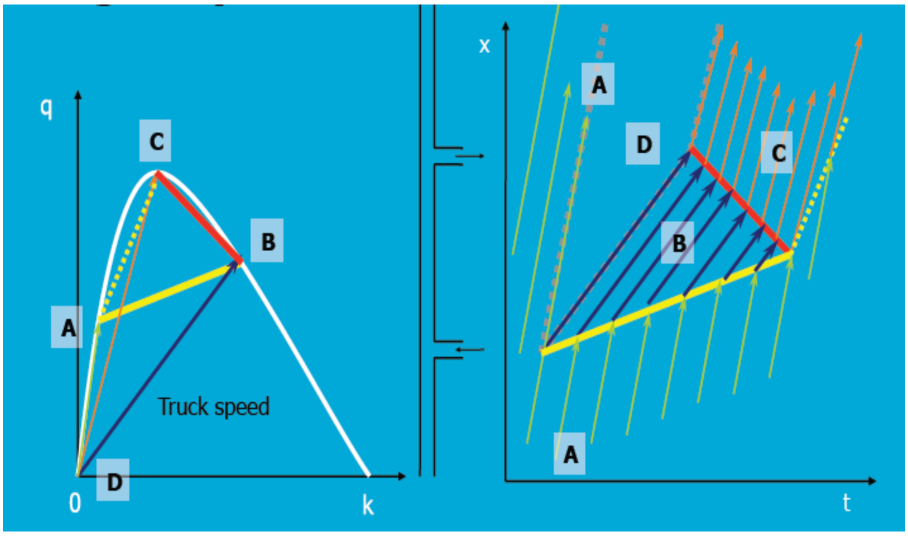


<u>Moving bottlenecks</u>

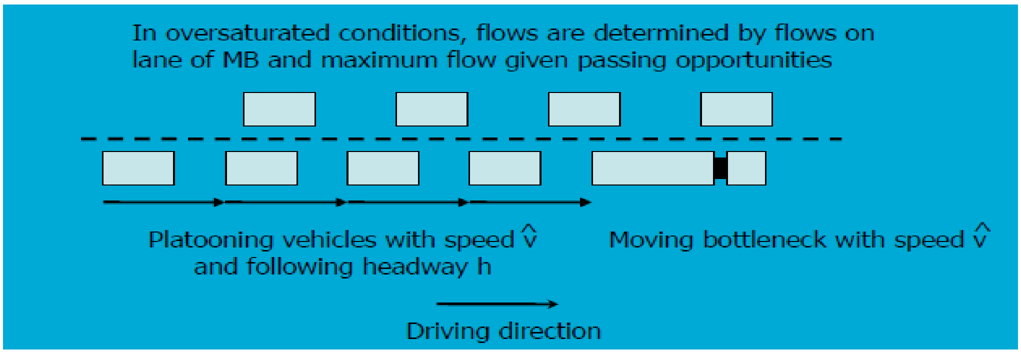
Fundamental diagram at the MB

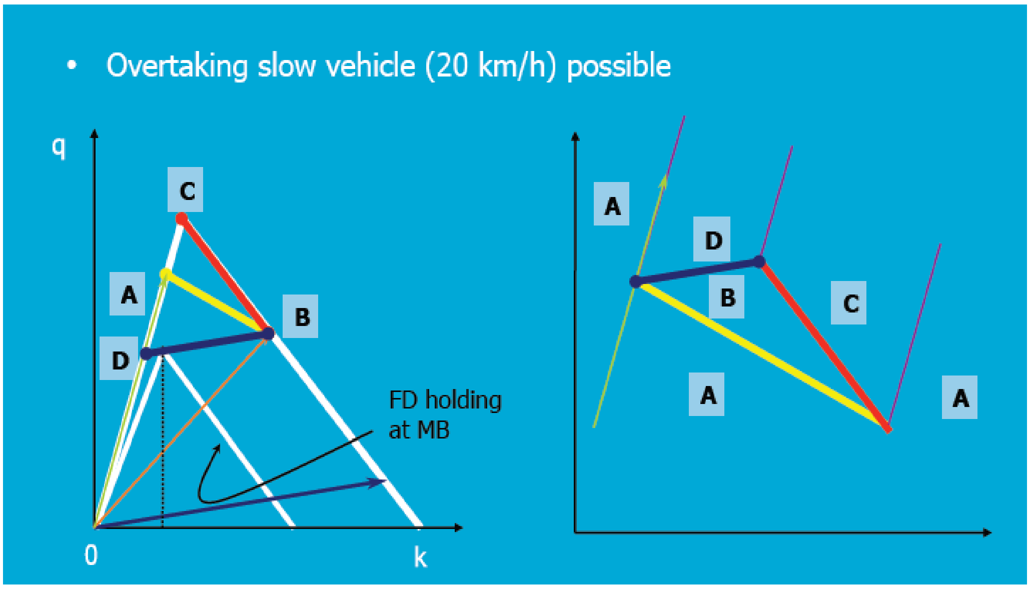


- Assume no overtaking opportunities.
- Headway h is determined by speed of MB.
- Since q = 1/h, the flow upstream of the MB is determined by the speed of the MB.
- Upstream flow can be determined from FD easily.



- Now assume overtaking possibilities at the MB.
- Example: two-lane motorway (one lane operating at capacity).





Queuing Theory

The most straightforward approach to model traffic dynamics is probably the use of queuing theory. In queuing theory we keep track of the number of vehicles in a queue (*n*). A queue starts whenever the flow to a bottleneck is larger than the bottleneck capacity, where the cars form a virtual queue. The outflow of the queue is given by the infrastructure (it is the outflow capacity of the bottleneck, given by *C*), whereas the inflow is the flow towards the bottleneck (*q*) as given by the traffic model. In an equation, this is written as:

$$dn = q(t)\,dt - C(t)\,dt$$

The number of vehicles in the queue (*n*; *dn* stands for the change in the number of vehicles in the queue) will evolve in this way until the queue has completely disappeared. Note that both the inflow and the capacity are time dependent in the description. For the inflow, this is due to the random distribution pattern of the arrival of the vehicles. Vehicles can arrive in platoons or there can be large gaps in between two vehicles. The capacity is also fluctuating.

On the one hand, there are vehicle-to-vehicle fluctuations. For instance, some drivers have a shorter reaction time, hence a shorter headway leading to a higher capacity. On the other hand, on a larger scale, the capacities will also depend on road or weather conditions (e.g. wet roads, night-time).

Figure 2 shows how the number of vehicles in the queue, n, fluctuates with time for a given inflow and outflow curve. The disadvantage of the queuing theory is that the queues have no spatial dimension, and they do not have a proper length either (they do not occupy space). Other models, which overcome these problems, are discussed below.

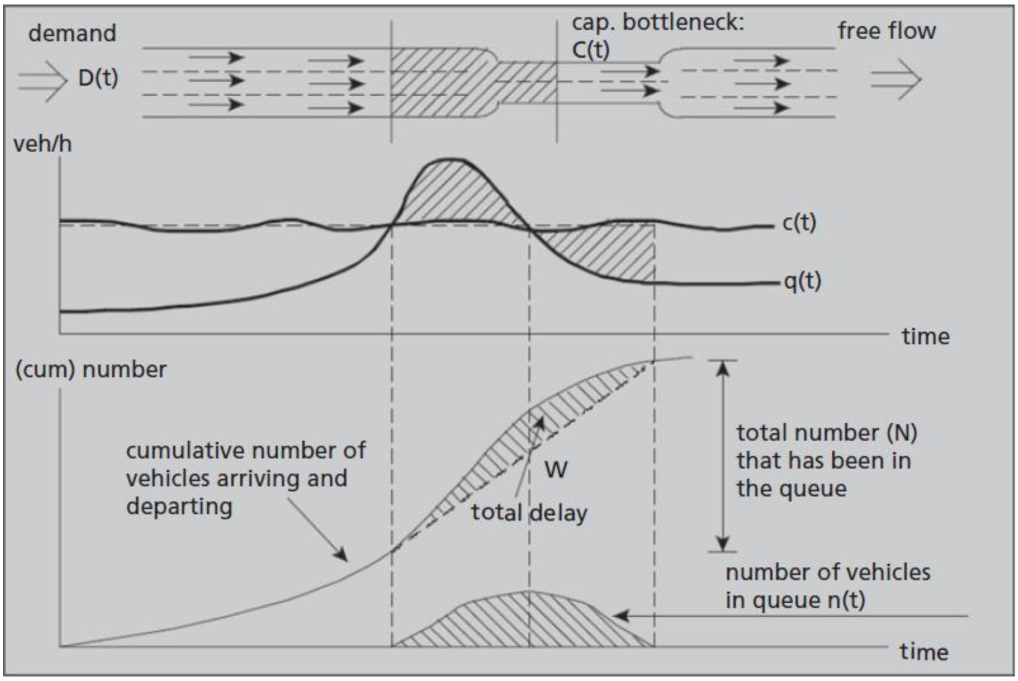
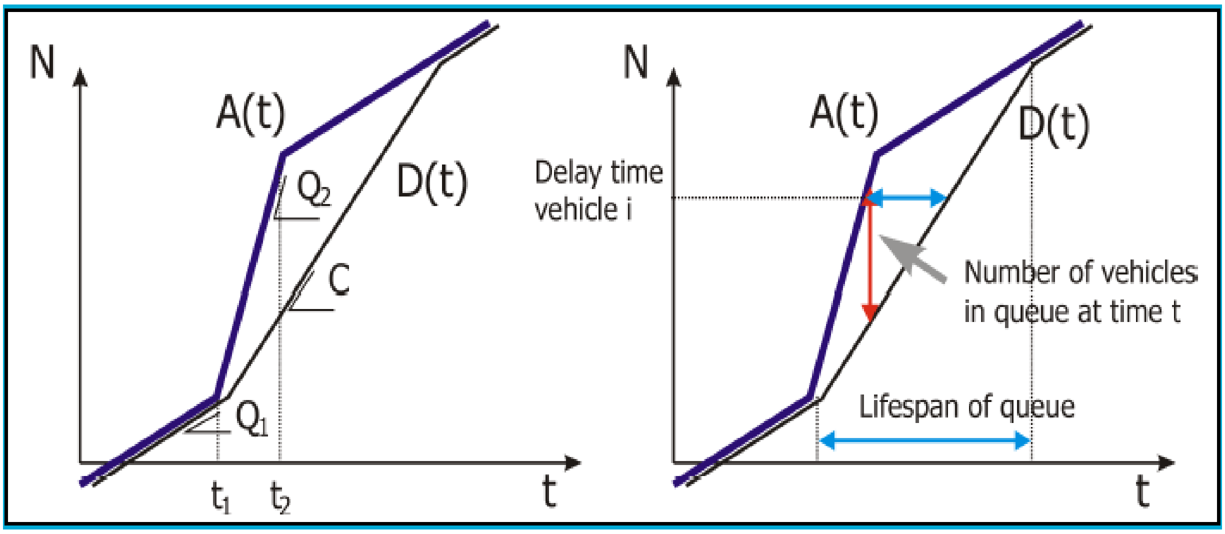


Figure 2 Functioning of Queuing Theory.

Queuing Models



Computations queuing model

- Maximum length of the queue

• From graph: occurs at t2:

$$Max = A(t_2) - D(t_2) = \int_{t_1}^{t_2} (Q_2 - C)\,dt = (t_2 - t_1)(Q_2 - C)$$

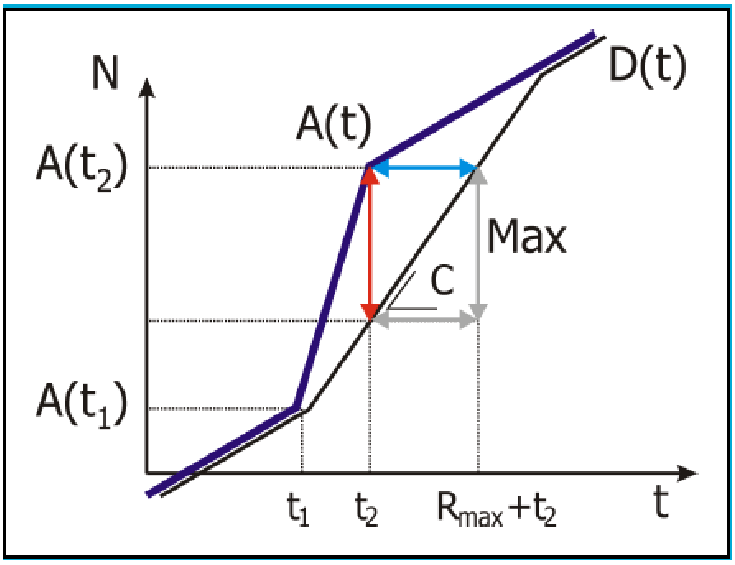
• Number of vehicles in the queue at t2

• Maximum delay (at time t2)

$$D^{-1}[A(t_2)] - A^{-1}[A(t_2)] = D^{-1}[A(t_2)] - t_2$$

From figure:

$$C \cdot R_{max} = Max \quad \rightarrow \quad R_{max} = \frac{Max}{C} = (t_2 - t_1) \cdot \left(\frac{Q_2 - C}{C} \right)$$



• Duration of congestion
• Intersection of arrival and departure curve

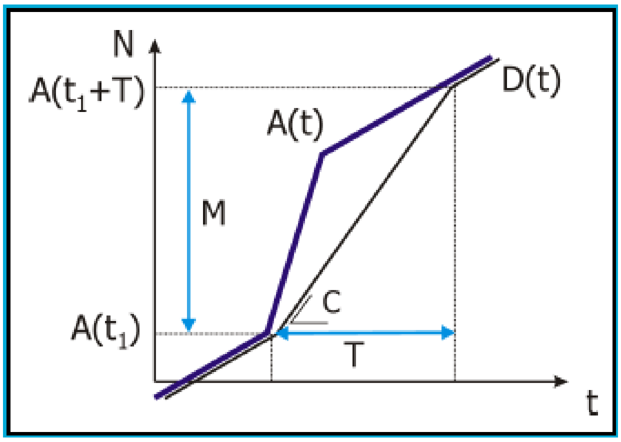
$$A(t) = A(t_2) + Q_1 (t - t_2) \quad \text{for} \quad t > t_2$$
$$D(t) = A(t_1) + C \cdot (t - t_1) \quad \text{for} \quad t > t_1$$

Yields:

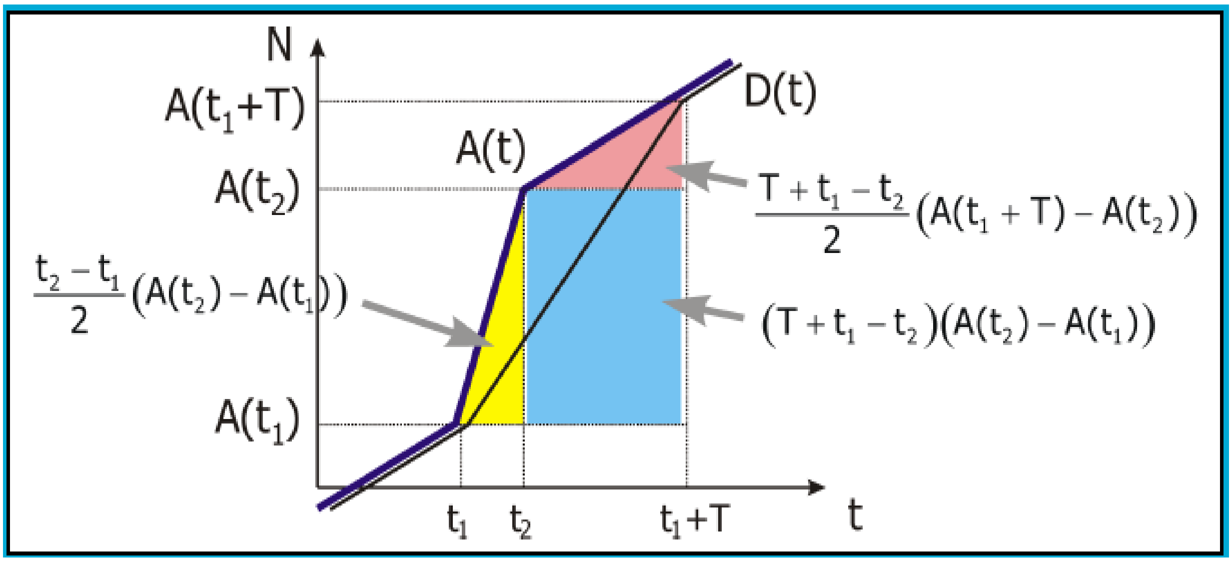
$$A(t_2) + Q_1 (T - t_2) = A(t_1) + C \cdot (T - t_1)$$
$$\Leftrightarrow$$
$$T = \frac{A(t_2) - A(t_1) + Ct_1 - Q_1 t_2}{C - Q_1} = \frac{(t_2 - t_1) Q_2 + Ct_1 - Q_1 t_2}{C - Q_1}$$

• Total number of vehicles that have been in the queue

$$M = C \times T$$
$$= C \times \frac{(Q_2 - Q_1) t_2 - (Q_2 - C) t_1}{C - Q_1}$$



• Total collective loss



• Total collective loss
• Sum of three areas

$$\frac{1}{2}(T-(t_2-t_1))C\cdot T+\frac{1}{2}T(Q_2(t_2-t_1))$$

• Subtract fourth area

$$-\frac{1}{2}C\cdot T\cdot T$$

Yields total collective loss

$$R=\frac{t_2-t_1}{2}(Q_2-C)T=\frac{Max}{2}T \qquad R=\frac{Max}{2}T=\frac{Max}{2}\frac{M}{C}=\frac{M}{2}R_{max}$$

• Mean loss time

$$R_{mean}=\frac{R}{M}=\frac{1}{2}R_{max}$$