Ch-2 Linear programming

2-1 Introduction to linear programming

A linear form means a mathematical expression of the type ($a_1x_1+a_2x_2+.....+a_nx_n$) where $a_1,a_2,\ldots,a_n$ are constants and x_1, $x_2\ldots x_n$ are variables. The term programming refers to the process of determining particular programs or plans of action. So, linear programming (LP) is one of the most important optimization (maximization/ minimization) techniques

The methods applied for solving a linear programming problem are basically simple problems: a solution can be obtained by a set of simultaneous equations. However a unique solution for a set of simultaneous equations in n-variables (x_1, $x_2\ldots x_n$), at least one of them is non-zero, can be obtained if there are exactly n relations. When the number of relations is greater than or less than n, a unique solution does not exist but a number of trial solution can be found.

In various practical situation, the problems are seen in which the number of relations is not equal to the number of variables and many of the relations are in the form of inequalities ($\leq$ or $\geq$) to maximize or minimize a linear function of the variables subject to such conditions. Such problems are known as linear programming problem (LPP).

Definition: the general LPP calls for optimizing (maximizing / minimizing) a linear function of variables called the 'objective function' subject to a set of linear equations and / or inequalities called the ' constraints' or' restrictions'.

2-2General form of LPP

A mathematical model is formulated for general problem of allocating resources to activities. In particular, this model is to select the values for x_1, $x_2 \ldots x_n$ so as to maximize or minimize.

$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

Subject to restrictions

$a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_{1n} (\leq \text{ or } \geq) b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_{1n} (\leq \text{ or } \geq) b_2$

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$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots + a_{mn}x_{1n} (\leq \text{ or } \geq) b_m$

2-3 Applications of linear programming

1- Personnel assignment problem.
2- Transportation problem.
3- Efficiency on operation of system of dams.
4- Optimum estimation of executive compensation.
5- Agriculture applications.
6- Military applications.
7- Production management.
8- Marketing management.
9- Manpower management.
10- Physical description.

2-4 Advantages of linear programming techniques.

1- It helps us in making the optimum utilization of productive resources.

2- The quality of decisions may also be improved by linear programming techniques.

3- Provides practically solutions.

Ex1

If a factory produce two products, the factory has two lines let the number of the first product is x1 if we want to determine the quantity for this product under the circumstances of the factory as it has 48 work hours weekly for one machine, the product x1 needs 2 hours to complete.

The above words can be expressed mathematically by the following equation.

$2x1=48$…(1)

Eq-1 is mathematical program give the solution:

$X1=24$,

And that means utilization all the available hours.

But if we don't want to utilize all the available hours we can write it:

$2x1\leq 48$…(2)

Eq-2 agree with the our hypothesis which we can find many feasible solutions one of them is $x1=24$,

For the second product x2 we can put another equation

If it needs four hours to complete the quantity for this product will be:

$2x1+4x2=48$……(3)

The solutions are:

$x1=0$, $x2=12$

$x1=24$, $x2=0$

If we don't want to utilizes all the available hours

$2x1+4x2 \leq 48$…(4)

Which has many solutions one of them equation 3 solution as:

x1=2, x2=1,

x1=6, x2=3,

x1=5, x2=4

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If another machine and the two products must pass throw it after machine1

The available work hours is 84 weekly

Product 1 needs 4 hours , product 2 needs 2 hours

The equation will be:

$4x1+2x2=84$…(5)

Not all the available:

$4x1+2x2 \leq 84$…(6)

For the two machines :

$2x1+4x2 \leq 48$

$4x1+ 2x2 \leq 84$….(7)

We can add a not have constraint that the product 2 quantity must not below 7, as

$X2 \geq 7$…(8)

So the constraint's will be:

$2x1 + 4x2 \leq 48$

$4x1+ 2x2 \leq 84$ …(9)

$X2 \geq 7$

Eq-9 solution may have negative values for x1, x2 which are not acceptable so we must add another constraint $x1, x2 \geq 0$.

$2x1 + 4x2 \leq 48$

$4x1+ 2x2 \leq 84$

$X2 \geq 7$

$x1, x2 \geq 0$…..(10)

the above equations give as the whole constraints we must depend to find the solution, they will be many solutions but which we must prefer.

The objective function will give us the way to select the optimal solution

So if we have the net profit for product 1 is C1 and C2 for product 2

The objective function will be:

$Z=C1X1+C2X2$…..(11)

let $C1=1$, $C2=2$,

$Z=X1+2X2$

So the mathematical model which give us the maximum profit is:

Maximize $Z=X1+2X2$

Subject to:

$2x1 + 4x2 \leq 48$

$4x1+ 2x2 \leq 84$

$X2 \geq 7$

$x1, x2 \geq 0$…..(12)

But if C1, C2 represent costs that objective function will be minimize the cost.

<u>Ex 2</u>

A company produces to product A and B which possess raw materials 400 quintals and 450 labor hours. It is known that 1 unit of product A requires5. Quintals of new materials and 10 man hours and yield a profit of $ 45. Product b requires 20 quintals of new materials, 15 man hours and yields a profit of $ 80. Formulate the lpp.

The Solution

Let

X1 be the number of units of product a.

X2 be the number of units of product b

	Product a	Product b	Availability
Raw materials	5	20	400
Man hours	10	15	450
Profit	$ 45	$ 80	

Hence

Maximize $z=45x1+80x2$ subject to

$5x1+ 20x2 \leq 400$,

$10x1 + 15x2 \leq 450$,

$x1 \geq 0$, $x2 \geq 0$.

Ex 3

A firm manufactures 3 products a,b and c. the profits are $ 3, $ 2 and $ 4 respectively. The firm gas 2 machines and below is given the required processing time in minutes for each machine on each product.

	Products		
Machine	A	B	C
X	4	3	5
Y	2	2	4

Machine x and y have 2000 and 2500 machine minutes. The firm must manufacture 100 a's, 200b's and 50 c's type, but not more than 150 a's.

The Solution

Let

X1 be the number of units of product a

X2 be the number of units of product b

X3 be the number of units of product c

	Products			
Machine	A	B	C	Availability
X	4	3	5	2000
Y	2	2	4	2500
Profit	3	2	4	

Max $z=3x1+2x2+4x3$

St

$4x1+3x2+5x3\leq 2000$

$2x1+ 2x2+4x3\leq 2500$

$100\leq x1\leq 150$

$X2 \geq 200$

$X3\geq 50$

Ex 4

A company has 3 operational department weaving, processing and packing with the capacity to produce 3 different types of clothes that are suiting shirting and woolen yielding with the profit of $ 2, $ 4, and $ 3 per meters respectively. 1 m suiting requires 3 minutes in weaving 2 mins in processing in packing similarly 1m of shirting requires 4 mins in weaving 1 min in processing 3 min in packing while 1m of woolen requires 3min in each department is 60, 40 and 80 ours for waeving, processing and packing department respectively formulate a lpp to find the product to maximize the profit.

The Solution

Let

X1 be a number of units of suiting

X2 be a number of units of shirting

X3 be a number of units of woolen

	Suiting	Shirting	Woolen	Available time
Weaving	3	4	3	60
Processing	2	1	3	40
Packing	1	3	3	80
Profit	2	4	3	

Maximize $z=2x1+4x2+3x3$ subject to

$3x1 +4x2+3x3 \leq 60$

$2x1+x2+3x3 \leq 40$

$X1 +3x2+3x3 \leq 80$

$X1 \geq 0,\ x2 \geq 0, x3 \geq 0$

Ex 5

Abc company products both interior and exterior paints from 2 raw materials m1 and m2. The following table produces basic of problem.

	Exterior paint	Interior paint	Availability
M1	6	4	24
M2	1	2	6
Profit per ton	5	4	

A market survey indicates that daily demand for interior paint cannot exceed that for exterior paint by more than 1 ton. Also maximum daily demand for interior paint is 2 tons. Formulate lpp to determine the best product mix of interior and exterior paints that maximizes that daily total profit.

The Solution

Let

X1 be the number of units of exterior paint.

X2 the number of units of interior paint

Maximize $z=5x1+4x2$ subject to

$6x1+4x2\leq 24$

$X1+2x2 \leq 6$

$X2-x1\leq 1$

$X2\leq 2$

$X1 \geq 0$

$X2 \geq 0$

Ex6

A manufacture produces 3 module(I ,II, and III) of a certain product he uses 2 raw materials A, and B of which 4000 and 6000 units respectively are available the raw materials per unit of 3 models are given below:

Raw materials	I	II	III
A	2	3	5
B	4	2	7

The labor time for each minutes of model 1 is twice that of model II and thrice of model III. The entire labor force of factory can produce and equivalent Of 2500 units of model I . a model I. a model survey indicates that the minimum demand of 3 models is 500, 500 and 375 units respectively. However the ratio of number of units produced must be equal to 3:2:5. Assume that profits per unit of model are 60, 40 and 100 respectively. Formulate a lpp.

The Solution

Let

X1 be the number of units of model I

X2 be the number of units of model Ii

X3 be the number of units of model Iii

Raw	I	Ii	Iii	Availability
A	2	3	5	4000
B	4	2	7	6000
Profit	60	40	100	

$X1+1/2x2+1/3x3 \leq 2500$ labor time

$X1 \geq 500$, $x2 \geq 500$, $x3 \geq 375$ minimize demand

The given ratio is $x1:x2/5=k$

$X1=3k$, $x2=2k$, $x3=5k$

$X2=2k \rightarrow k=x2/2$

Therefore $x1=3x2/2 \rightarrow 2x1=3x2$ similarly $2x3=5x2$

Maximize $z=60x1+40x2+100x3$

Subject to $2x1+3x2+3x2+5x3 \geq 4000$

$4x1 +2x2+7x3 \geq 6000$

$X1+1/2\ x2+1/3x3 \geq 2500$

$2x1= 3x2$

$2x3=5x2$

And $x1 \geq 500$, $x2 \geq 500$, $x3 \geq 375$