2.5. Live Load Distribution Factors

Live load distribution factor (DF) is used because the moving load (truck or tandem) cannot be concentrated on one exterior or interior girder. Thereby, presence of deck slab leads to distribute the live load into all supporting girders because the deck slab acts as a wide plate. The girder under the wheel line is subjected to a main fraction of load while the rest of fraction is participated by the adjacent girders.

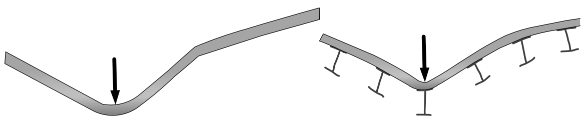

Figure 2.10: Deflection of Beams and Deck under moving Load

The (DF) values of moment differ from that used for shear. Also, the values of interior girder are different from that of exterior girder. In general, there are values for one (single) loaded lane and two or more (multiple) loaded lanes. They (DFs) are expressed as:

- DF_{mi}: for bending moment in the interior girders g_{mi}
- DF_{vi}: for shear in the interior girders g_{vi}
- DF_{me}: for bending moments in the exterior girders g_{me}
- DF_{ve}: for shear in the exterior girders g_{ve}

So, the greater value governs in each interior and exterior girder. However, in case of precast prestress concrete girders, the greatest among the four values is governing.

$$M_{LL+IM} = [M_{Mo}(1 + IM) + M_{Ln}]DF_m$$
$$V_{LL+I} = [V_{Mo}(1 + IM) + V_{Ln}]DF_v$$

2.5.1. AASHTO LRFD Tables

The (DF) values can be determined from AASHTO LRFD Tables where equations are already adopted and multiple presence factor is included $(m = 1)$ but with local stipulations. Other cases, lever rule method is applicable where (m) value is required.

N_g: number of girders.

L: length of span (ft).

S: spacing of girders (ft).

h_d: thickness of deck slab (in).

K_g: longitudinal stiffness parameter (in^4).

e: transforming factor.

d_e: distance from exterior girder center to the inside edge of curb or barrier (ft).

The longitudinal stiffness parameter (K_g) shall be taken as:

$$K_g = n(I_g + A_g.e_g{}^2)$$
$$n = E_g/E_d$$

where:

E_g: modulus of elasticity of girder material (ksi).
E_d: modulus of elasticity of deck material (ksi).
A_g: area of girder (in^2).
I_g: moment of inertia of the basic girder (in^4).
e_g: distance between the centers of gravity of the basic girder and deck (in).

Table 2.5: Distribution of Live Load per Lane for Concrete Deck on Steel or Concrete Beams [AASHTO LRFD Tables 4.6.2.2.2 and 4.6.2.2.3]

Location	Action	Loaded Lanes	Equation	Range of Applicability
Interior	Moment	Single	$g_{mi1} = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12Lh_d{}^3}\right)^{0.1}$	$N_g \geq 4$ $20 \leq L \leq 240$ $3.5 \leq S \leq 16.0$ $4.5 \leq h_d \leq 12.0$ $10^4 \leq K_g \leq 7\text{x}10^6$
		Multiple	$g_{mi2} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12Lh_d{}^3}\right)^{0.1}$	
			Lever Rule if $N_g = 3$	
	Shear	Single	$g_{vi1} = 0.36 + \frac{S}{25}$	$N_g \geq 4$ $20 \leq L \leq 240$ $3.5 \leq S \leq 16.0$ $4.5 \leq h_d \leq 12.0$
		Multiple	$g_{vi2} = 0.2 + \frac{S}{12} - \left(\frac{S}{35}\right)^{2.0}$	
			Lever Rule if $N_g = 3$	
Exterior	Moment	Single	Lever Rule	$-1.0 \leq d_e \leq 5.5$
		Multiple	$g_{me2} = e_m g_{mi2}$ $e_m = 0.77 + d_e/9.1$	
			Lever Rule if $N_g = 3$	
	Shear	Single	Lever Rule	
		Multiple	$g_{ve2} = e_v g_{vi2}$ $e_v = 0.6 + d_e/10$	
			Lever Rule if $N_g = 3$	

2.5.2. <u>Lever Rule</u>

The lever rule is an analytical tool similar to determining the reaction at the supports of a simple beam with or without a loaded overhang. This method assuming there is hinge at the interior support. Thus, the (DF) of the exterior girder is the reaction (R_e). The axle load (P) is assumed to equal one-unit weight and the presence factor (m) almost for single lane loaded.

$$R_e = X.R/S$$
$$R = 1 \rightarrow R_e = X/S$$
$$DF = mR_e$$
$$\rightarrow g_{me1} = g_{ve1} = 1.2(R_e)$$

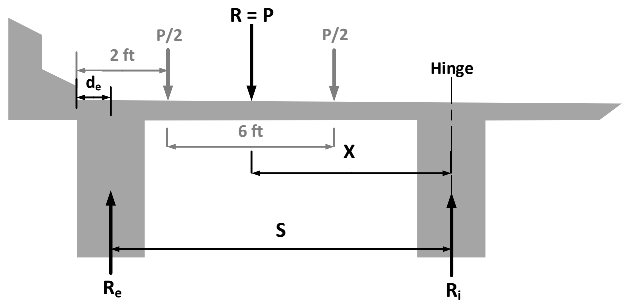

Figure 2.11: Lever Rule for Exterior Girder

2.5.3. Special Approach for Exterior Girders

A special analysis for determination of the (DFs) for the exterior girder depending on assuming that the entire cross section rotates as a rigid body about the longitudinal centerline of the bridge. The reaction (R) on the exterior girder is calculated in terms of number of lanes loaded simultaneously but with increments of one lane a time and the presence factor (m) is taken into account. The value of (R) are given by:

$$R = \frac{N_L}{N_g} + \frac{x_e \sum_1^{N_L} e}{\sum_1^{N_g} x^2}$$

where:

N_L: number of loaded lanes.

N_g: number of beams/girders supporting the deck.

x_e: eccentricity of the exterior girder from the center of gravity of the pattern of girders.

e: eccentricity of design truck or lane load from the center of gravity of the pattern of girders.

x: horizontal distance from the center of gravity of pattern of girders to each girder.

2.5.4. Distribution Factors for Fatigue Limit State

Distribution factors for fatigue (DF_f) are required for checking the effects of fatigue on a bridge girder, the fatigue load is placed in a single lane.

AASHTO specification stated that multiple presence factors (m) are not to be used for the fatigue load limit check because this load is calculated for only one design truck. Thus, (DF_f) are obtained from bending moment and shear for one lane loaded with dividing on (1.2) which is the (m) for one lane loaded and embedded in those expressions.

$$DF_{mf} = g_{m1}/1.2$$

$$DF_{vf} = g_{v1}/1.2$$

2.5.5. Distribution Factors for Deflection Limit State

Distribution factors for deflection (DF_Δ) are required to control deformation of beams and girders supporting a deck. This factor is calculated as one value for all those structures.

$$DF_\Delta = mN_L/N_g$$

2.5.6. Span Length for Distribution Factor

The effective length used for calculation of live load distribution factor is alike the length used to calculate the force itself and as tabulated below.

Table 2.6: Span Length for Live Load Distribution Factor Calculation

Force Effect	Length (L) ft
Positive bending moment	The length of span for which bending moment is being calculated
Negative bending moment near interior supports of continuous spans	The average length of two adjacent spans
Negative bending moment other than near interiors supports of continuous spans	The length of span for which bending moment is being calculated
Shear	The length of span for which bending moment is being calculated
Exterior reaction	The length of the exterior span
Interior reaction of continuous span	The average length of two adjacent spans

Example 2.7:

For the cross section of a highway bridge having a simple span of 85 ft, as shown below. It consists of an 8.5 in. thick R.C. deck (including ½ in. thick integral wearing surface) supported on and composite with AASHTO Type IV prestressed concrete girders that are spaced at 7′-8″ on centers. Assuming composite construction, determine the live load distribution factors for an interior girder for bending moment and shear.

The following data are given:

Cross-sectional area of girder (A_g) = 789 in^2.

Bending moment of inertia of girder (I_g) = 260730 in^4.

Neutral axis from the extreme bottom fibers (y_b) = 24.73 in.

Compressive strength of concrete (f_c'): Deck/Girder = 4.5/6 ksi

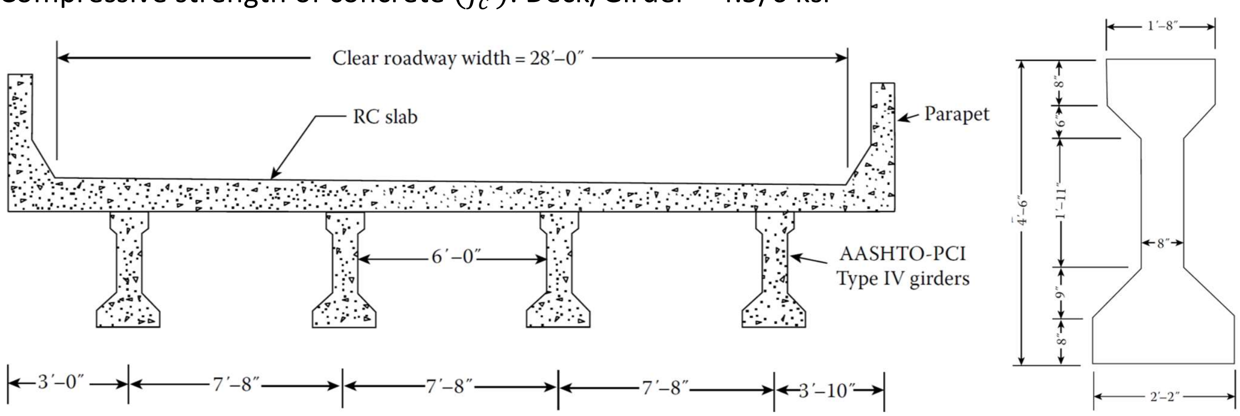

Figure 2.12: Details for Example 2.7

Solution 2.7

Check the applicability criteria:

$N_g \geq 4 \qquad N_g = 4 \ \therefore \text{OK}$

$20 \leq L \leq 240 \qquad L = 85 \text{ ft} \ \therefore \text{OK}$

$3.5 \leq S \leq 16.0 \qquad S = 7.67 \text{ in} \ \therefore \text{OK}$

$4.5 \leq h_d \leq 12 \qquad h_d = 8.5 - 0.5 = 8 \text{ in} \ \therefore \text{OK}$

$E_c = 33000\left(\Upsilon_c{}^{1.5}\right)\sqrt{f_c'}$

$n = E_g/E_d = \sqrt{6.0}/\sqrt{4.5} = 1.155$

$I_g = 260730 \text{ in}^4$

$A_g = 789 \text{ in}^2$

$e_g = 29.27 + 1 + (½)(8) = 34.27 \text{ in}$

$K_g = n\left(I_g + A_g . e_g{}^2\right) = 1.155[260730 + 789(34.27)^2 = 1.371\text{x}10^6 \text{ in}^4$

$1\text{x}10^4 \leq K_g \leq 7\text{x}10^6 \qquad K_g = 1.371\text{x}10^6 \text{ in}^4 \ \therefore \text{OK}$

Thus, the trial cross section satisfies the design stipulations.

$N_L = INT(w/12) = INT(28/12) = 2$

$\because N_L = 2 \rightarrow \therefore$ check both g_{i1} and g_{i2}

Live load distribution factor for moment:

$$g_{mi1} = 0.06 + (S/14)^{0.4}(S/L)^{0.3}\left(K_g/12Lh_d{}^3\right)^{0.1}$$
$$= 0.06 + (7.667/14)^{0.4}(7.667/85)^{0.3}(1.371\text{x}10^6/12(85)(8^3)^{0.1} = 0.481$$
$$g_{mi} = 0.075 + (S/9.5)^{0.6}(\text{S/L})^{0.2}\left(K_g/12Lh_d{}^3\right)^{0.1}$$
$$= 0.075 + (7.667/9.5)^{0.6}(7.667/85)^{0.2}(1.371\text{x}10^6/12(85)(8^3)^{0.1} = 0.674$$

$\therefore DF_{mi} = 0.674$

Live load distribution factor for shear:

$$g_{vi} = 0.36 + S/25$$
$$= 0.36 + 7.667/25 = 0.667$$
$$g_{vi2} = 0.2 + S/12 - (S/35)^{2.0}$$
$$= 0.2 + 7.667/12 - (7.667/35)^{2.0} = 0.791$$

$\therefore DF_{vi} = 0.791$

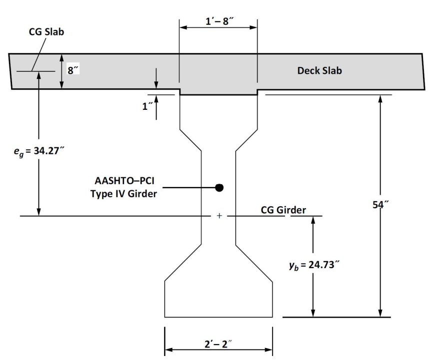

Figure 2.13: Calculation of (e_g) for Example 2.7