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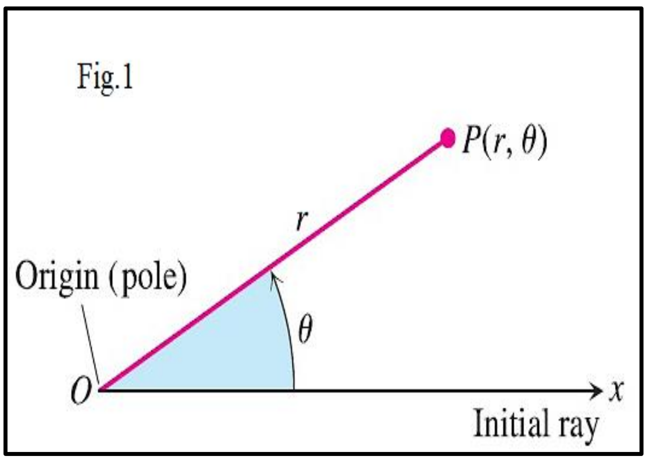
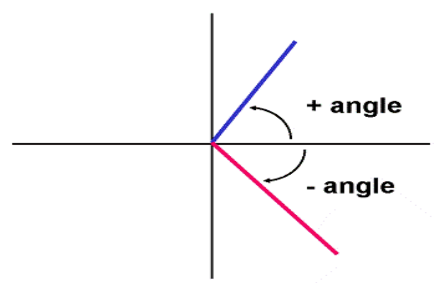
Polar coordinates

1- Definition of Polar Coordinates

The coordinate of any point can be representing by polar form or in terms of pair (r, **θ)**. Fix an origin **O** (called the pole) and an initial ray (x-axis) from **O**. Then each point P can be located by assigning to it a polar coordinate pair (**r, θ**) as shown in figure1 below in which **r** gives the directed distance from origin **O** to **P** and θ gives the directed angle from the initial ray (x-axis) to ray OP.

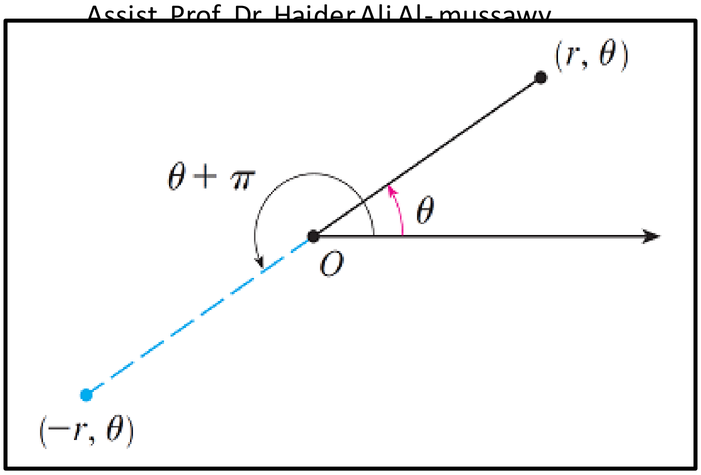


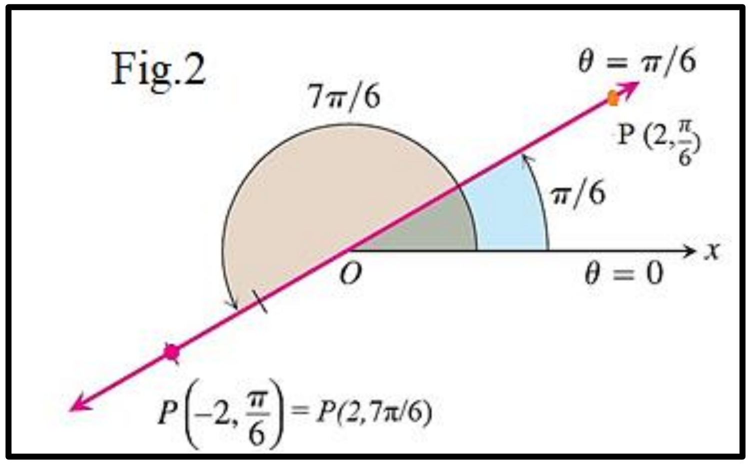
- **θ** is (+ve) if it is measured counter clock wise.
- **θ** is (-ve) if it is measured clock wise.



Representation of any point by polar coordinates starts by drawing ray of the angle **(θ),** then specify the point on the ray according to value of **r.**

The negative value of r means that the distance (OP) is in opposite direction of angle ray. Fig.2

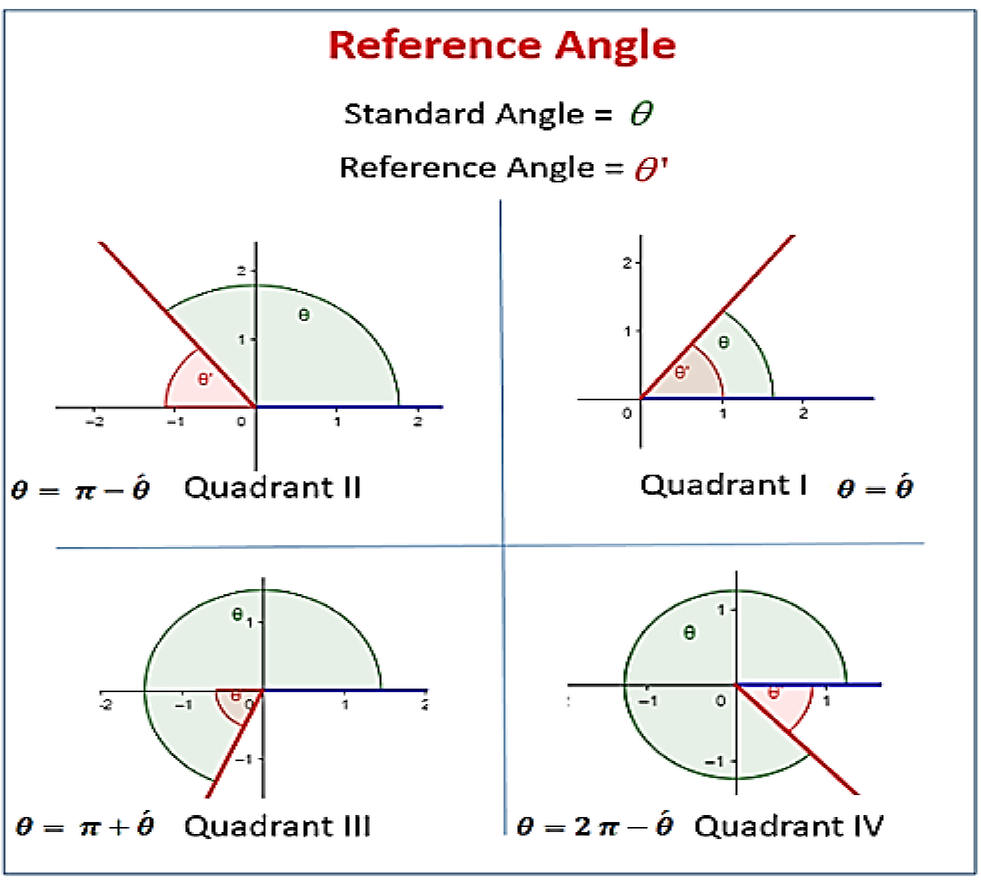




In the Cartesian coordinate system (x,y) every point has only one representation, but in the polar coordinate system each point has infinitely many pairs of polar coordinates due to multi cycles of this angle, therefore:-

$\theta = \theta \pm 2n\pi$

Also **(r, θ)** = $(-r, \theta \pm \pi)$



<u>Example1:</u> Find all the polar coordinates of the point P $(2,\frac{\pi}{6})$

Solution: <u>for **r =2**</u> **, θ =** $\frac{\pi}{6}$ **, θ =** $\frac{\pi}{6}$ **and θ =** $\frac{\pi}{6} \pm \mathbf{2n\pi}$

P $(2,\frac{\pi}{6})$, P $(2,-\frac{11\pi}{6})$, and P $(2,\frac{\pi}{6} \pm \mathbf{2n\pi})$ $(2,\frac{\pi}{6} - 10\pi)$

<u>for **r =-2**</u> **, θ =** $\frac{7\pi}{6}$ **, θ =** $\frac{\pi}{6}$**- π =** $-\frac{5\pi}{6}$ **, θ =** $\frac{7\pi}{6} \pm \mathbf{2n\pi}$ **and** $-\frac{5\pi}{6} \pm \mathbf{2n\pi}$

P $(-2,\frac{7\pi}{6})$, P $(-2,-\frac{5\pi}{6})$, P $(-2,\frac{7\pi}{6} \pm \mathbf{2n\pi})$ and $(-2,-\frac{5\pi}{6} \pm \mathbf{2n\pi})$

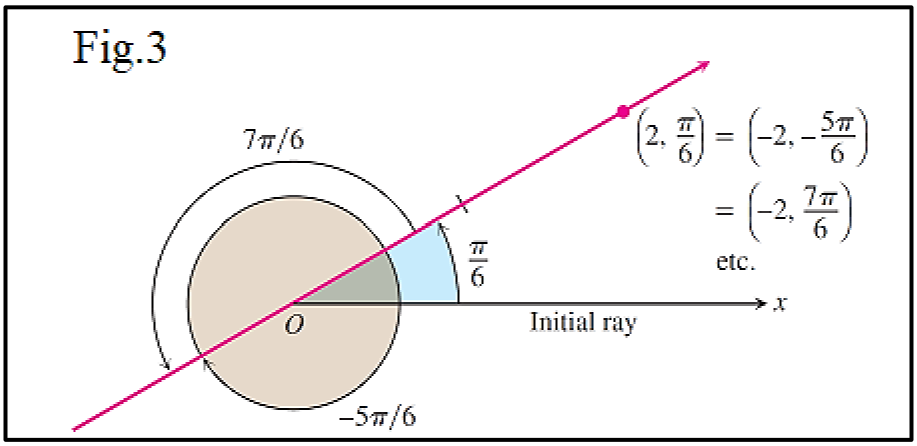


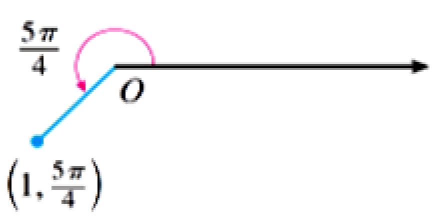
Fig.3

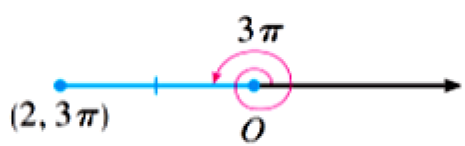
<u>Example 2:</u> Plot the points for the following polar coordinates:

(a) (1, 5π/4) (b) (2, 3 π)

a) (1, 5π/4) = $(1,\ \frac{\pi}{4} - \frac{4}{4}\pi)$ $(1, -\frac{3}{4}\pi)$ $= (\frac{\pi}{4}, -1)$

Third quarter = $\theta + \pi = \frac{5}{4}\pi \rightarrow \theta = \frac{5}{4}\pi - \pi = \frac{\pi}{4}$





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Home work1: Plot the points for the following polar coordinates, and then find all the polar coordinates of each point.

(1) (2, -2π/3) **(2) (-3, -π/4)** **(3) (2, $\frac{\pi}{2}$)**

Home work2: Graph the sets of points whose polar coordinates satisfy the following conditions.

1) $1 \le r \le 2$ and $0 \le \theta \le \frac{\pi}{2}$
2) $-3 \le r \le 2$ and $\theta = \frac{\pi}{2}$
3) $\frac{2\pi}{3} \le \theta \le \frac{5\pi}{6}$

2- Relating Polar and Cartesian Coordinates

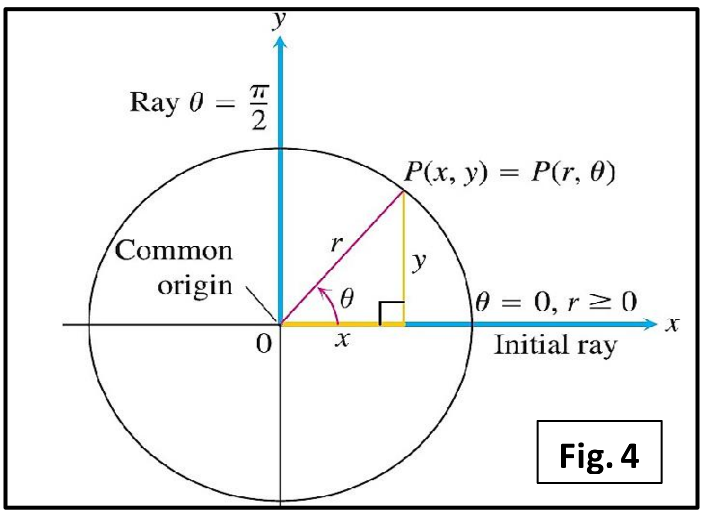


Fig. 4

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x}$$

Example3: Convert the point (2; π/3) from polar to Cartesian coordinates.
P(x,y) = p(1,

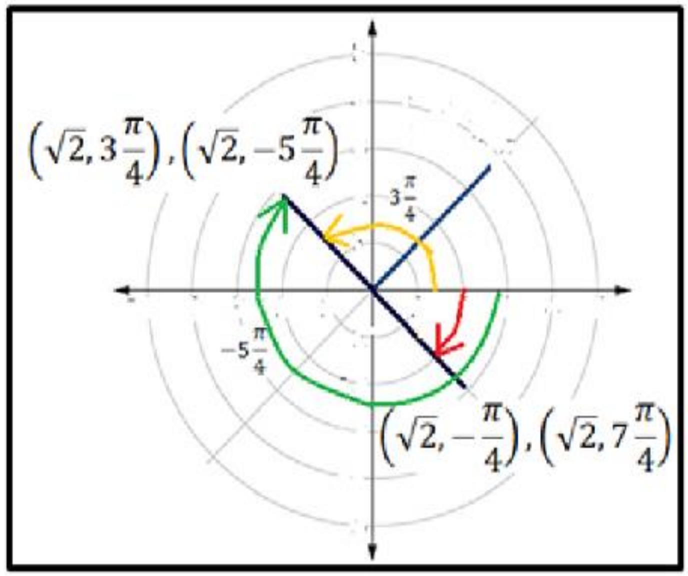
$$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2 \cdot \frac{1}{2} = 1 \qquad y = r\sin\theta = 2\sin\frac{\pi}{3} = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$$

Therefore, the point is $(1, \sqrt{3})$ in Cartesian coordinates.

Example 4: Represent the point with Cartesian coordinates (1, - 1) in terms of polar coordinates.

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (-1)^2} = \mp\sqrt{2}$$

$$tan\theta = \frac{y}{x} = \frac{-1}{1} = -1 \rightarrow \theta = \tan^{-1} -1 = 45 = \frac{\pi}{4}$$



Since the point (1, - 1) lies in the fourth quadrant, the possible answer is the following polar coordinates as shown in figure 5

For r = + $\sqrt{2}$ $\left(\sqrt{\mathbf{2}}, -\frac{\pi}{4}\right), \left(\sqrt{\mathbf{2}}, \mathbf{7}\frac{\pi}{4}\right)$

For r = - $\sqrt{2}$ $\left(-\sqrt{\mathbf{2}}, \mathbf{3}\frac{\pi}{4}\right), \left(-\sqrt{\mathbf{2}}, -\mathbf{5}\frac{\pi}{4}\right)$

Example 5: Replace the following Cartesian equations by equivalent polar equations.

1) x= 1 2) x^2= 4y 3) $x^2 + (y-3)^2$=9

Solution

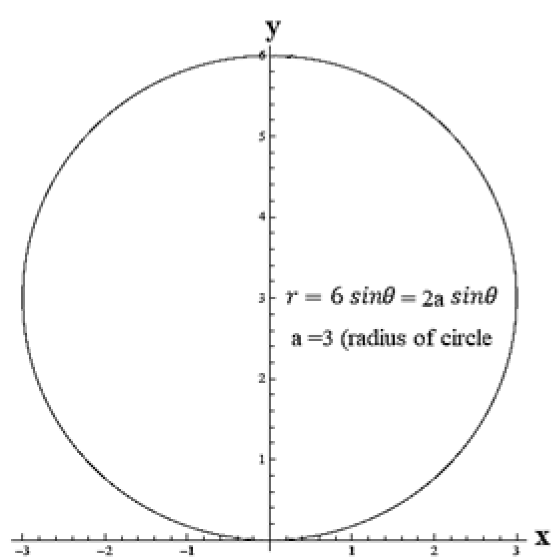
$r\cos\theta = 1 \rightarrow r = \frac{1}{\cos\theta} = \sec\theta$

1) x^2= 4y

Solution: We use the formulas x= $r\cos\theta$ and y= $r\sin\theta$:

$$(rcos\theta)^2 = 4\,rsin\theta \rightarrow r^2cos^2\theta = 4rsin\theta$$

$$r = 4\frac{sin\theta}{cos^2\theta} = 4\ \frac{1}{\cos\theta}.\ \frac{sin\theta}{\cos\theta} = r = 4\,sec\theta\,.\,tan\theta$$



2) $x^2 + (y-3)^2$=9

Solution

$x^2 + y^2 - 6y + 9$ =9

$x^2 + y^2 - 6y = 0$

$r^2 - 6\ rsin\theta = 0$

$\rightarrow\ r(r - 6\ sin\theta) = 0$

$\rightarrow r = 0\ \ or\ (r = 6\ sin\theta\ )$

Example 6: Replace the following polar equations by equivalent Cartesian equations and identify their graphs.

1) $\boldsymbol{rcos\theta = -4}$

$x = -4$ The graph: vertical line through $x = -4$

2) $\boldsymbol{r^2 = 4rcos\theta}$

$x^2 + y^2 = 4\ x$

$x^2 - 4\ x + 4 + y^2 = 4$

$(x-2)^2 + y^2 = 4$ The graph is circle, radius 2, center (h,k) = (2,0)

3) $r = \frac{4}{2cos\theta - sin\theta}$

$$2rcos\theta - r\sin\theta = 4$$

$2x - y = 4 \rightarrow$ $y = 2x - 4 = 0$ The graph is line $slope\ m = 2$ y- intercept $b = -4$

Homework 3: Replace the following polar equations by equivalent Cartesian equations and identify their graphs.

1) $r^2cos^2\theta - r^2sin^2\theta = 1$ 2) $r = 1 + 2rcos\theta$