

Engineering Mechanics

(1st Semester)

Syllabus

1. Basic Concepts, Analysis of Forces
2. Concepts of Moments and Couples
3. Resultant of Force System
4. Equilibrium
5. Analysis of Structures: Analysis of Truss
6. The Centroid and Center of Gravity
7. Moment of Inertia.

Text book

1. ENGINEERING MECHANICS

Third Edition 2002, A. HIGDON and W. STILS

1.1 Introduction

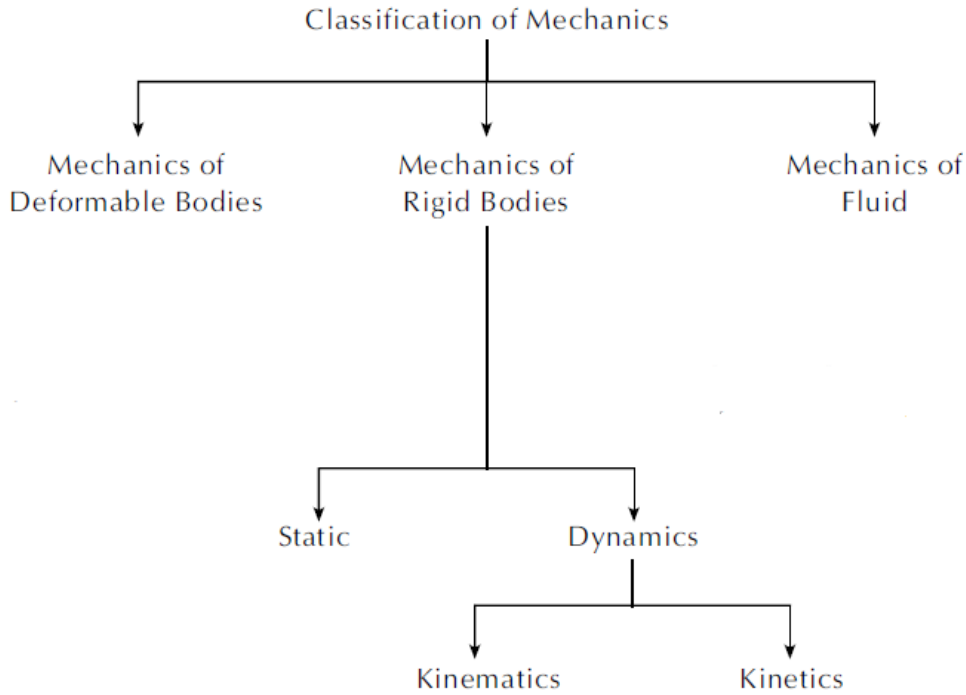
Mechanics: is a branch of the physics which deals with the study of the effect of force system acting on a particle or a rigid body which may be at rest or in motion.

Engineering Mechanics can be subdivided into three branches:

A. Rigid- body mechanics the body is stay in the same shape after applying the forces (no deformation are considered in the body) and this branch is divided into two areas: static and dynamics.

B. Deformable-body mechanics

C. Fluid mechanics.



Static Mechanics:

It is the study of the effect of force system acting on a particle or rigid body which is at rest.

Dynamic Mechanics:

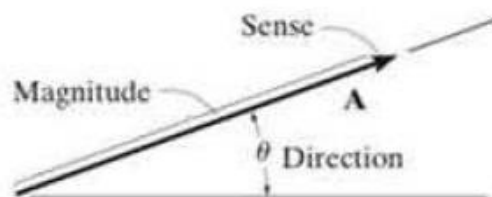
It is the study of the effect of force system acting on a particle or rigid body which is in motion.

1.2 Basic Concepts:

Particle: it is defined as an entity having considerable mass but negligible dimension.

Rigid Body: A solid body having considerable mass as well as dimension.

Vector Quantities: are the quantities which have magnitude and direction, such as force, weight, distance, speed, displacement, acceleration and velocity.

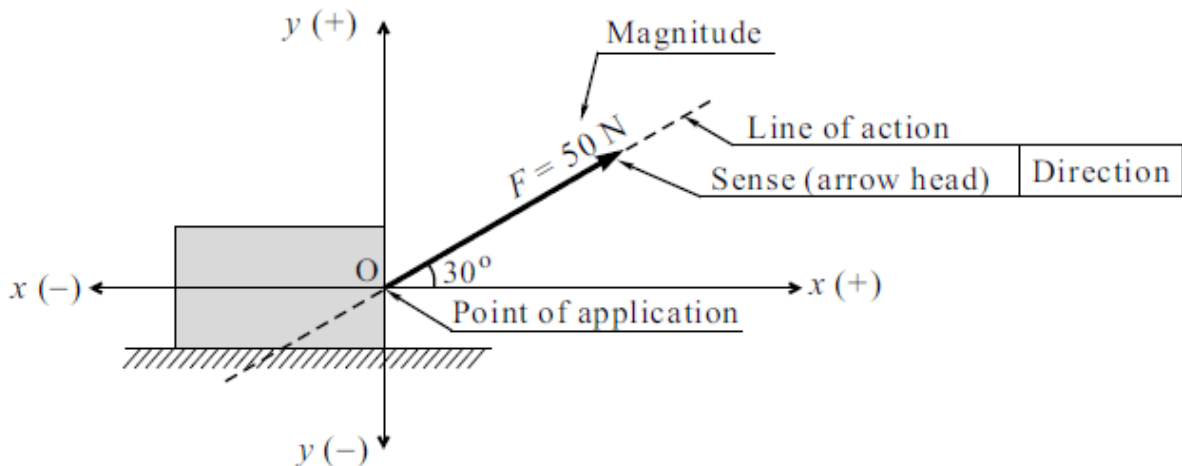


Scalar Quantities: are the quantities which have only magnitude, such as: time, size, sound, density, light and volume.

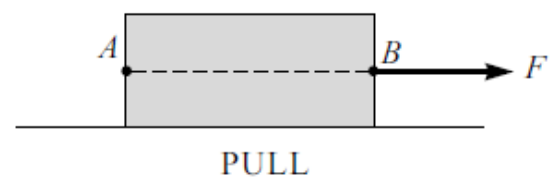
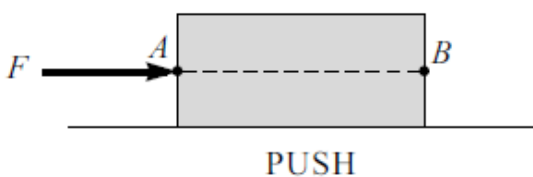
Force: is an action that changes, or tends to change, the state of motion of the body upon which it acts. In general, force is considered as a "push" or "pull" exerted by one body on another.

A complete description of a force must include its:

1. Magnitude
2. Direction and sense.
3. Point of action.

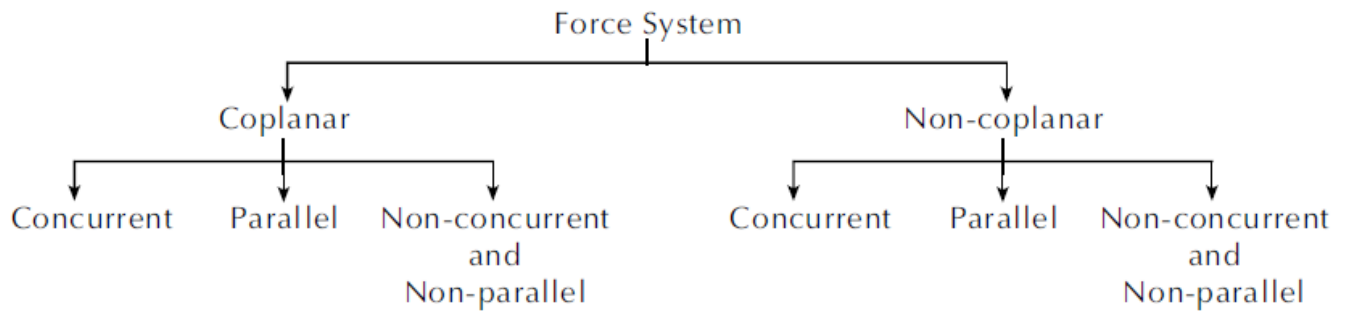


Principle of Transmissibility of Force: It states that the condition of equilibrium or uniform motion of rigid body will remain unchanged if the point of application of a force acting on a rigid body is transmitted to act at any other point along its line of action.



1.3 Force System

Is a number of forces acting in a given situation and can be classified according to the arrangement of the line of action of the forces on the system.



<p>Coplanar Concurrent Force System</p>	<p>Coplanar Parallel Force System</p>	<p>Coplanar Non Concurrent and Non Parallel Force System</p>
<p>Non Coplanar Concurrent Force System</p>	<p>Non Coplanar Parallel Force System</p>	<p>Non Coplanar Non Concurrent and Non Parallel Force System</p>

* Concurrent: all forces pass through a point.

** Coplanar: in the same plane.

***Parallel: parallel line of action.

****Collinear: common line of action.

1.4 Units and their Relations:

QUANTITY	DIMENSIONAL SYMBOL	SI UNITS		U.S. CUSTOMARY UNITS		
		UNIT	SYMBOL	UNIT	SYMBOL	
Mass	M	Base units	kilogram	kg	slug	—
Length	L		meter	m	foot	ft
Time	T		second	s	second	sec
Force	F		newton	N	pound	lb

Quantity	Unit of Measurement (FPS)	Equals	Unit of Measurement (SI)
Force	lb		4.448 N
Mass	slug		14.59 kg
Length	ft		0.304 8 m

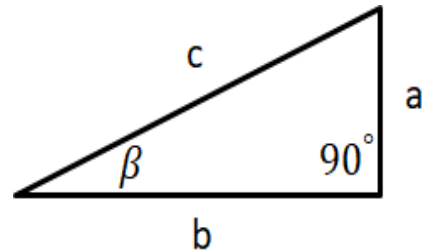
1.5 Trigonometric Relations

A. Right Angle's Triangles

$$\sin \beta = \frac{a}{c}$$

$$\cos \beta = \frac{b}{c}$$

$$\tan \beta = \frac{a}{b}$$



B. Oblique Triangle

1. Sine Law

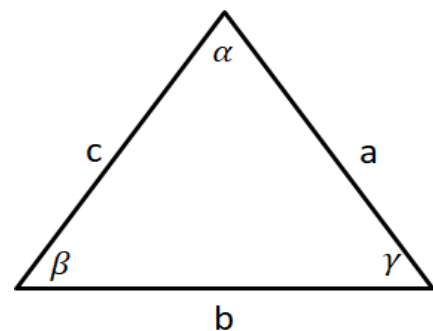
$$\frac{a}{\sin \beta} = \frac{b}{\sin \alpha} = \frac{c}{\sin \gamma}$$

2. Cosine Law

$$a^2 = b^2 + c^2 - 2bc \cos \beta$$

$$b^2 = a^2 + c^2 - 2ac \cos \alpha$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$



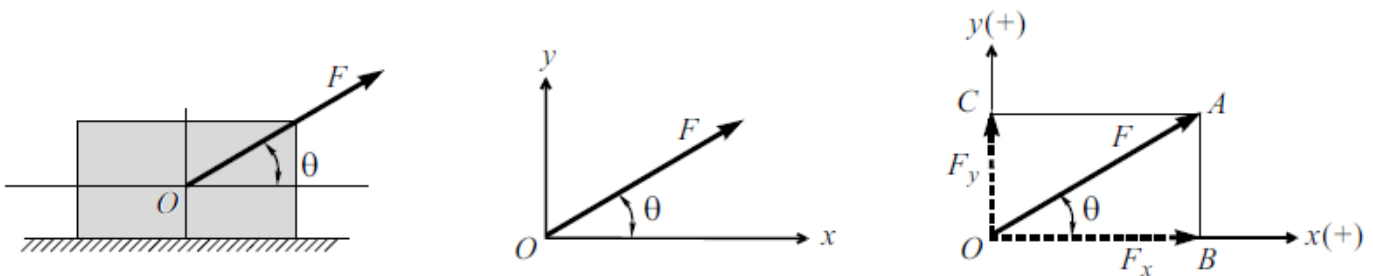
1.6. Composition and Resolution of Force.

There are two common problems in statics involve either finding the resultant force, knowing its components, or resolving a known force into two components.

1.6.1 Finding the Components of a Force (Resolution of Force)

the process of breaking the force into a number of components, which are equivalent to the given forces is called resolution of force.

A. Resolving a force into rectangular components

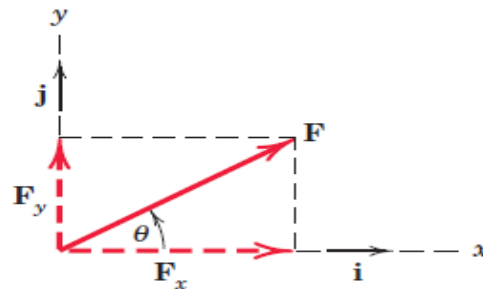


Two Dimensional Force System

Let the force (F) shown below with the direction (θ); we can resolve this force into two components:

1-Horizontal Component (F_x) which lies on X-axis.

2-Vertical Component (F_y) which lies on Y-axis.



$$F_X = F \cos \theta$$

$$F_Y = F \sin \theta$$

$$F = \sqrt{F_X^2 + F_Y^2}$$

In vector expression:-

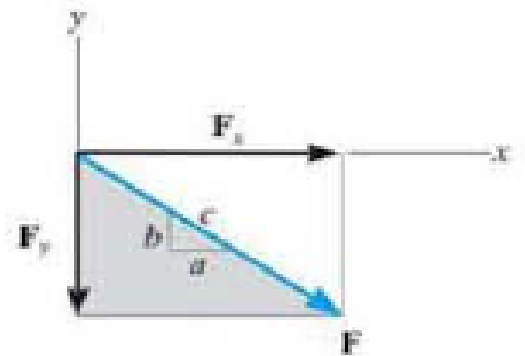
$$F = F_X i + F_Y j = F \cos \theta i + F \sin \theta j$$

$$\theta = \tan^{-1} \frac{F_Y}{F_X}$$

Instead of using the angle however the direction of \mathbf{F} can also be defined using a small slope triangle such as shown in figure below. Since this triangle and the larger shaded triangle are similar, the proportional length of the sides gives:

$$F_X = F \frac{a}{c}$$

$$F_Y = F \frac{b}{c}$$



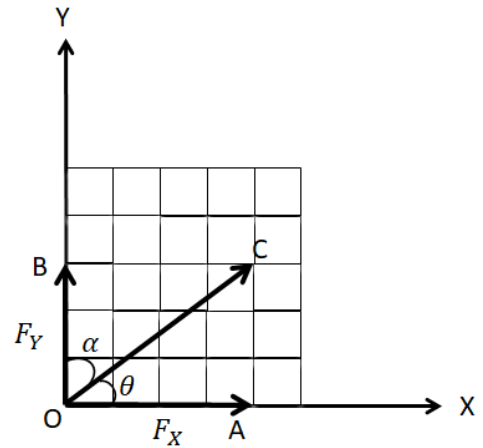
Or in another way;

$$F_x = OA = F \cos \theta = \left(\frac{4}{5}\right) F =$$

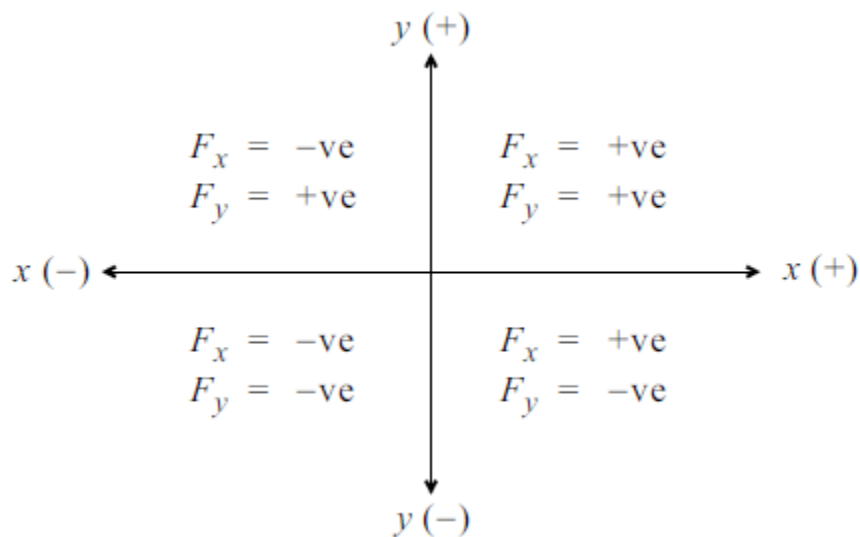
0.8 F to the right through O

$$F_y = OB = F \sin \theta = \left(\frac{3}{5}\right) F =$$

0.6 F upward through O

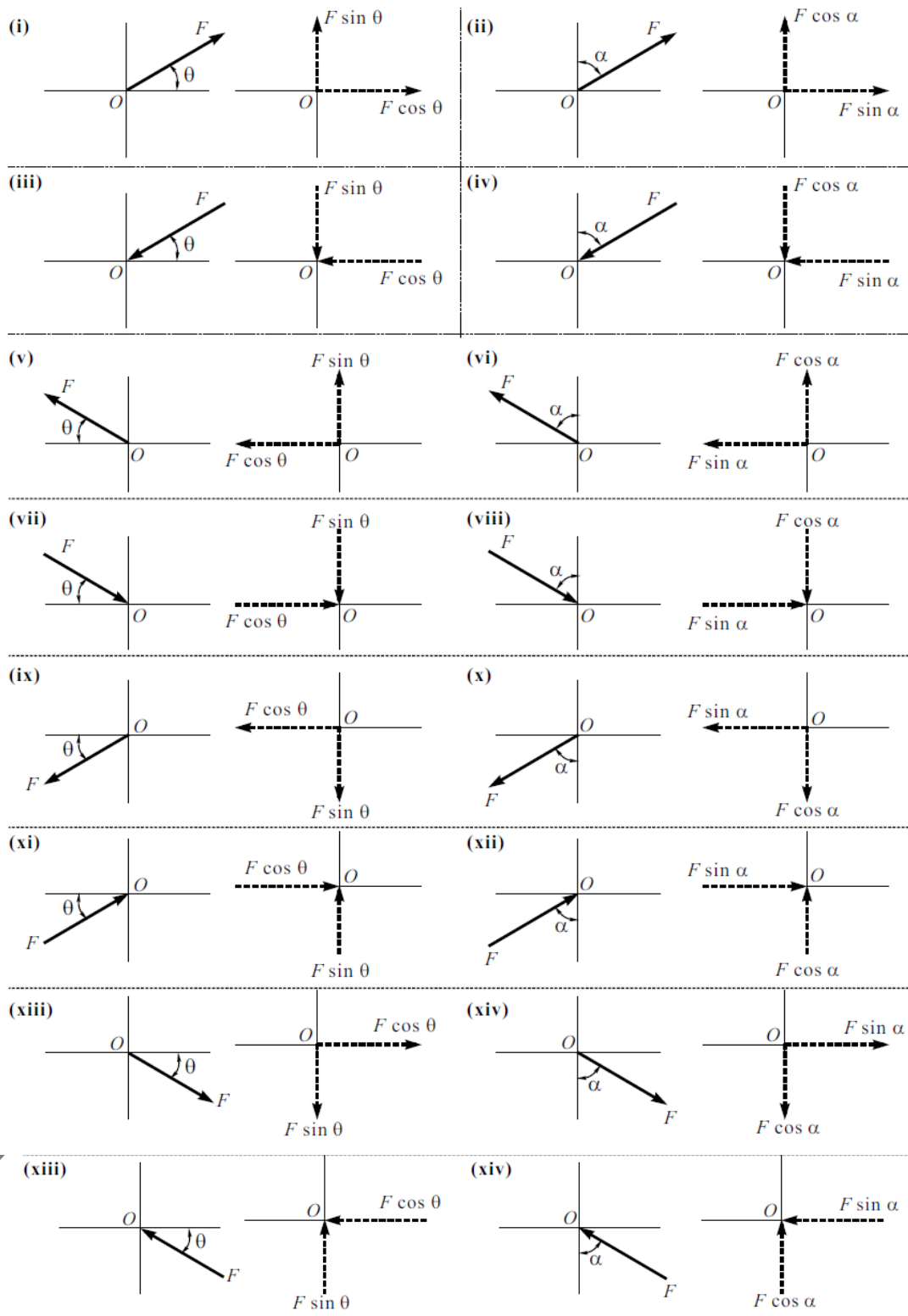


Note (1)

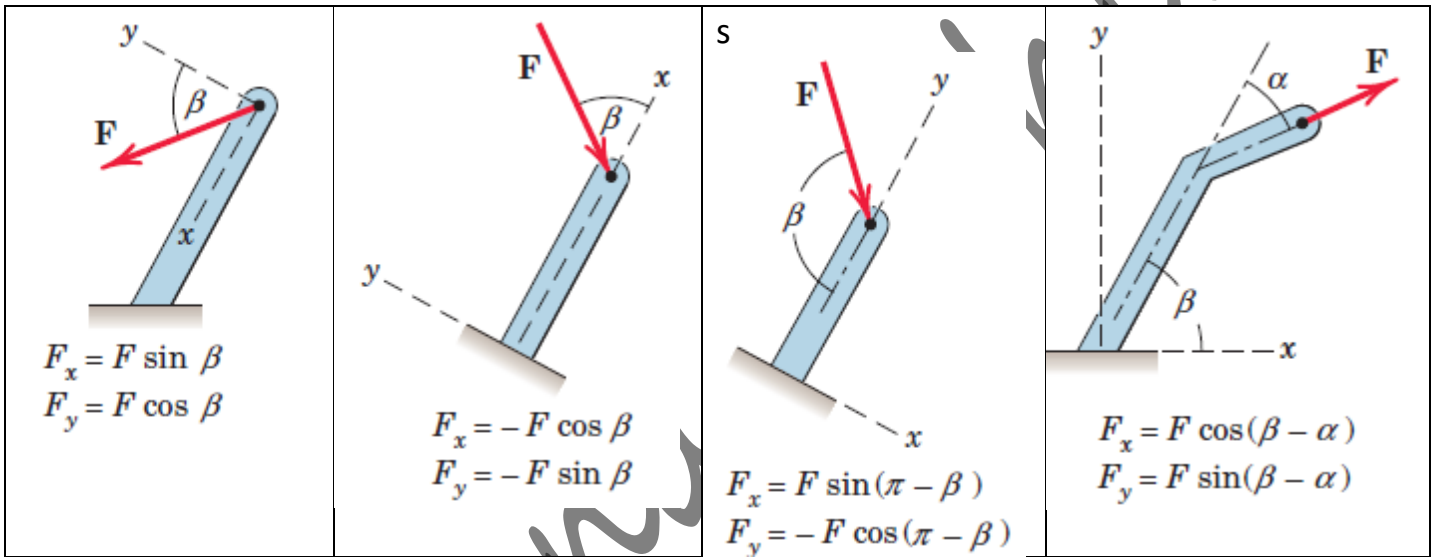


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Note(2)



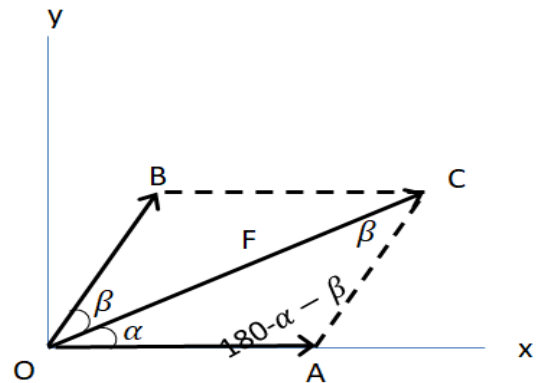
Note(3): Dimensions are not always given in horizontal and vertical directions, angles need not be measured counterclockwise from the X-axis, and the origin of coordinates need not be on the line of action of a force. Therefore, it is essential that we be able to determine the correct components of a force no matter how the axes are oriented or how the angles are measured. The figure below suggests a few typical examples of force resolution in two dimensions.



B. Resolving a force into nonrectangular components

$$\frac{OA}{\sin \beta} = \frac{F}{\sin(180 - \alpha - \beta)}$$

$$F = \sqrt{OA^2 + OB^2 - 2(OA)(OB) \cos(180 - \alpha - \beta)}$$



Examples

Example (1)

The direction of the force (**P**) is (**30°**), find the horizontal components if the vertical components is (**30N**).

Solution:-

From the diagram shown:

$$F_Y = 30N \quad \uparrow$$

$$F_Y = F \sin \theta$$

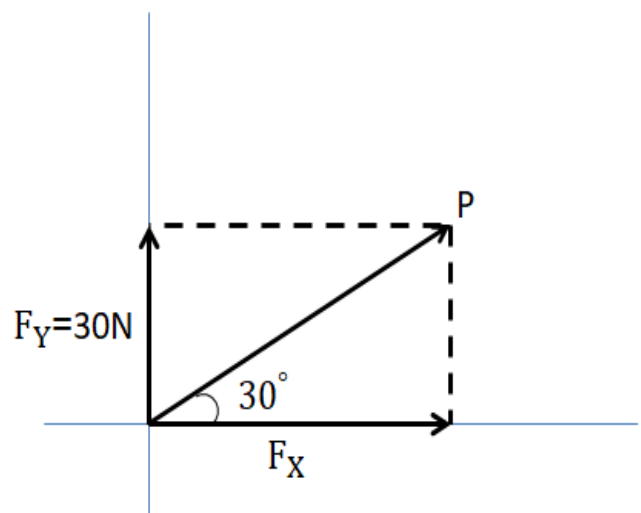
$$30 = P \sin \theta$$

$$30 = P \cdot 0.5$$

$$P = 60N$$

$$F_X = F \cos \theta = 60 \cos 30$$

$$F_X = 60 \times \frac{\sqrt{3}}{2} = 30\sqrt{3}N \quad \rightarrow$$



Example(2)

Determine the magnitude and direction of force (**P**), if the horizontal and vertical components are (**20N**),(**40N**) respectively.

Solution:

$$F = \sqrt{(F_X)^2 + (F_Y)^2}$$

$$F = \sqrt{(20)^2 + (40)^2} = \sqrt{400 + 1600} = \sqrt{2000} = 44.72\text{N}$$

$$\theta = \tan^{-1}\left(\frac{F_Y}{F_X}\right) = \tan^{-1}\left(\frac{40}{20}\right) = 63.43^\circ$$

Example(3)

Find the two components of the force (**100 N**) if:

$$\theta = 30^\circ, 120^\circ, 270^\circ$$

Solution:

$$\theta = 30^\circ$$

$$F_x = F \cdot \cos \theta =$$

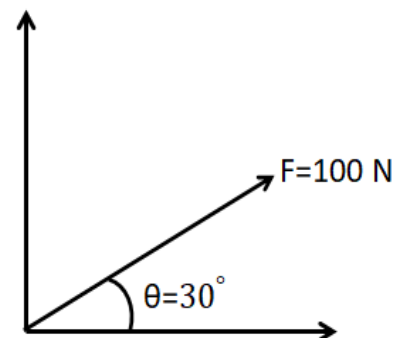
$$100 \times \cos 30 = 100 \times \frac{\sqrt{3}}{2} =$$

$$50 \sqrt{3} \text{ N} \rightarrow$$

$$F_y = F \cdot \sin \theta =$$

$$100 \times \sin 30 = 100 \times 0.5 =$$

$$50 \text{ N} \uparrow$$



$$\theta = 120^\circ$$

$$F_x = F \cdot \cos \theta =$$

$$100 \times \cos 120 =$$

$$100 \times (-0.5) = -50 \text{ N} \quad \leftarrow$$

$$F_y = F \cdot \sin \theta =$$

$$100 \times \sin 120 = 100 \times \frac{\sqrt{3}}{2} =$$

$$50\sqrt{3} \text{ N} \quad \uparrow$$

$$\theta = 270^\circ$$

$$F_x = F \cdot \cos \theta =$$

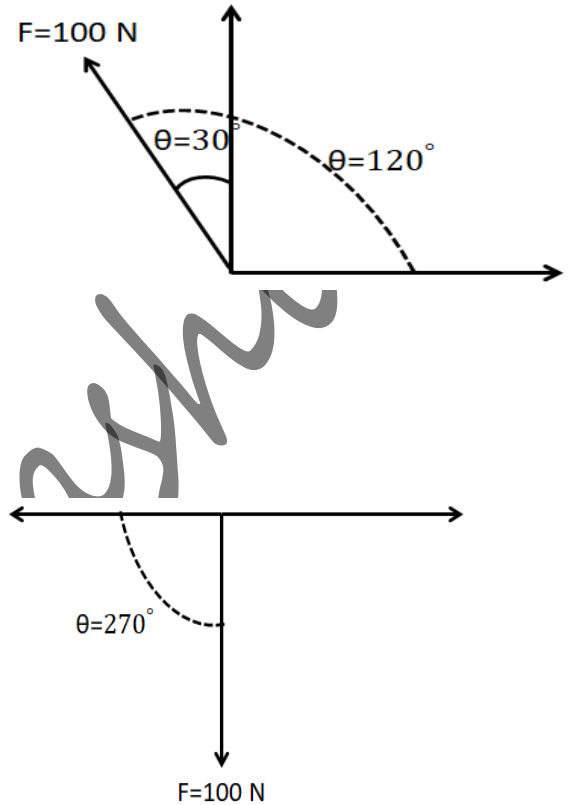
$$100 \times \cos 270 = 100 \times 0 =$$

$$0$$

$$F_y = F \cdot \sin \theta =$$

$$100 \times \sin 270 =$$

$$100 \times (-1) = -100 \text{ N} \quad \downarrow$$



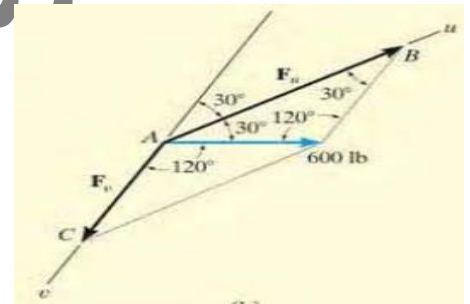
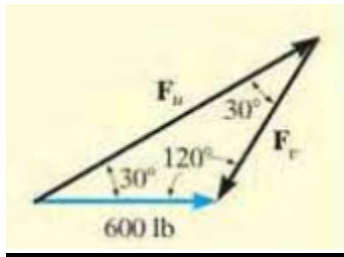
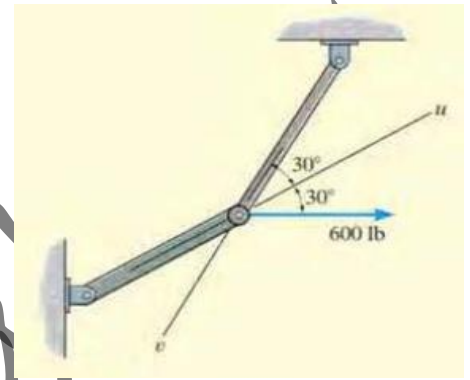
Example(4)

Resolve the horizontal **600 lb** force shown in figure into components acting along the **u** and **v** axes and determine the magnitudes of these components.

Solution:-

$$\frac{F_u}{\sin 120} = \frac{600}{\sin 30} \quad F_u = 1039 \text{ lb}$$

$$\frac{F_v}{\sin 30} = \frac{600}{\sin 30} \quad F_v = 600 \text{ lb}$$



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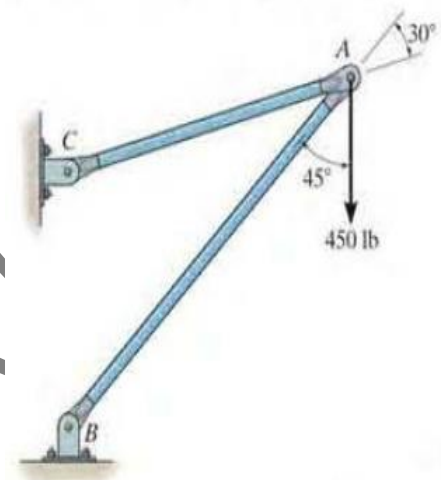
Example(5)

The force **F=450 lb** acts on the frame. Resolve this force into components acting along members **AB** and **AC**,and determine the magnitude of each components.

Solution:-

$$\frac{F_{AB}}{\sin 105} = \frac{450}{\sin 30} \quad F_{AB} = 869 \text{ lb}$$

$$\frac{F_{AC}}{\sin 45} = \frac{450}{\sin 30} \quad F_{AC} = 636 \text{ lb}$$



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Example (6)

The forces F_1 , F_2 , and F_3 , all of which act on point A of the bracket, are specified in three different ways. Determine the x and y components of each of the three forces.

Solution:-

The components of F_1 are

$$F_{1x} = 600 \cos 35^\circ = 491 \text{ N} \rightarrow$$

$$F_{1y} = 600 \sin 35^\circ = 344 \text{ N} \uparrow$$

The components of F_2 are

$$F_{2x} = -500 \left(\frac{4}{5}\right) = -400 \text{ N} \leftarrow$$

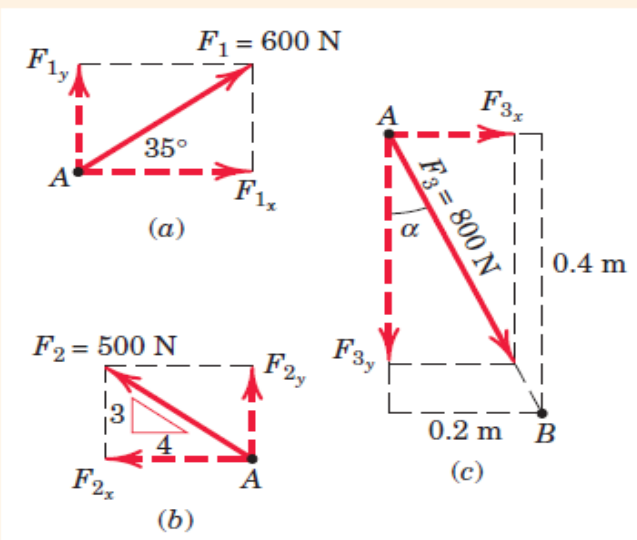
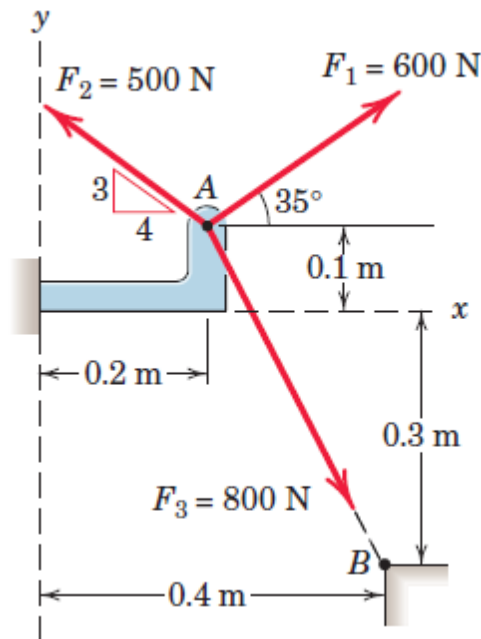
$$F_{2y} = 500 \left(\frac{3}{5}\right) = 300 \text{ N} \uparrow$$

The components of F_3 are

$$\alpha = \tan^{-1}\left(\frac{0.2}{0.4}\right) = 26.6^\circ$$

$$F_{3x} = F_3 \sin \alpha = 800 \sin 26.6^\circ = 358 \text{ N} \rightarrow$$

$$F_{3y} = -F_3 \cos \alpha = -716 \text{ N} \downarrow$$



Example(7)

Determine the x and y components of F_1 and F_2 acting on the boom shown in figure.

Solution:-

By the parallelogram law. F_1 is resolved into x and y components, Fig (b), since F_{1x} acts in the -x direction, and F_{1y} acts in the +y direction, we have.

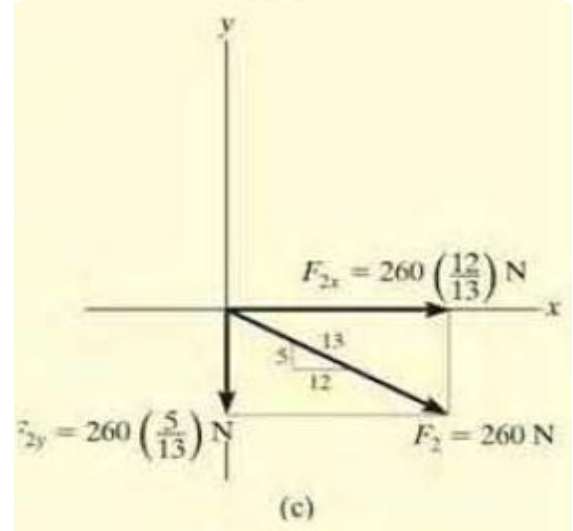
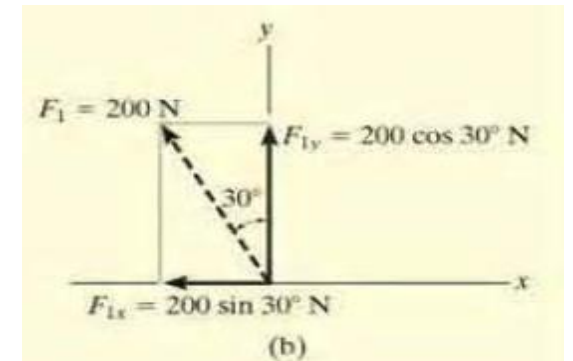
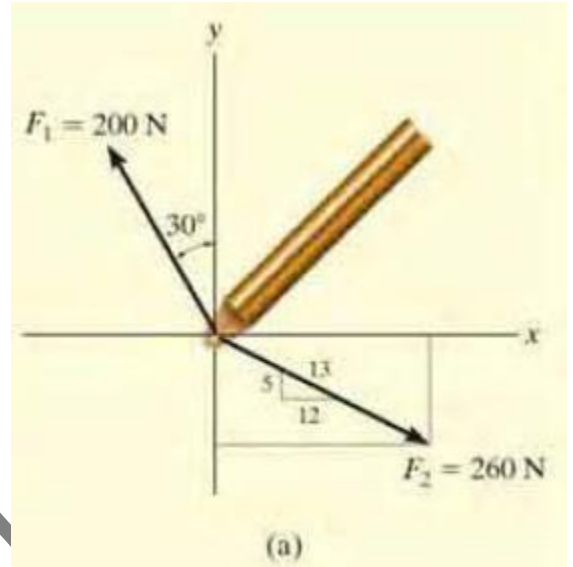
$$F_{1x} = -200 \sin 30 = -100 \text{ N} \leftarrow$$

$$F_{1y} = 200 \cos 30 = 173 \text{ N} \uparrow$$

$$\frac{F_{2x}}{260} = \frac{12}{13} \quad F_{2x} = 260 \left(\frac{12}{13} \right) = 240 \text{ N} \rightarrow$$

similarly

$$F_{2y} = 260 \left(\frac{5}{13} \right) = 100 \text{ N} \downarrow$$



Example(8)

The **500 N** force **F** is applied to the vertical pole as shown in figure.

1. Determine the components of the force **F** along the \hat{x} and \hat{y} axis.
2. Determine the components of the force **F** along the x and y axis.

Solution:-

1. From Fig (b)

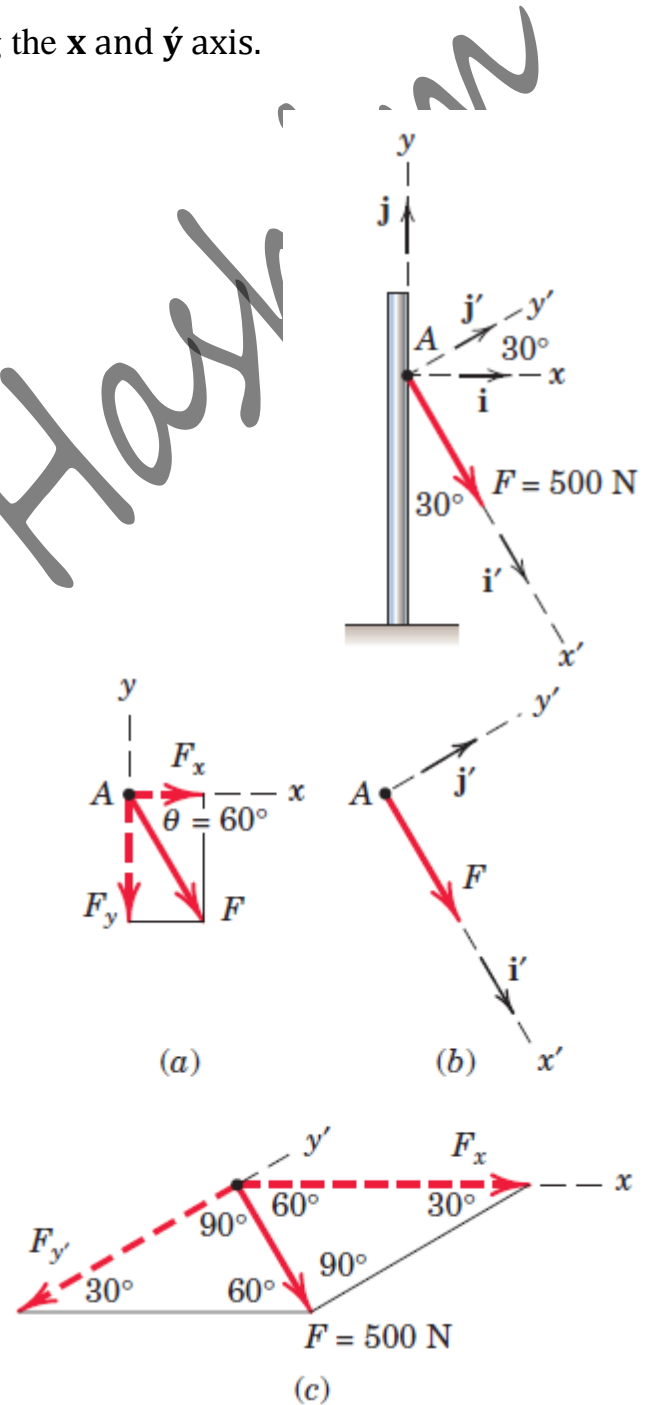
$$F_{\hat{x}} = 500\text{ N} \rightarrow F_{\hat{y}} = 0$$

2. The components of **F** in the x and y directions are nonrectangular and are obtained by completing the parallelogram as shown in fig (c). The magnitudes of the components may be calculated by the law of sines. Thus,

$$\frac{|F_x|}{\sin 90} = \frac{500}{\sin 30} \quad |F_x| = 1000\text{N}$$

$$\frac{|F_y|}{\sin 60} = \frac{500}{\sin 30} \quad |F_y| = 866\text{N}$$

$$= -866\text{ N} \downarrow F_x = 1000\text{ N} \rightarrow F_y$$



Three Dimensional Force System

Resolving a force into rectangular components

The force \mathbf{F} acting at point O in figure has the rectangular components F_x , F_y , F_z , where

$$F_x = F \cos \theta_x \quad \cos \theta_x = \frac{F_x}{F}$$

$$F_y = F \cos \theta_y \quad \cos \theta_y = \frac{F_y}{F}$$

$$F_z = F \cos \theta_z \quad \cos \theta_z = \frac{F_z}{F}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

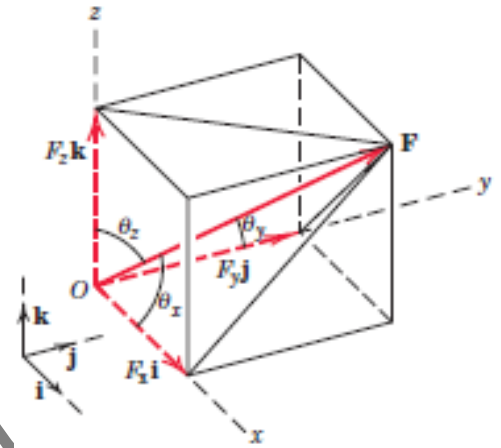
In vector expression:-

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k}$$

$$\mathbf{F} = F(\mathbf{i} \cos \theta_x + \mathbf{j} \cos \theta_y + \mathbf{k} \cos \theta_z)$$

Note:-

The cosine of θ_x , θ_y and θ_z are called direction cosine.



Example(9)

A force \mathbf{F} with a magnitude of **100 N** is applied at the origin \mathbf{O} of the axes $\mathbf{x-y-z}$ as shown. The line of action of \mathbf{F} passes through a point \mathbf{A} whose coordinates are **3 m, 4 m, and 5 m**. Determine the \mathbf{x} , \mathbf{y} , and \mathbf{z} scalar components of \mathbf{F} .

Solution:-

$$\text{Length of } OA = \sqrt{3^2 + 4^2 + 5^2} = 7.07$$

$$\cos \theta_x = \frac{3}{7.07}$$

$$\cos \theta_y = \frac{4}{7.07}$$

$$\cos \theta_z = \frac{5}{7.07} = 0.707$$

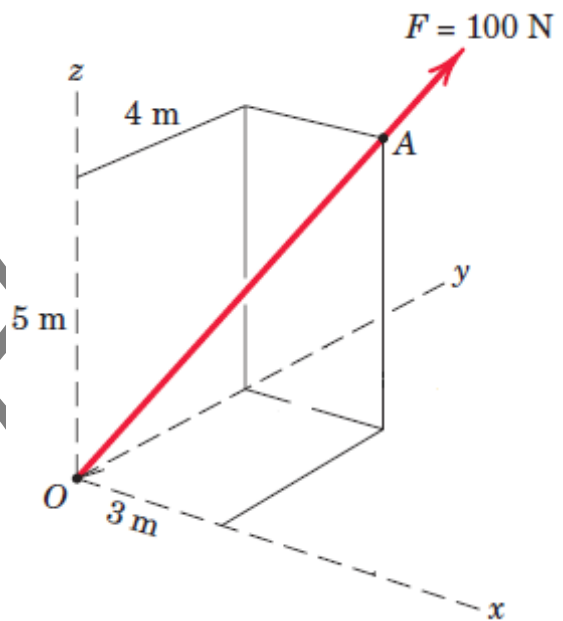
$$F_x = F \cos \theta_x = 100 \frac{3}{7.07} = 42.4$$

$$F_y = F \cos \theta_y = 100 \frac{4}{7.07} = 56.6$$

$$F_z = F \cos \theta_z = 100 \frac{5}{7.07} = 70.7$$

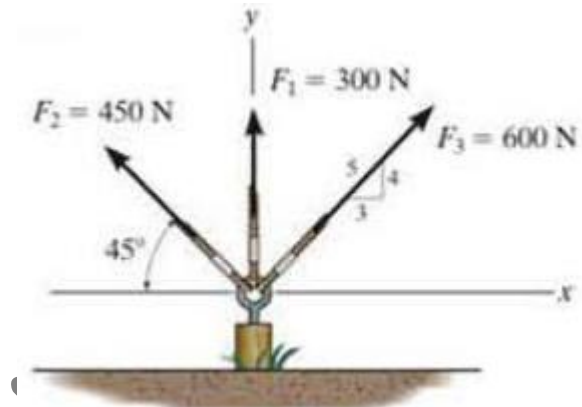
To express the force as a vector

$$\vec{F} = 100 \left(\frac{3}{7.07} \vec{i} + \frac{4}{7.07} \vec{j} + \frac{5}{7.07} \vec{k} \right)$$

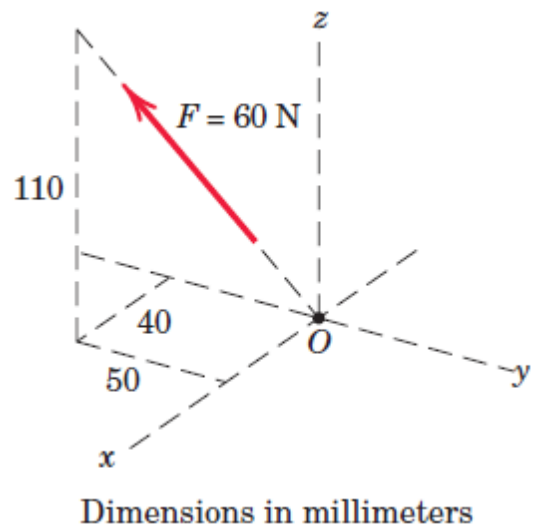


Home Work(1)

1. Resolve each force acting on the post into its x and y components.



2. Express \mathbf{F} as a vector in terms of the unit vectors \mathbf{i} , \mathbf{j} , and \mathbf{k} . Determine the angle between \mathbf{F} and the y-axis.



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