

3.3 Terrestrial solar radiation intensity (Irradiance) in W/m²

When solar radiation enters the atmosphere, part of it is absorbed at the higher atmospheric layers (200–300 km altitude) where most of harmful radiation (x-ray and gamma ray) is absorbed. As the radiation further penetrates the atmosphere the rays of longer wave length is absorbed, most of them in the ultra-violet region. Ozone layer (15–40 km altitude) protects the Globe form ultra-violet ray. As a result the radiation loses some of its thermal energy. The solar radiation undergoes further attenuation by water vapor and dust suspended in air.

Solar radiation can be divided into two main parts: –

a– Direct Beam Radiation (I_{Bn}): It is the direct solar radiation without any attenuation or diffusion and measured perpendicular to solar ray.

b– Diffused Radiation (I_D): It is the part that is re-emitted by the atmosphere in all directions.

Air Mass (m): It is a hypothetical term to measure the amount of attenuation of solar radiation by atmospheric air. Air mass is defined as the ratio of the actual distance traversed by the solar beam in the atmospheric air to the minimum hypothetical distance if sun would be at the zenith (Fig. 3.1). Accordingly air mass is related to zenith angle (z) as follows: –

$$m = \frac{1}{\cos(z)} \quad 80^\circ > z \geq 0^\circ \quad 3.5$$

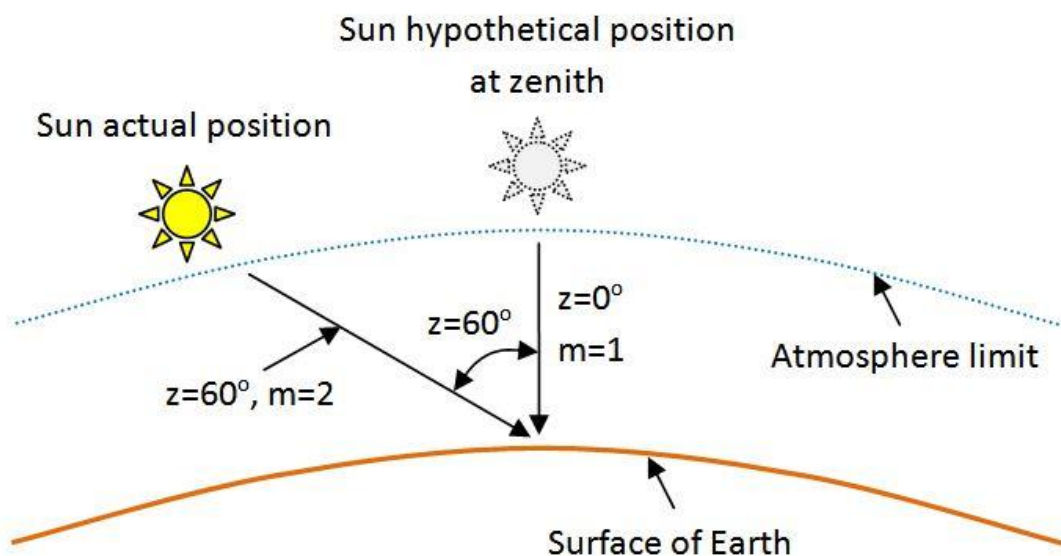


Fig. 3.1: The relation between air mass and zenith angle.

3.3.1 Irradiance on horizontal surfaces

The irradiance incident on a horizontal surface is the sum of the vertical component of direct beam radiation and the diffused radiation. The direct beam radiation intensity in (W/m^2) can be measured or calculated by suitable empirical formulas. The most well known formula is that suggested by ASHRAE and known as (Clear Sky Model). In this model the direct beam radiation I_{Bn} in (W/m^2) is estimated from the following formula: –

$$I_{Bn} = \frac{A}{e^{B/\sin(\alpha)}} = \frac{A}{e^{Bm}} \quad 3.6$$

The empirical coefficients A and B are unique for every month and are taken from table (3.2). The component of I_{Bn} that is intercepted by a horizontal surface is found as follows: –

$$I_B = I_{Bn} \sin(\alpha) \quad 3.7$$

The diffused solar radiation received by the horizontal surface can also be estimated by ASHRAE model as follows: –

$$I_D = C \times I_{Bn} \quad 3.8$$

The coefficient C is also taken from table 3.2 for every month.

Accordingly, the amount of solar radiation incident on a horizontal surface is the sum of beam and diffused components, as follows:

$$I = I_B + I_D \quad 3.9$$

The values of (n) in table 3.2 are calculated for the 21st of each month that is considered as the characteristic day in ASHRAE model.

Table (3.2): The coefficients used in ASHRAE model

Month	A (W/m ²)	B	C	n
January	1230	0.142	0.058	21
February	1215	0.144	0.06	52
March	1186	0.156	0.071	80
April	1136	0.18	0.097	111
May	1104	0.196	0.121	141
June	1088	0.205	0.134	172
July	1085	0.207	0.136	202
August	1107	0.201	0.122	233
September	1151	0.177	0.092	264
October	1192	0.16	0.073	294
November	1221	0.149	0.063	325
December	1233	0.142	0.057	355

3.3.2 Irradiance on tilted surfaces

Most of devices and system incorporating solar energy rely on collection surfaces inclined at a suitable angle above horizon in order to maximize collection efficiency. The beam radiation I_b received by a surface tilted at S is the component of the direct beam radiation I_{Bn} projected on that surface, accordingly: –

$$I_b = I_{Bn} \cos(i) \quad 3.10$$

Where (i) is the incidence angle calculated by eq. 2.10.

The diffused radiation received by a tilted surface comes from two sources; namely, the atmospheric air and the surrounding ground. The diffused radiation from the air I_d is estimated by the following equation: –

$$I_d = I_D \left(\frac{1 + \cos(s)}{2} \right) \quad 3.11$$

Whereas the diffused radiation emitted by the surrounding ground I_r is calculated by the following equation: –

$$I_r = \rho(I_B + I_D) \left(\frac{1 - \cos(s)}{2} \right) \quad 3.12$$

ρ is the surrounding ground reflectivity, a value less than 1 and depends on the nature of the ground ($\rho=0.2$ for dark soil, $\rho=0.9$ for snow).

Hence, the total irradiance I_T incident on a tilted surface is the summation of beam and diffused components; namely: –

$$I_T = I_b + I_d + I_r \quad 3.13$$

Figure 3.2 shows all component of terrestrial irradiance.

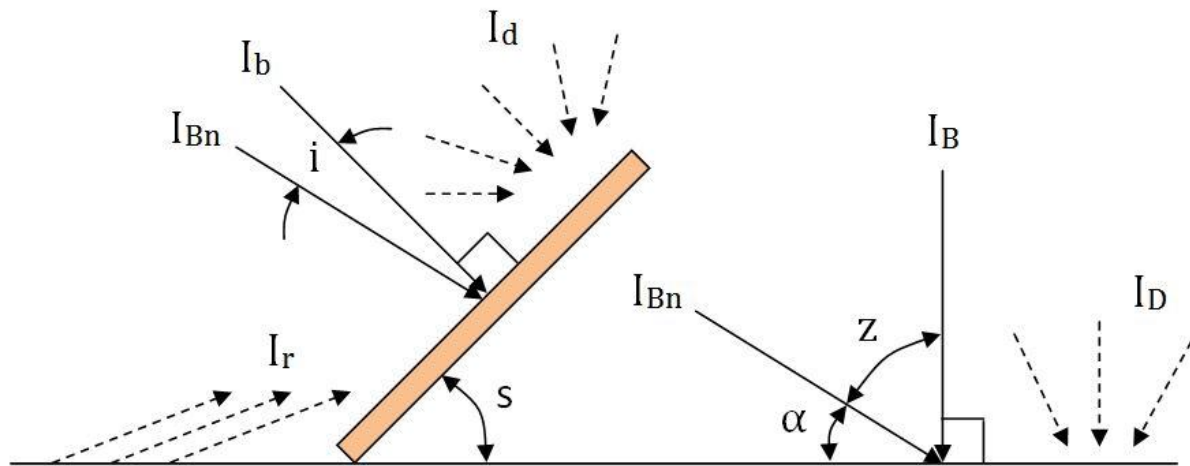


Fig. 3.2: Components of terrestrial solar radiation.

Ex. 3.2 Calculate the total irradiance incident on a surface tilted at 30° above horizon at 2:15 p.m. local time on the 21st of March for the position (34°N , 42°E). Assume that the ground reflectivity is 0.33. (For March: $A=1186$, $B=0.156$, $C=0.071$)

Sol.

$$n=31+28+21$$

$$n=80$$

$$\beta=(360/364)\times(80-81)$$

$$\beta=-0.989$$

$$\text{EQT}=9.87\times\sin(2\times-0.989)-7.53\times\cos(-0.989)-1.5\times\sin(-0.989) \quad \text{EQT}=-7.84 \text{ minutes}$$

$$\text{ST}=14.25+(42-45)/15+(-7.84)/60$$

$$\text{ST}=13.919=1:55 \text{ p.m.}$$

$$h=15^\circ[13.919-12]$$

$$h=28.785^\circ$$

$$\delta=0^\circ \quad (\text{spring equinox})$$

$$\sin(\alpha)=\sin(34)\sin(0)+\cos(34)\cos(0)\cos(28.785)$$

$$\sin(\alpha)=0.726 \quad \alpha=46.6^\circ$$

$$I_{Bn}=1186/\exp(0.156/0.726)$$

$$I_{Bn}=956.6 \text{ W/m}^2$$

$$\cos(i) = \sin(34-30)\sin(0) + \cos(34-30)\cos(0)\cos(28.785) \quad \cos(i) = 0.874 \quad i = 29^\circ$$

$$I_b = 956.6 \times 0.874 \quad I_b = 836 \text{ W/m}^2$$

$$I_D = 956.6 \times 0.071 \quad I_D = 67.9 \text{ W/m}^2$$

$$I_d = 67.9 \times (1 + \cos(30)) / 2 \quad I_d = 63.3 \text{ W/m}^2$$

$$I_B = 956.6 \times 0.726 \quad I_B = 694.5 \text{ W/m}^2$$

$$I_r = 0.33 \times (694.5 + 67.9) \times (1 - \cos(30)) / 2 \quad I_r = 16.8 \text{ W/m}^2$$

$$I_T = 836 + 63.3 + 16.8 \quad \underline{I_T = 916.1 \text{ W/m}^2}$$

3.4 Terrestrial solar radiation quantity (Irradiation) in J/m²

3.4.1 Irradiation on horizontal surfaces

The amount of terrestrial solar radiation accumulated in a period of time (usually one day) depends on the metrological conditions like the purity of air and existence of clouds and dust. The daily amount of solar radiation incident on a horizontal surface in a specific day in the year is usually taken from metrological tables which are averaged values of many years. The value is usually averaged for one month and denoted as (\bar{H}) with units of (kJ/m²day).

Clearness Index K_T

It is the ratio of terrestrial solar radiation quantity incident on a horizontal surface during one day to its extraterrestrial counterpart; namely: –

$$K_T = \frac{H}{H_0} \quad 3.14$$

In the same manner, the monthly average clearness index is defined as follows: –

$$\bar{K}_T = \frac{\bar{H}}{\bar{H}_0} \quad 3.15$$

Where \bar{H}_0 is calculated from eq. 3.4 using the characteristic days of each month from table 3.1.

The clearness index is a measure of the attenuation of solar radiation in the atmosphere with values less than unity. High values of \bar{K}_T indicate low attenuation and low diffused radiation with high beam radiation. Empirical formulas can be used to relate diffused radiation with clearness index like the following relation: –

$$\frac{\bar{H}_D}{\bar{H}} = 1.39 - 4.027\bar{K}_T + 5.531\bar{K}_T^2 - 3.108\bar{K}_T^3 \quad 3.16$$

The monthly average value of the daily amount of beam radiation incident on a horizontal surface \bar{H}_B is thus found as follows: –

$$\bar{H}_B = \bar{H} - \bar{H}_D \quad 3.17$$

3.4.2 Irradiation on tilted surfaces

To estimate irradiation on a tilted surface, eq. 3.13 is recalled: –

$$I_T = I_b + I_d + I_r$$

$$I_T = I_{Bn} \cos(i) + I_D \left(\frac{1 + \cos(s)}{2} \right) + \rho(I_B + I_D) \left(\frac{1 - \cos(s)}{2} \right)$$

$$I_T = I_B \frac{\cos(i)}{\cos(z)} + I_D \left(\frac{1 + \cos(s)}{2} \right) + \rho I \left(\frac{1 - \cos(s)}{2} \right) \quad 3.18$$

Beam Radiation and total radiation tilt factors

The ratio of the cosines of incidence and zenith angles in eq. 3.18 is called Beam Radiation Tilt Factor (R_B), it follows that: –

$$R_B = \frac{\cos(i)}{\cos(z)} = \frac{I_b}{I_B} \quad 3.19$$

In the same manner, the Total Radiation Tilt Factor is defined as follows: –

$$R = \frac{I_T}{I} \quad 3.20$$

Dividing eq. 3.18 by I and knowing that $I_B = I - I_D$ eq. 3.18 can be written in terms of tilt factors as follows: –

$$R = \frac{I_T}{I} = R_B \left(1 - \frac{I_D}{I}\right) + \frac{I_D}{I} \left(\frac{1+\cos(s)}{2}\right) + \rho \left(\frac{1-\cos(s)}{2}\right) \quad 3.21$$

Eq. 3.21 is in terms of instantaneous values (irradiance). It can be written in terms of amount values (irradiation) as follows: –

$$\bar{R} = \frac{\bar{H}_T}{\bar{H}} = \bar{R}_B \left(1 - \frac{\bar{H}_D}{\bar{H}}\right) + \frac{\bar{H}_D}{\bar{H}} \left(\frac{1+\cos(s)}{2}\right) + \rho \left(\frac{1-\cos(s)}{2}\right) \quad 3.22$$

The value of \bar{R}_B can be evaluated by integrating R_B for a daytime length. The result for a tilted surface facing south is: –

$$\bar{R}_B = \frac{\cos(L-s)\sin(\delta)\sin(h'_s) + \frac{\pi}{180}h'_s\sin(L-s)\sin(\delta)}{\cos(L)\cos(\delta)\sin(h_s) + \frac{\pi}{180}h_s\sin(L)\sin(\delta)} \quad 3.23$$

h'_s is evaluated as the minimum value from the following two equations: –

$$\cos(h'_s) = -\tan(L)\tan(\delta)$$

$$\cos(h'_s) = -\tan(L-s)\tan(\delta)$$

While h_s is the sunset hour angle for a horizontal surface.