Engineering analysis

and numerical.

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References :-

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2- "Advanced engineering mathematics" ; C. Ray Wyline , Lous C. Barrett , fifth edition 1982 .

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3. "Numerical methods" ; P. Kandasamy , 2008 .

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داس

Contents:-

chapter one

Determinants and matrices.

Chapter 1 :-

Intoduction of determinants and matrices.

Consider the following set of equations:-

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
\vdots \qquad\quad \vdots \qquad\qquad & \vdots \\
a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n
\end{aligned}
$$

تدوين

In matrix notation :-

$$
\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}
=
\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}
\Rightarrow AX = b
$$

where $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{12} \\ a_{21} & a_{22} & \cdots & a_{21} \\ \vdots & & & \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$

* If $b \neq 0$ they are called non-homogeneous, if $b = 0$ the equation is called homogeneous.

(case i)

If $b \neq 0$ and $|A| \neq 0$ then we have a unique solution $X = A^{-1} b$

Determinants:-

If A is a square matrix then det. A or $|A|$ is a number calculated from A and found as follows.

$$A \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ then } |A| = ad - bc$$

$$\text{If } A = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 3 \\ 3 & 0 & 5 \end{bmatrix} \end{array} \Rightarrow |A| = 2\begin{vmatrix} 5 & 3 \\ 0 & 5 \end{vmatrix} - 3\begin{vmatrix} 1 & 3 \\ 3 & 5 \end{vmatrix} + 4\begin{vmatrix} 1 & 5 \\ 3 & 0 \end{vmatrix}$$

$$\Rightarrow |A| = 2(25) - 3(5-9) + 4(-15) = 50 + 12 - 60 = \underline{\underline{2}}$$

H.w If $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 5 & 6 \\ 7 & -8 & 9 \end{bmatrix}$, find det.(A)?

ans: 240

Ex

Evaluate the determinant $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 0 \\ 2 & 0 & 1 & 3 \\ 1 & 2 & 1 & 0 \end{bmatrix}$

Solution

$$\begin{array}{c} \begin{matrix} + & - & + & - \end{matrix} \\ \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 0 \\ 2 & 0 & 1 & 3 \\ 1 & 2 & 1 & 0 \end{bmatrix} \end{array} = 0 \begin{bmatrix} 0 & 2 & 0 \\ 0 & 1 & 3 \\ 2 & 1 & 0 \end{bmatrix} - 1 \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix}$$

$$+2 \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 3 \\ 1 & 2 & 0 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= 0 - \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix} \end{array} + 2 \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 3 \\ 1 & 2 & 0 \end{bmatrix} \end{array} - 3 \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix} \end{array}$$

$$= -\left[(-3) - 2(-3)\right] + 2\left[-6\right] - 3\left[(-2) + 2(4)\right]$$

$$= -3 - 12 - 18 = \underline{\underline{-33}}$$

Solution of Linear equations :-

(A) Direct method :-

1. Cramer rule.
2. matrix inversion method. في المرحلة الاولى

حذف

3. Gauss-elimination method. [Row- operater]
4. Gauss-Jordan elimination method.
5. Triangularization method or factorization method or decomposition method.

(B) Indirect method :-

1. Jacobi method of iteration.
2. Gauss-seidel method of iteration.

Ⓐ Direct method :-

✱ الطريقة الاولى Cramer والثانية matrix inversion تم شرحها بالتفصيل في المرحلة الاولى .

③ Gauss-elimination method (Row operater method) :-

شروط الطريقة :-
① المصفوفة مربعة
② $|A| \neq 0$

① For the matrix $[a_{ij}/b_i]$ $i = 1, 2, ---n$, $j = 1, 2, ---m$

② Multiply the first row R_1 by $\frac{-a_{i1}}{a_{11}}$ and add the ith row for $i = 1, 2, 3 ---n$

③ Repeat the step (2) for second row (R_2) to $(n-1)$ th row.

④ We will get $\boxed{\text{upper}}$ (العليا) triangular matrix. $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$

⑤ Solve for X_n from the n.th equation and solve for $X_{n-1}, ---, X_1$

Ex/ Use Gauss elimination method to solve the following linear equation :-

$$2X_1 + 4X_2 - 8X_3 = 6$$
$$-X_1 - 3X_2 + 6X_3 = 4$$
$$5X_1 + 7X_2 - 2X_3 = 24$$

شروط الطريقة :-
① $A_{3\times3}$ المصفوفة مربعة
② $|A| \neq 0$

Solution

$$\begin{array}{ccc} \oplus & \ominus & \oplus \end{array}$$
$$\begin{bmatrix} 2 & 4 & -8 \\ -1 & -3 & 6 \\ 5 & 7 & -2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 24 \end{bmatrix}$$

✱ $\because |A| = 2(6-42) - 4(2-30) - 8(-7+15) =$

$= -72 + 112 - 64 = -24 \neq 0$

upper tr

$$\left[\begin{array}{ccc|c} 2 & 4 & -8 & 6 \\ -1 & -3 & 6 & 4 \\ 5 & 7 & -2 & 24 \end{array}\right] \quad R_2 = R_2 + R_1\left(\frac{1}{2}\right)$$

$$\left[\begin{array}{ccc|c} 2 & 4 & -8 & 6 \\ 0 & -1 & 2 & 7 \\ 5 & 7 & -2 & 24 \end{array}\right] \quad R_3 = R_3 + R_1\left(\frac{-5}{2}\right)$$

$$\left[\begin{array}{ccc|c} 2 & 4 & -8 & 6 \\ 0 & -1 & 2 & 7 \\ 0 & -3 & 18 & 9 \end{array}\right] \Rightarrow R_3 = R_3 + (R_2)(-3)$$

$$\left[\begin{array}{ccc|c} 2 & 4 & -8 & 6 \\ 0 & -1 & 2 & 7 \\ 0 & 0 & 12 & -12 \end{array}\right]$$

Now we have the eq's:-

$$2X_1 + 4X_2 - 8X_3 = 6 \quad \text{--- (1)}$$

$$-X_2 + 2X_3 = 7 \quad \text{--- (2)}$$

$$12X_3 = -12 \quad \text{--- (3)}$$

$\therefore \boxed{X_3 = -1}$; $\boxed{X_2 = -9}$; $\boxed{X_1 = 17}$