2.Concepts of Moments and Couples

2.1 Moment of Force (Torque)

The moment of a force is the ability of the force to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force.

Mathematically:

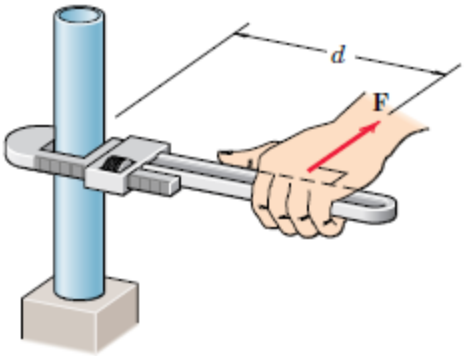
The moment of a force= The applied force **X** Perpendicular distance

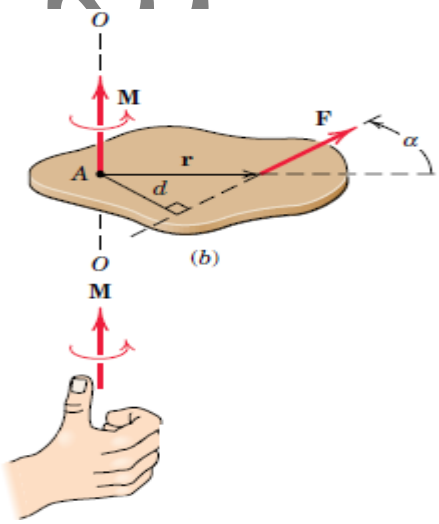
M=F.d

M= The moment of a force (N.m)

F= Applied force (N)

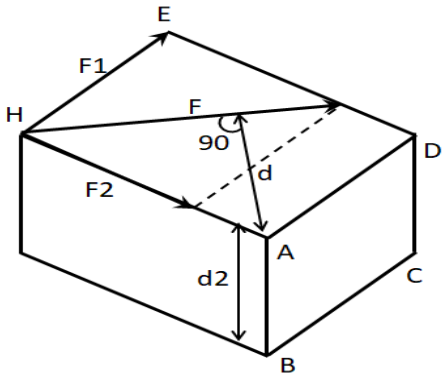
d= Perpendicular distance between the point of action of the force and moment center.





Notes:-

1.If the force does not lie in a plane perpendicular to the moment axis, then the force may be resolved into two components, one parallel to the axis and the other is in plane perpendicular to the axis.



For Example:-

The moment of F about the axis BC in the above figure, may be formed by resolving it into two components:-

The moment of the component F1 about BC=0 F1// BC

The moment of the components F2 about BC = F2 d2

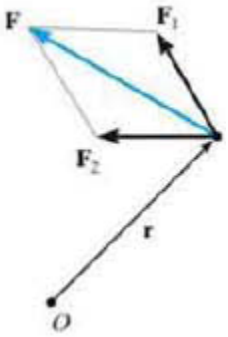
Then the moment of force F about BC=F2 d2

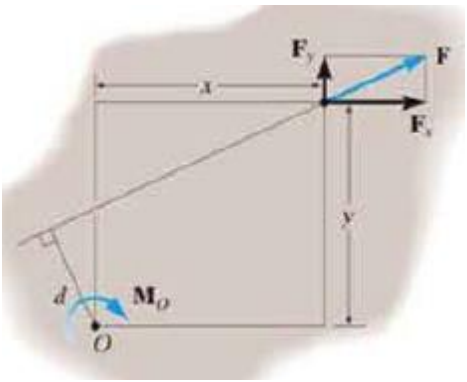
2.1.1 Principle of Moments

It states that the moment of a force about a point is equal to the sum of the moments of the components of the force about the point.

For example:-

Consider the moment of the force F and two of its components about point O as shown in figure below.





$since\ F = F_1 + F_2$

$M_0 = F.r = (F_1 + F_2)r = F_1 r + F_2 r$

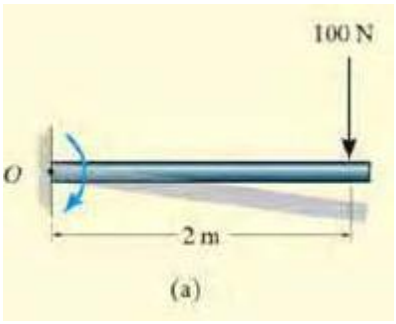
For two dimensional problems, above figure, we can use the principle of moments by resolving the force into its rectangular components and then determine the moment using scalar analysis, thus,

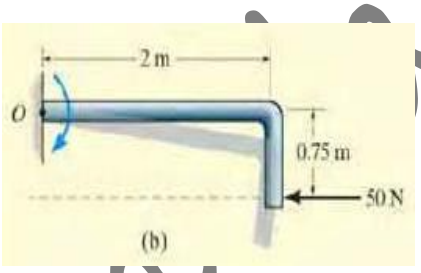
$M_O = F_x y - F_y x$

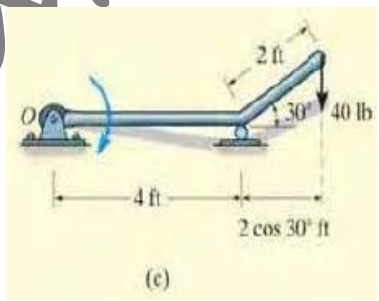
Examples

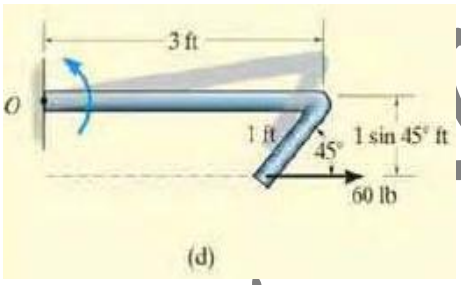
Example (1):-

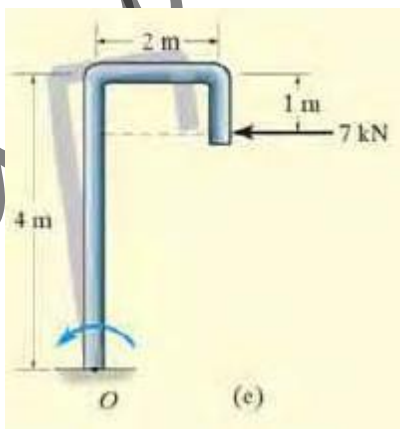
For each case illustrated in the figure, determine the moment of the force about point O.











Solution:-

Fig (a) $M_O = (100\ \text{N})(2\text{m}) = 200\ \text{N.m}$

Fig (b) $M_O = (50\ \text{N})(0.75\ \text{m}) = 37.5\ \text{N.m}$

Fig (c) $M_O = (40\ \text{lb})(4\ \text{ft} + 2\cos 30\ \text{ft}) = 299\ \text{lb.ft}$

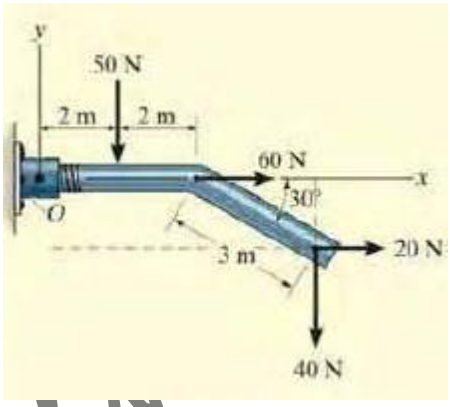
Fig (d) $M_O = (60\ \text{lb})(1\sin 45\ \text{ft}) = 42.4\ \text{lb.ft}$

Fig (e) $M_O = (7\ \text{kN})(4\text{m} - 1\text{m}) = 21\ \text{kN.m}$

Example(2):-

Determine the moment of the force system about point O.

Solution:-



$$M_O = \sum M\,d$$

$$M_O = (-50\ \text{N})(2\text{m}) + (60\ \text{N})(0) +$$

$$20\text{N}(3\sin 30\ \text{m}) - (40\ \text{N})(4\text{m} + 3\cos 30\ \text{m}) =$$

$$-344\ \text{N.m} = 344\ \text{N.m}$$

Example (3):-

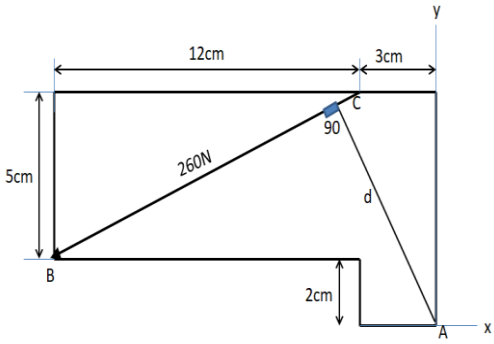
A.Determine the moment of the 260N force shown in figure with respect to point A when

1.the force is resolved into components at B

2.the force is resolved into components at C

B.by means of the principle of moments determine the perpendicular distance from the force to point A.

Solution:-



$$A.F_x = 260\frac{12}{13} = 240\text{N} \leftarrow$$

$$F_y = 260\frac{5}{13} = 100\text{N} \downarrow$$

$$1\text{-}\sum M_A = 240(2) + 100(15) = 1980\ \text{N.cm}$$

$$2\text{-}\sum M_A = 240(7) + 100(3) = 1980\ \text{N.cm}$$

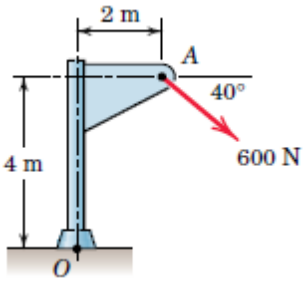
B. The moment of force = ∑moment of its components

$$260(d) = 1980 \rightarrow d = \frac{1980}{260} = 7.61\ cm$$

Example(4):-

Calculate the moment about the base point O of the 600 N

Solution:-



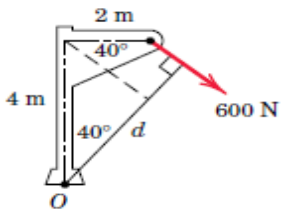
The moment arm to the 600N force is

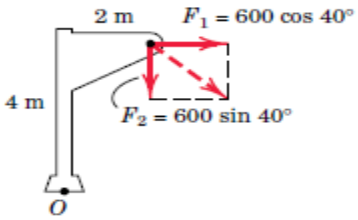
$d = 4\cos 40 + 2\sin 40 = 4.35\ m$

$\mathrm{M_O} = 600(4.35) = 2610\ \mathrm{N.m}$

Another method by replacing the force by its rectangular components at A

$\mathrm{M_O} = (600\cos 40)\mathrm{N}(4\mathrm{m}) + (600\sin 40)\mathrm{N}(2\mathrm{m})$ =2610 N.m

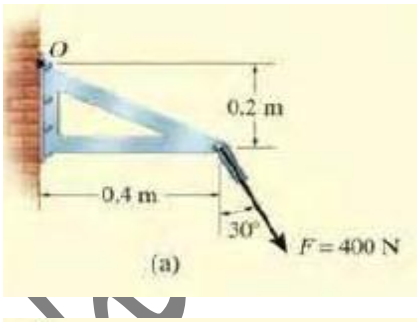




Example(5):-

Force F acts at the end of the angle bracket shown in figure. Determine the moment of the force about point O.

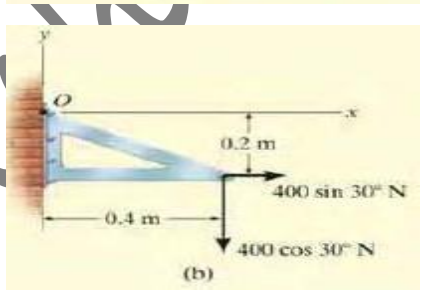
Solution:-



The force is resolved into x and y components as shown in Fig.(b),then

$M_O = 400 \sin 30N(0.2\ m) - 400 \cos 30\ N(0.4m) =$

$-98.6\ N.m = 98.6\ N\ m$



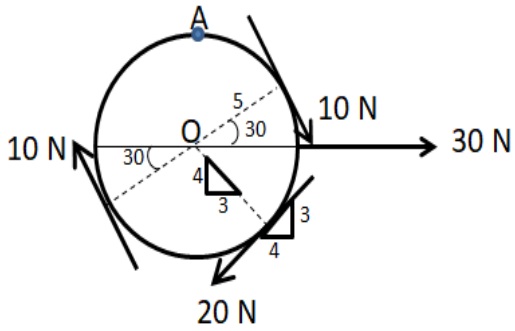
Example(6):-

Determine the moment of the force system

A. with respect to O

B. with respect to A

Solution:-



A.

$M_O = 10(5) + 10(5) + 20(5) = 200\ N.m$

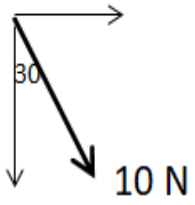
B.

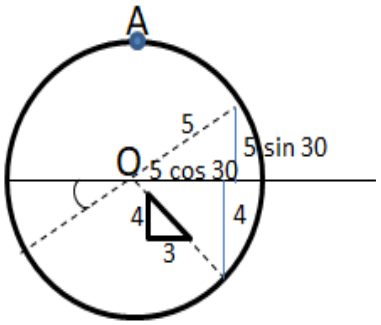
$M_A = 10 \cos 30\ (5 \cos 30) - 10 \sin 30(5 -$

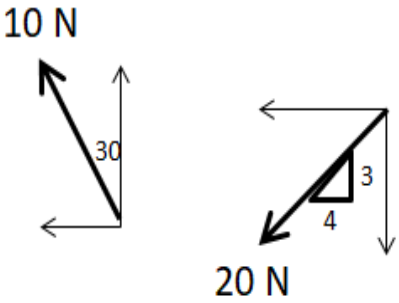
$5 \sin 30) + 10 \cos 30(5 \cos 30) + 10 \sin 30(5 +$

$5 \sin 30) - 30(5) + 20\frac{4}{5}(5 + 4) + 20\frac{3}{5}(3) =$

$130\ N.cm$





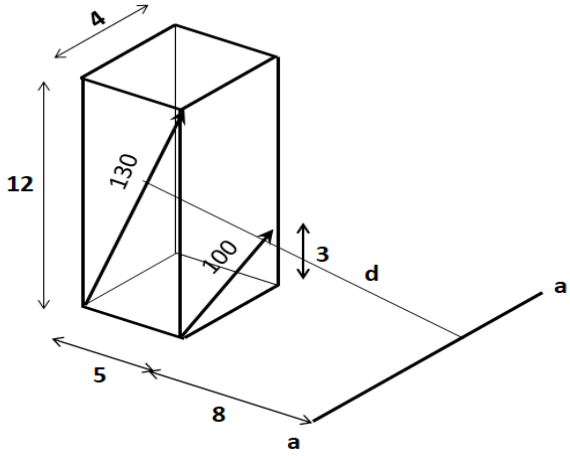


Example (7):-

A.Determine the moment produced by the forces with respect to a-axis

B.Determine the orthogonal distance from the 130N force to the a-axis.

Solution:-



A.Components of force 130N

$$130\frac{12}{\sqrt{12^2+5^2}} = 130\frac{12}{13} = 120 \text{ N}$$

$$130\frac{5}{\sqrt{12^2+5^2}} = 130\frac{5}{13} = 50 \text{ N}$$

Components of force 100 N

$$100\frac{3}{\sqrt{4^2+3^2}} = 100\frac{3}{5} = 60 \text{ N}$$

$$100\frac{4}{\sqrt{4^2+3^2}} = 100\frac{4}{5} = 80 \text{ N}$$

$$\sum M_{a-a} = 120(13) + 60(8) = 1560 + 480$$
$$= 2040 \text{ N.cm}$$

B.

moment of a force $= \sum$ moments of its components

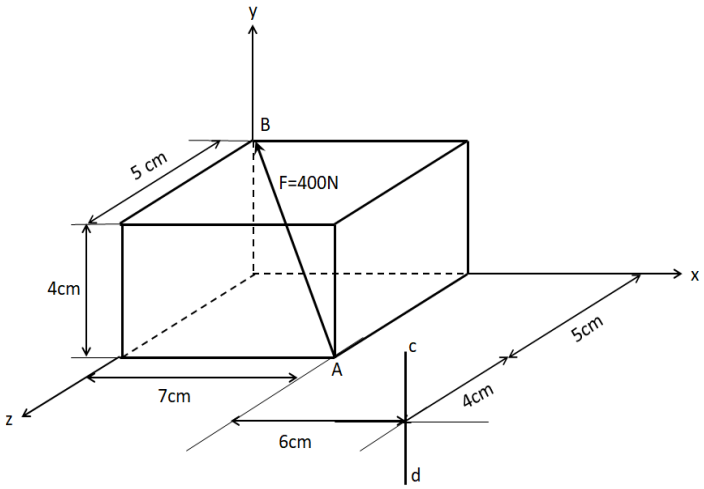
130(d)=1560

$$d = \frac{1560}{130} = 12cm$$

Example(8):-

Determine the moment of force F with respect to the vertical line cd.

Solution:-



$$AB = \sqrt{4^2 + 5^2 + 7^2} = 9.49 \text{ cm}$$

$$\cos\theta_x = \frac{7}{9.49}$$

$$\cos\theta_y = \frac{4}{9.49}$$

$$cos\theta_z = \frac{5}{9.49}$$

$$F_x = F\cos\theta_x = -400\frac{7}{9.49} = -295N$$

$$F_y = F\cos\theta_y = 400\frac{4}{9.49} = 168.6N$$

$$F_z = F\cos\theta_z = -400\frac{5}{9.49} = 211N$$

$$\sum M_{cd} = -295(4) + 211(6)$$
$$= 86N.cm$$

2.2 Couples:-

A couple is defined as two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d. Since the resultant force is zero, the only effect of a couple is to produce a rotation or tendency of rotation in a specified direction. The moment produced by a couple is called a couple moment.

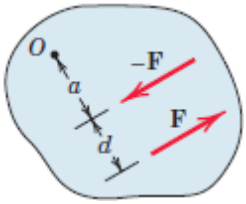
The moment of the couple is the algebraic sum of the moments of its forces about any axis perpendicular to the plane of the couple. The moment is the same for all axis perpendicular to the plane.

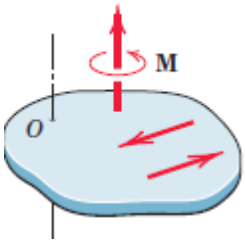
Consider the action of two equal and opposite forces **F** and **–F** a distance **d** apart, as shown in figure. These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis perpendicular to their plane and passing through any point such as O in their plane is the couple M. This couple has a magnitude.

M= F(a+d)-Fa

Or

M=Fd



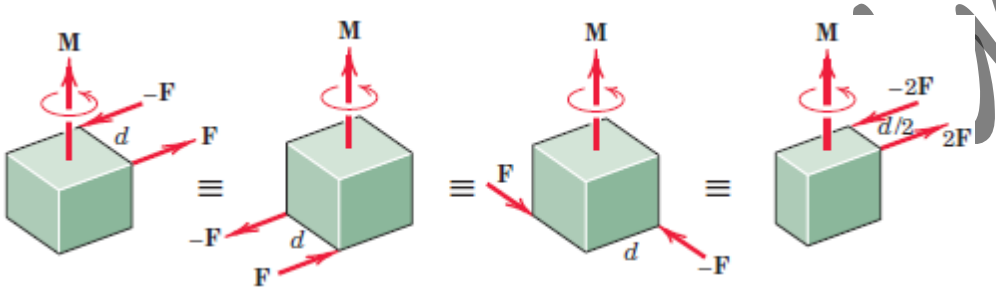


Note especially that the magnitude of the couple is independent of the distance **a** which locates the forces with respect to the moment center O.It follows the moment of a couple has the same value for all moment centers and then the the vector M is not a localized vector and can be moved to any parallel position.

2.2.1Equivalent Couples

If two couples produce a moment with the same magnitude and direction, then these two couples are equivalent.

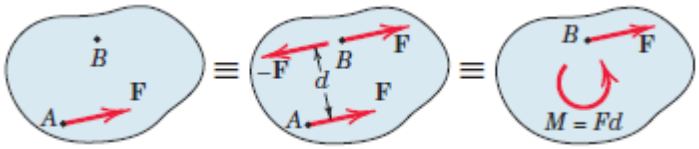
The figure below shows four different configuration of the same couple M.In each of the four cases, the couples are equivalent.



2.2.2Force-Couple Systems

We can represent the dual effect of any force (the tendency to push or pull the body in the direction of the force, and rotate the body about any fixed axis which does not intersect the line of the force) by replacing the given force by an equal parallel force and a couple.

In the figure below the given force F acting at point A is replaced by an equal force at point B and the counterclockwise couple M=F.d.



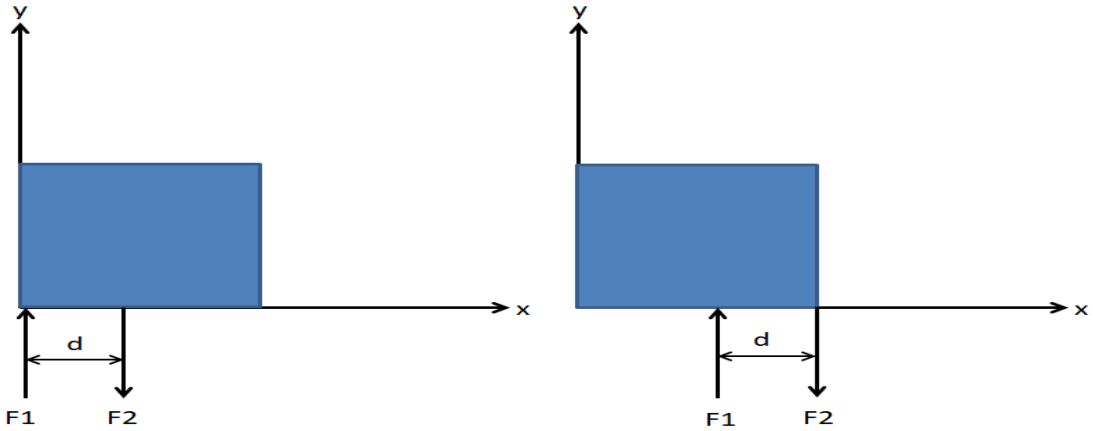
Note :-

A resultant couple moment is simply the vector sum of all couple moments of the system.

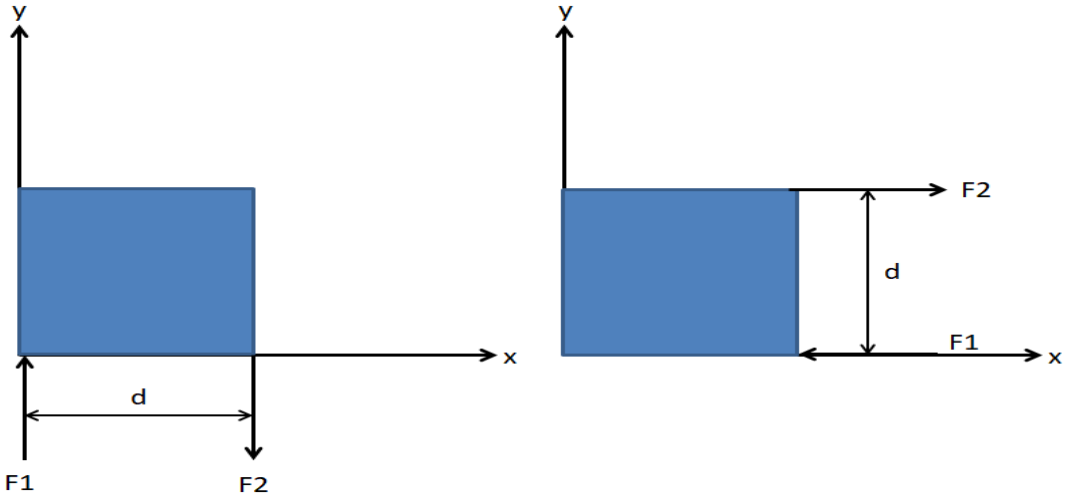
2.2.3Transformation of a Couple

Transformations of a couple are operations on the couple that do not change any of its characteristics (its magnitude, direction and the sense of rotation).

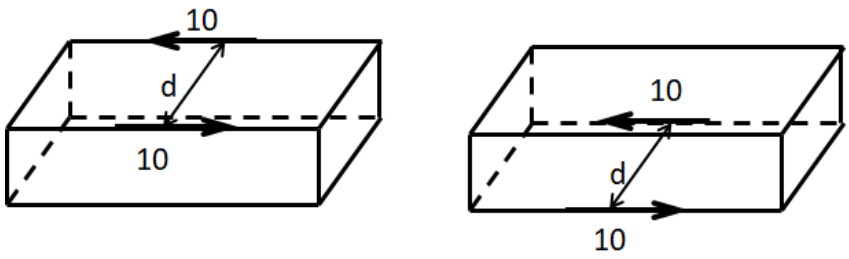
1. The couple is moved to a parallel position in its plane.



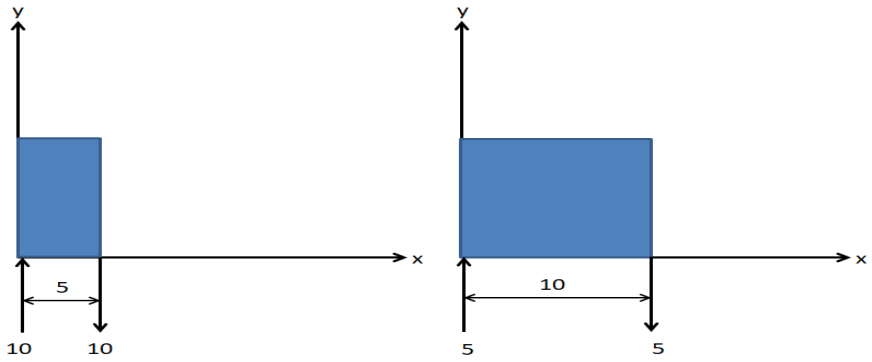
2. The couple is rotate in its plane.



3. The couple is moved to a parallel plane.



4. The couple moments are equaled if the product of the force multiplied by the distance is the same for each couple.



Examples

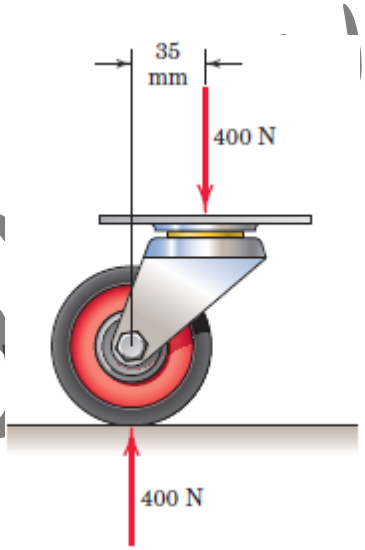
Example(1):-

The caster unit is subjected to the pair of 400N forces shown. Determine the moment associated with these forces.

Solution:-

M=F.d

$$M = 400\frac{35}{1000} = 14\ N.m$$



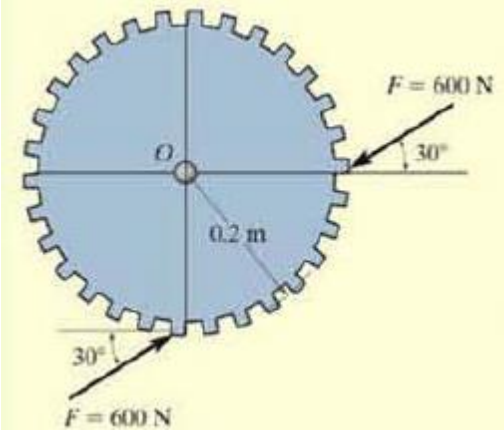
Example (2):-

Determine the magnitude and direction of the couple moment acting on the gear.

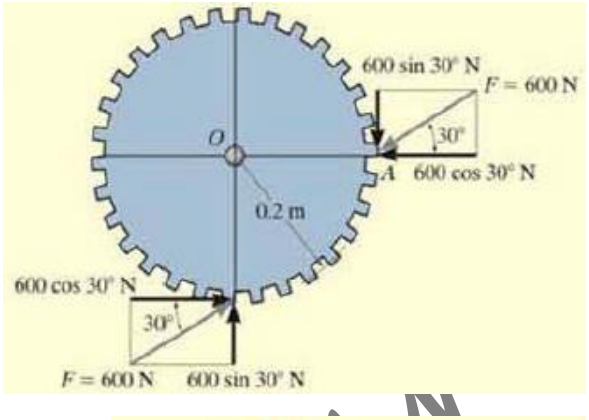
Solution:-

Method 1

The couple moment can be determined by summing the moments of these force components about any point.

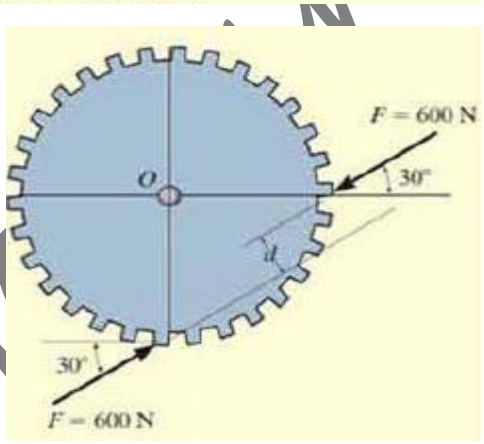


$$M = \sum M_O = (600 \cos 30\ \text{N})(0.2\text{m}) - (600 \sin 30\ \text{N})(0.2\ \text{m}) = 43.9\ \text{N.m}$$



Method 2

M=F.d



Example (3):-

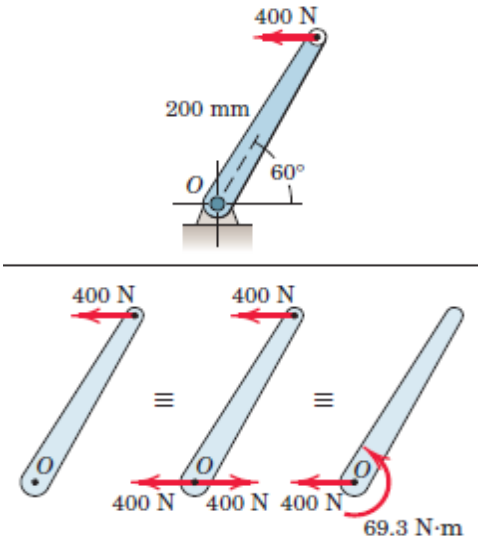
Replace the horizontal 400 N force acting on the lever by an equivalent system consisting of a force at O and a couple.

Solution:-

We apply two equal and opposite 400 N forces at O and identify the CCW couple

M=F.d=400 (0.2 sin 60)=69.3 N.m

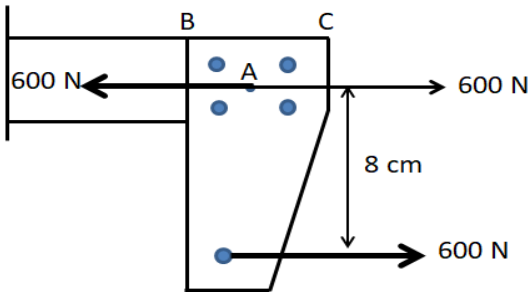
Thus, the original force is equivalent to the 400N at O and the 69.3 N.m couple.



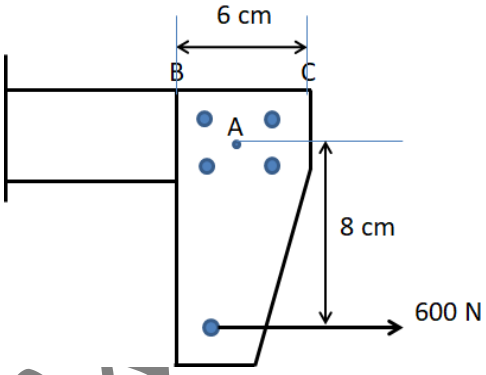
Example(4):-

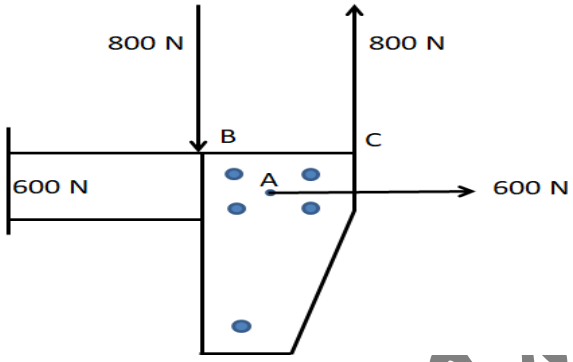
Replace the 600N force by a force through A and a couple whose forces act vertically through point B and C.

Solution:-



600 x 8=4800



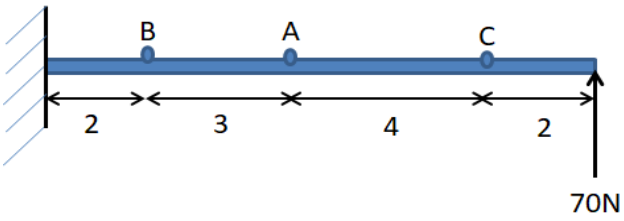


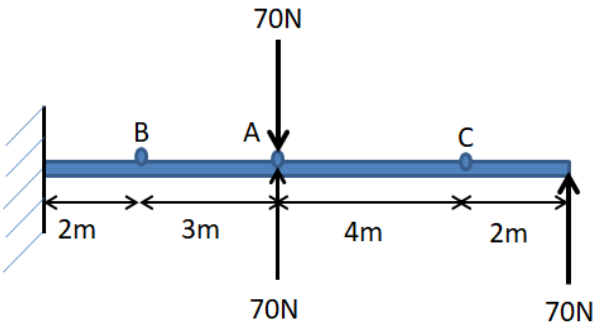
$$\frac{4800}{6} = 800\ \text{N}$$

Example(5):-

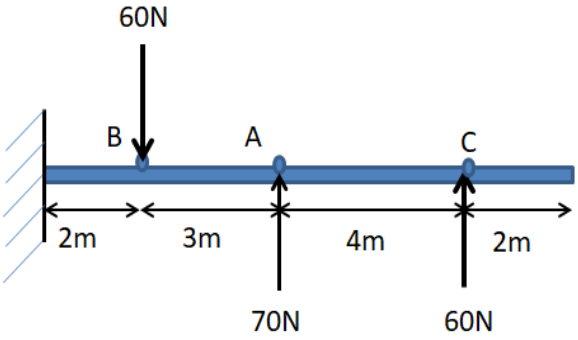
By transformation of a couple replace the 70N force by a force through A and a couple whose forces act vertically through B and C.

Solution:-





70 NX 60 cm=4200 N cm

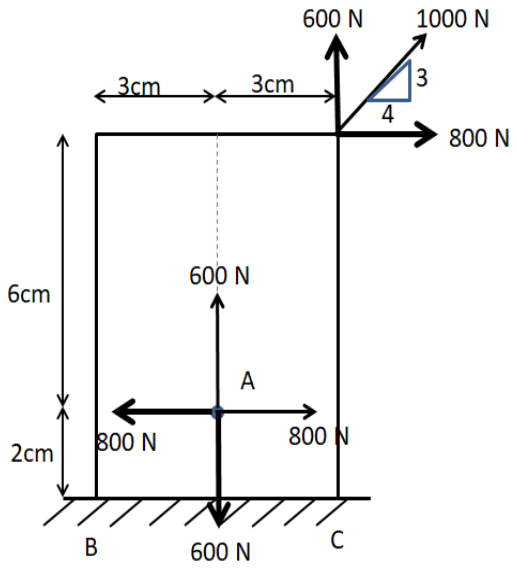


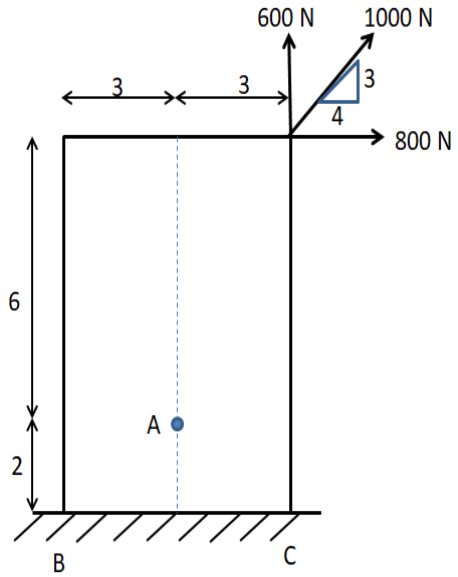
$$\frac{4200\ \text{N.cm}}{70\text{cm}} = 60\ \text{N}$$

Example(6):-

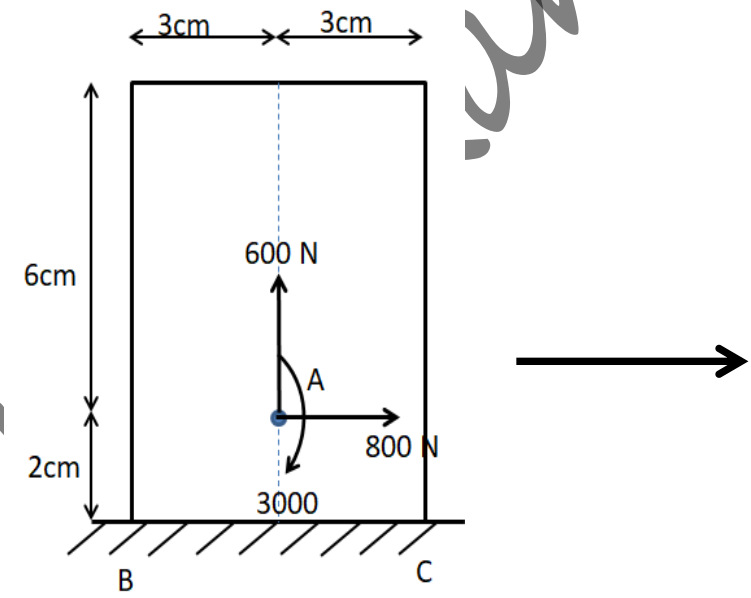
Replace the 1000N force by a force through A and couple whose forces act vertically through B and C.

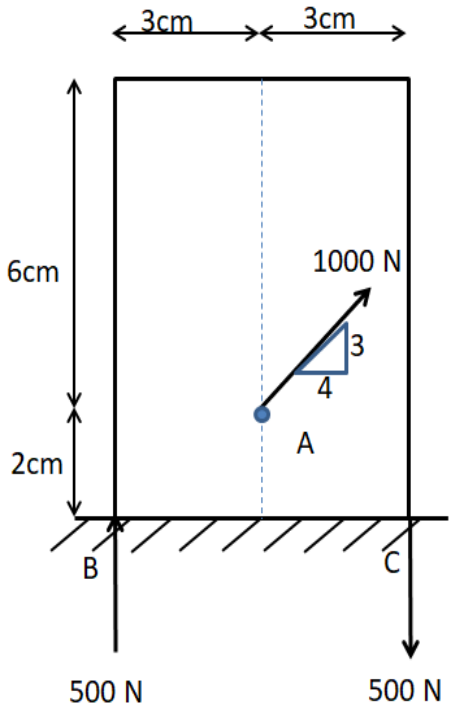
Solution:-





$$\sum M = 800X6 - 600X3 = 3000\ N.cm$$





$$\frac{3000\ N.cm}{6\ cm} = 500\ N$$

Example(7):-

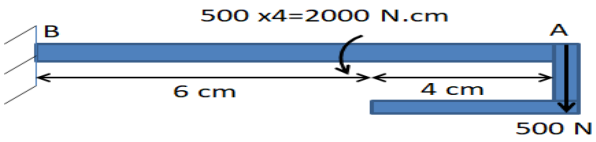
Replace the 500N force by:-

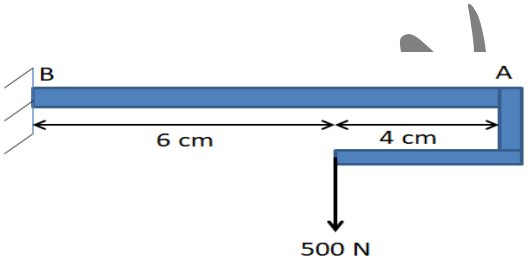
A. A force through A and a couple.

B. A force through B and a couple.

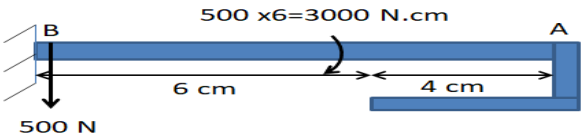
Solution:

A.





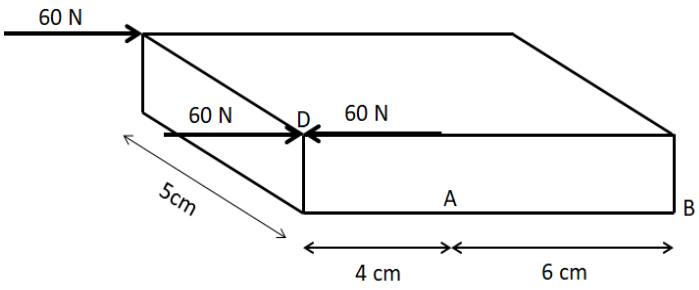
B.



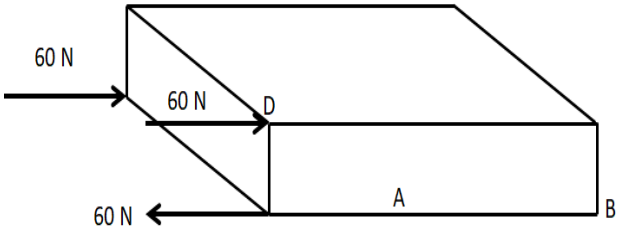
Example (8):-

Replace the 60N force shown in figure by a force through D and a couple whose forces act horizontally through A and B.

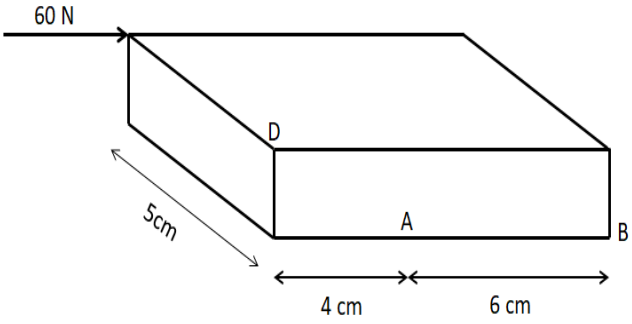
Solution:-

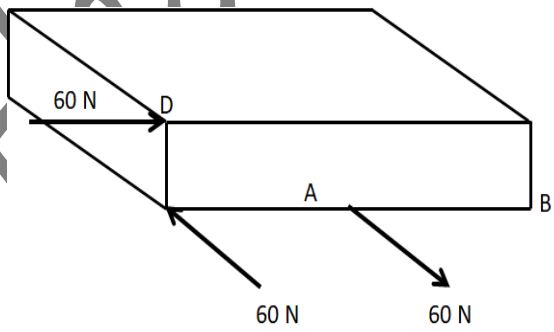


1

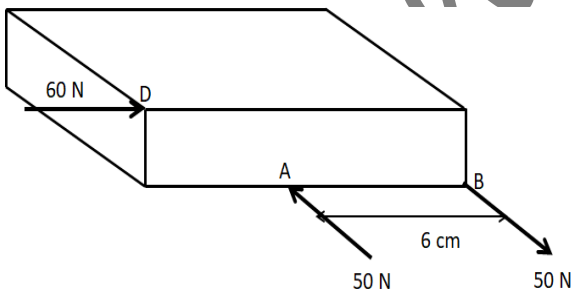


2





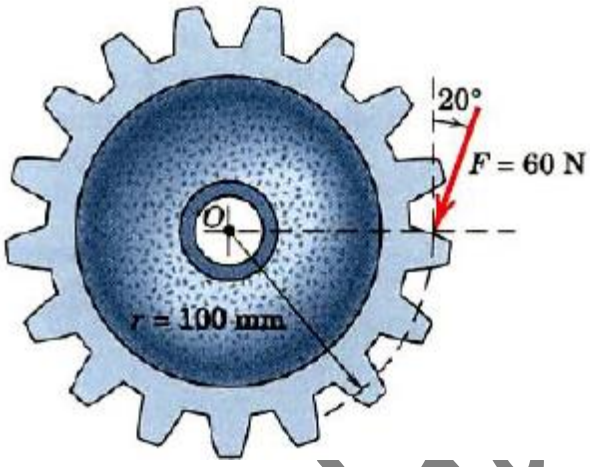
3



4

Home Work(2)

Q1.Find the moment of the force about point O



Q2. The force **F** acts on the end of the pipe at B. Determine (a) the moment of this force about point A (b) the magnitude and direction of a horizontal force, applied at C, which produces the same moment.

Given:

$F = 70$ N

$a = 0.9$ m

$b = 0.3$ m

$c = 0.7$ m

$\theta = 60$ deg