

# Solar Energy Engineering

A course intended for the senior students

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## Chapter One Introduction

### 1.1 Types of Energy Sources

The sources of energy can be classified into two broad categories:–

- Fossil Sources
- Renewable Sources

**Fossil sources** comprise the traditional types of fuels, mainly, crude oil, natural gas and coal. These types will be depleted in the near future and the humanity should seek alternative sources. The traditional fuels also produce harmful byproducts that pollute the environment.

**Renewable sources** are always compensated by the nature and are clean sources with no harmful pollutants. There are many types of renewable energies, the most important are: –

- Solar energy
- Wind energy
- Geothermal energy
- Sea waves energy
- Biomass energy

The most important type of the above five types is **solar energy**. Extensive research efforts are devoted to study, invent and test devices and systems that are operated on solar energy.

## 1.2 Advantages of solar energy

- Available everywhere.
- Simple to harness.
- Systems require the least maintenance.
- Suitable for variety of applications.

## 1.3 Disadvantages of solar energy

- Low heat flux (which requires large collection areas).
- Not available at night and cloudy weathers (which requires effective storage means).
- Inherently changing with time and location.
- Systems are more expensive than in fossil fuels.

## 1.4 Applications of solar energy

Solar energy can be utilized in two major branches: –

- **Solar photovoltaic cells** at which the solar radiation is directly converted to DC current or can be stored in batteries. Solar cells are appropriate for low-power applications such as lighting and small electronic devices; however, large areas of cell fields can produce considerable amounts of power. The cell efficiency in general is less than 20%.
- **Solar thermal systems** at which the solar radiation is converted to thermal energy that can be carried by working fluids or stored in suitable materials. Solar thermal systems are more efficient than solar cells and can be utilized in various fields, such as: –
  - ❖ Domestic and industrial water heating
  - ❖ Air-conditioning and refrigeration
  - ❖ Distillation and desalination
  - ❖ Drying of food crops
  - ❖ Electric power generation

## 1.5 The sun

The sun is an average star with a diameter of 1.39 million km which is 109 times the diameter of earth at the equator. Energy is generated at the core of the sun by nuclear fusion where four Hydrogen atoms are united to form one Helium atom. An extreme small residue of hydrogen matter is converted to a huge amount of energy keeping the sun surface at a temperature of 6760 K.

## Chapter Two

# Solar Geometry

### 2.1 Earth orbit

Earth circulates the sun in an elliptical orbit with variable distance between the sun and earth. The minimum center to center distance occurs on the 21<sup>st</sup> of December which is 144.48 million km and the maximum distance is 154.3 million km on the 21<sup>st</sup> of June. The complete revolution takes 365.25 days. The Earth also rotates about itself one revolution per day. The axis of earth rotation about itself is inclined 23.45° to the Azimuthal Axis. Azimuthal Axis is the axis perpendicular to the plane intersecting the centers of sun and earth, called the Azimuthal plane (Fig. 2.1). This inclination causes the occurrence of seasons on earth.

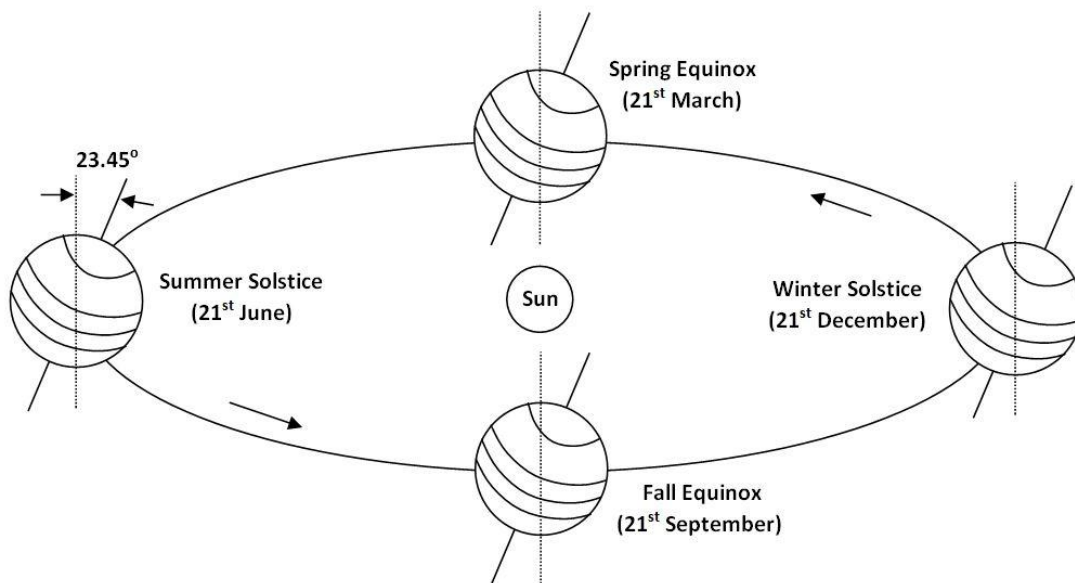


Fig. (2.1): Orbit of Earth around the sun showing Solstice and Equinox points.



### **Latitude angle (L)**

It is the angle intercepted between certain location on Earth and the equator. Latitudes, by convention, take positive values in the northern hemisphere and negative values at the southern hemisphere. Significant latitudes are: – (Equator  $L=0$ ), (Tropic of Cancer  $L=23.45^\circ$ ), (Tropic of Capricorn  $L=-23.45^\circ$ ), (Arctic Circle  $L=66.55^\circ$ ) and (Antarctic Circle  $L=-66.55^\circ$ ). Iraq lies between  $L=29.1^\circ$  and  $L=37.3^\circ$ . For Baghdad  $L=33.3^\circ$ .

### **Longitude angle (Long)**

It is the angle between the certain location on Earth and Greenwich Line near London. Negative longitudes lie west of Greenwich Line and positive longitudes to the east. Latitudes and longitudes constitute the geographic coordinates of Earth surface. Iraq lies between  $Long=38.8^\circ$  and  $Long=48.5^\circ$ . For Baghdad  $Long=44.4^\circ$ .

### **Hour angle (h)**

This angle is used to replace the temporal hours by spatial angles. It is defined as the angle between the location of a point at certain time and the location of the same point at the solar noon. Solar noon is located exactly between sunrise and sunset. Since the earth takes 24 hours to circumference a complete revolution about itself ( $360^\circ$ ) then the hour angle can be determined as follows: –

$$h = 15[ST - 12] \qquad 2.2$$

Where  $ST$  denotes the solar time in 24–hours–format starting from 0 to 12 before solar noon and 13 to 24 in the afternoon. As a result the hour angle takes negative values in the morning hours and positive values in the afternoon.

### **Solar time (ST) and local time (LT)**

Solar time is the actual time related to the rotation of earth about itself. Therefore solar time is unique for every location on earth, or more precisely, for each and every longitude. For practical reasons, it is difficult to use solar time for official timing of regions having different longitudes and belong to one country. Therefore, the world is divided into 24 standard time zones. Each time zone (TZ) has a single time that is approximated for the entire zone and valid for entire country. For large countries more than one time zone is used in timing. The time that is used in a certain time zone is called local time (LT). The error between the local and solar times must be accounted for in solar energy calculations. Another error is caused by

some irregular motion of Earth around the sun. Earth does not move in a constant speed. It accelerates and decelerates along the year causing an error of several minutes each day. The error of irregular motion can be estimated by the following equation called (the equation of time): –

$$EQT = 9.87 \sin(2\beta) - 7.53 \cos\beta - 1.5 \sin\beta \quad 2.3$$

EQT is calculated from eq. (2.3) in minutes.  $\beta$  is calculated as follows: –

$$\beta = \frac{360}{364} (n - 81) \quad 2.4$$

Where (n) is the day number in the year.

Consequently, the solar time (ST) corresponding to a certain longitude (Long) and certain local time (LT) of one of the standard time zones (TZ) is given as follows:–

$$ST = LT + \frac{\text{Long} - \text{TZ}}{15} + \frac{EQT}{60} \quad 2.5$$

TZ in eq. (2.5) represents the nearest time zone of the longitude (Long) under consideration. TZ takes positive multiples of  $15^\circ$  west of Greenwich and negative multiples of  $15^\circ$  east to Greenwich. For Iraq,  $\text{TZ} = -45^\circ$  and for Baghdad  $\text{Long} = -44.4^\circ$ .

### **Sunrise, sunset and daytime length**

The solar noon is the mid time between sunrise and sunset. The total time between sunrise and sunset is called daytime length. The hour angle corresponding to the sunrise has a certain value and is denoted by  $h_s$  and is calculated by the following equation: –

$$\cos(h_s) = -\tan(L)\tan(\delta) \quad 2.6$$

The value of  $h_s$  is symmetrical around solar noon. To find the daytime length, the value of  $h_s$  is converted back to hours by dividing on 15 and then multiplied by 2 to account for the two periods around solar noon. Hence, daytime length (DL) in hours is calculated as follows: –

$$DL = \left(\frac{2}{15}\right) \cos^{-1}(-\tan(L)\tan(\delta)) \quad 2.7$$

**Ex. 2.1** Calculate the solar time in Baghdad city at 10:30 a.m. on the 7<sup>th</sup> of December. Then calculate daytime length, sunrise and sunset times for the same day.

**Sol.**

$$n=31+28+31+30+31+30+31+31+30+31+30+7$$

$$n=341$$

$$\beta=(360/364)\times(341-81)$$

$$\beta=257.14$$

$$EQT=9.87\times\sin(2\times 257.14)-7.53\times\cos(257.14)-1.5\times\sin(257.14) \quad EQT=7.42 \text{ minutes}$$

$$ST=10.5+(44.4-45)/15+7.42/60$$

$$\underline{\underline{ST=10.5836=10:35}}$$

$$\delta=23.45\times\sin((360/365)\times(341+284))$$

$$\delta=-22.79^\circ$$

$$DL=(2/15)\times\cos^{-1}(-\tan(33.3)\times\tan(-22.79))$$

$$\underline{\underline{DL=9.863 \text{ hours}}}$$

$$\text{Sunrise time} = 12-9.863/2 = 7.0685$$

$$\underline{\underline{\text{Sunrise at 7:04 a.m. solar time}}}$$

$$\text{Sunset time} = 12+9.863/2 = 16.931$$

$$\underline{\underline{\text{Sunset at 4:56 p.m. solar time}}}$$

### **Altitude angle ( $\alpha$ )**

It is the angle between the solar beam and the horizon. The complementary angle of ( $\alpha$ ) is called the Zenith angle (z) which is the angle between the solar beam and the vertical axis (Fig. 2.3). Altitude angle is given by the following equation: –

$$\sin(\alpha) = \cos(z) = \sin(L)\sin(\delta) + \cos(L)\cos(\delta)\cos(h) \quad 2.8$$

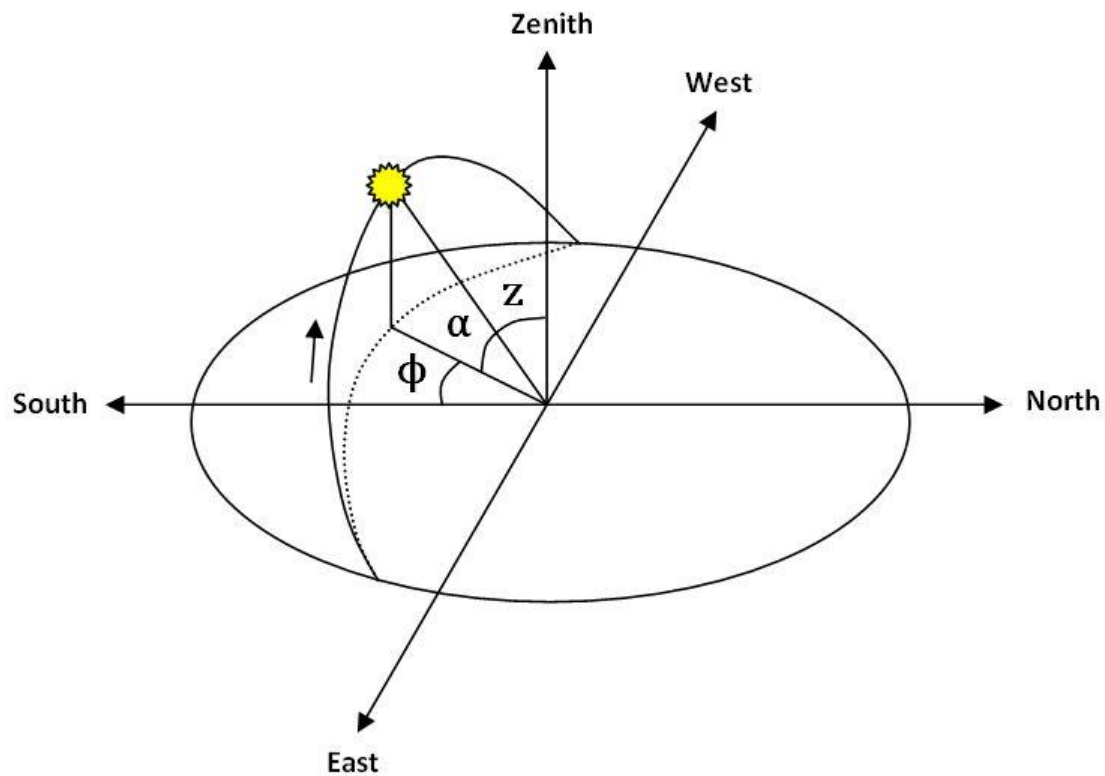


Fig. (2.3): Schematic view of the path of sun in the sky.

### Azimuth angle ( $\phi$ )

It is the angle between the horizontal projection of the solar beam and the south. In the northern hemisphere, Azimuth angle takes negative values before solar noon and positive value in afternoon hours. In southern hemisphere it is measured relative to the north direction with positive values before noon and negative values in the afternoon. It is calculated by the following equation: –

$$\sin(\phi) = \frac{\cos(\delta)\sin(h)}{\cos(\alpha)} \quad 2.9$$



## 2.3 Solar angles for tilted surfaces

It is a common practice in solar energy applications to use tilted surfaces facing geographic south in order to maximize the amount of collected radiation. The optimum tilt angle is unique for each latitudes and each date in the year. However it is sufficient and practical to use a single tilt angle for the entire heating season and another tilt angle for the cooling season. The rule of thumb is to add  $10^\circ$  to the latitude value in winter and subtract  $10^\circ$  in summer. For the location of Baghdad ( $33.3^\circ$  latitude) the optimum tilt angle in winter is  $43^\circ$  and in summer  $23^\circ$ .

### **Incidence angle (i)**

This angle is corresponding to the zenith angle of a horizontal surface. It is defined as the angle between the solar beam and the axis perpendicular to the tilted surface (Fig. 2.4). For a tilted surface facing south, it is given by the following equation: –

$$\cos(i) = \sin(L - s)\sin(\delta) + \cos(L - s)\cos(\delta)\cos(h) \quad 2.10$$

Where (s) is the surface tilt angle. For the tilted surfaces in the southern hemisphere the term (L-s) in eq. 2.10 is replaced by (L+s).

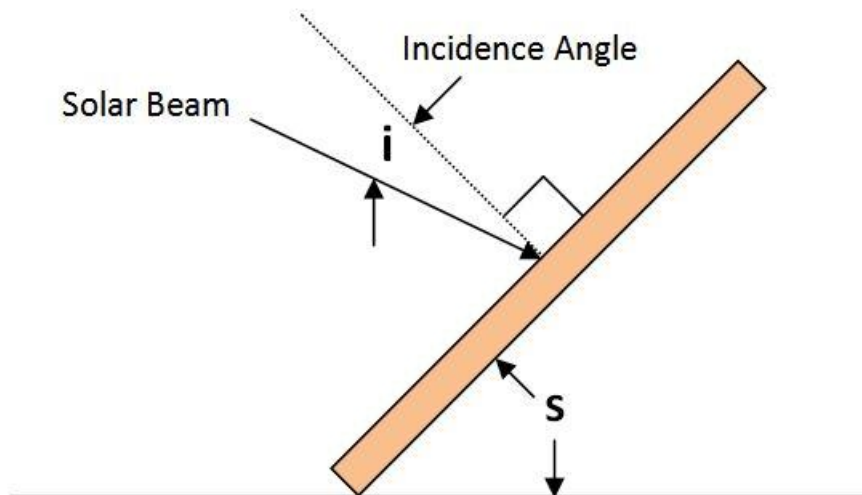


Fig. (2.4): Tilted surface with the incidence angle.

## Exposure period (EP) of a tilted surface

The daytime length of the horizontal surfaces has a corresponding concept for the tilted surfaces, called exposure period (EP). It represent the effective period at which the solar radiation hits the tilted surface, and is given by the following equation: –

$$EP = \left(\frac{2}{15}\right) \cos^{-1}(-\tan(L - s)\tan(\delta)) \quad 2.11$$

The value of (EP) from eq. 2.11 is compared with the value of (DL) from eq. 2.7 and the minimum value is considered as the exposure period of the tilted surface. For the southern hemisphere, the term (L-s) is also replaced by (L+s).

**Ex. 2.2** Find altitude and azimuth angles at 2:15 solar time on 21<sup>st</sup> of April for the location (50°N, 65°E). For the same day, time and location find also incident angle on a solar collector tilted 45° above horizon and the exposure time of that surface.

**Sol.**  $n=31+28+31+21$   $n=111$

$$\delta=23.45 \times \sin((360/365) \times (111+284)) \quad \delta=11.57^\circ$$

$$h=15^\circ[14.25-12] \quad h=33.57^\circ$$

$$\sin(\alpha)=\sin(50)\sin(11.57)+\cos(50)\cos(11.57)\cos(33.57) \quad \sin(\alpha)=0.805 \quad \alpha=53.61^\circ$$

$$\sin(\phi)=\cos(11.57)\sin(33.57)/\cos(53.61) \quad \sin(\phi)=0.917 \quad \phi=66.49^\circ \text{ (to the west)}$$

$$\cos(i)=\sin(50-45)\sin(11.57)+\cos(50-45)\cos(11.57)\cos(33.57)$$

$$\cos(i)=0.828 \quad i=34.10^\circ$$

$$EP=(2/15) \times \cos^{-1}(-\tan(50-45) \times \tan(11.57)) \quad EP=11.86 \text{ hours}$$

$$DL=(2/15) \times \cos^{-1}(-\tan(50) \times \tan(11.57)) \quad DL=10.12 \text{ hours}$$

Therefore the real exposure period is **10.12 hours**

## Chapter Three

# Solar Radiation Calculations

### 3.1 Extraterrestrial solar radiation intensity (Irradiance) in W/m<sup>2</sup>

The amount of solar radiation is inversely proportional to the distance it travels in the outer space. The solar radiation undergoes further attenuation upon entering the atmosphere. The rate of solar radiation intensity in W/m<sup>2</sup> (irradiance) measured on a surface perpendicular to the solar beam and placed just above the atmosphere when Earth is at its average distance from the sun is termed as **Solar Constant I<sub>sc</sub>** (recently called Total Solar Irradiance TSI). The most accurate and recent measured value of the solar constant is 1360.8 ± 0.5 W/m<sup>2</sup>. The value of irradiance above the atmosphere varies during the year by about ±3.5% due to the change of distance from the sun. Therefore, the instantaneous value of irradiance in W/m<sup>2</sup> on a surface perpendicular to the solar beam at any day in the year and termed as I<sub>o</sub> is given by the following equation: –

$$I_o = I_{sc} \left[ 1 + 0.033 \cos \left( \frac{360}{365} n \right) \right] \quad 3.1$$

Where n is the day number in the year and I<sub>sc</sub>=1360 for calculation purposes.

The value of I<sub>o</sub> is calculated on a surface perpendicular to solar beam. The amount can be calculated for a surface parallel to ground and placed outside atmosphere as follows: –

$$I_H = I_o \sin(\alpha) \quad 3.2$$

### 3.2 Extraterrestrial solar radiation quantity (Irradiation) in J/m<sup>2</sup>

The values of irradiance I<sub>o</sub> and I<sub>H</sub> can be integrated for a specified period of time from t<sub>1</sub> to t<sub>2</sub> to get the amount of solar radiation in (J/m<sup>2</sup>) accumulated during this period (H'<sub>o</sub>): –

$$H'_o = \int_{t_1}^{t_2} I_H dt = \int_{t_1}^{t_2} I_o \sin(\alpha) dt$$

The value of sin(α) can be substituted from eq. 2.8 to get: –

$$H'_o = \int_{t_1}^{t_2} I_o (\sin(L)\sin(\delta) + \cos(L)\cos(\delta)\cos(h))dt$$

The only unknown in the above equation is the hour angle (h). Therefore, the integration must be performed in terms of the hour angle rather than time. Since every 15° of the hour angle equals one hour (3600 s), then the time dt must be divided by 3600 and multiplied by 15 to get a value in degrees. Then, it must also be converted to radians by multiplying by (π/180). It follows that: –

$$dt = \frac{180 \times 3600}{\pi \times 15} dh$$

The resulting integral becomes: –

$$H'_o = \frac{180 \times 3600}{\pi \times 15} \int_{h_1}^{h_2} I_o (\sin(L)\sin(\delta) + \cos(L)\cos(\delta)\cos(h))dh$$

Performing the integration and taking care of the angle units, the result is: –

$$H'_o = \frac{12 \times 3600}{\pi} I_o \left[ \frac{\pi}{180} (h_2 - h_1) \sin(L)\sin(\delta) + \cos(L)\cos(\delta) [\sin(h_2) - \sin(h_1)] \right]$$

3.3

If the limits of integration  $h_1$  and  $h_2$  are substituted as the hour angles at sunrise and sunset, respectively, then eq. 3.3 gives the amount of solar radiation of a whole day incident on a horizontal surface outside the atmosphere ( $H_o$ ): –

$$H_o = \frac{24 \times 3600}{\pi} I_o \left[ \frac{\pi}{180} h_s \sin(L)\sin(\delta) + \cos(L)\cos(\delta) \sin(h_s) \right]$$

3.4

Where  $h_s$  are given by eq. 2.6. The amount  $H_o$  can be calculated for a characteristic day in each month and considered as the monthly average of the daily amount of solar radiation for that month. It is termed as  $\bar{H}_o$  and is equivalent to an average value of the daily amounts of all days in that month. The characteristic days of each month are listed in the following table: –

Table 3.1: Characteristic day of each month

| Month | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
|-------|----|----|----|----|----|----|----|----|----|----|----|----|
| Day   | 17 | 16 | 16 | 15 | 15 | 11 | 17 | 16 | 15 | 15 | 14 | 10 |

**Ex. 3.1** Calculate the direct and horizontal extraterrestrial irradiance on 11<sup>th</sup> of June at the solar noon for 30°N latitude. Calculate also the monthly average irradiation of June.

**Sol.**  $n=31+28+31+30+31+11$   $n=162$

$I_o=1360 \times [1+0.033 \times \cos(360 \times 162/365)]$   $I_o=1317.8 \text{ W/m}^2$

$\delta=23.45 \times \sin[(360/365) \times (162+284)]$   $\delta=23.08^\circ$

$\sin(\alpha)=\sin(30) \times \sin(23.08)+\cos(30) \times \cos(23.08) \times \cos(0)$   $\sin(\alpha)=0.9927$   $\alpha=83^\circ$

$I_H=1317.8 \times 0.9927$   $I_H=1308.1 \text{ W/m}^2$

11<sup>th</sup> of June is the characteristic day. Therefore it can be used to find  $\bar{H}_o$

$h_s=\cos^{-1}[-\tan(30) \times \tan(23.08)]$   $h_s=104.24^\circ$

$\bar{H}_o=(24 \times 3600/3.14159) \times 1317.8 \times [(3.14159/180) \times 104.24 \times \sin(30) \times \sin(23.08)$   
 $+ \cos(30) \times \cos(23.08) \times \sin(104.24)]$

$\bar{H}_o=40.911 \text{ MJ/m}^2$