3.Resultant of Force System

The resultant is a reprehensive force which has the same effect on the body as the group of forces it replaces. Or a simplest force which can replace the original forces system without changing its external effect on a rigid body.

When the resultant is zero, then the body is in equilibrium and the original force system in this case called a balanced.

The symbol of resultant force is: ⟶

The unit of resultant force is :Newton N

The resultant is applied for different type of forces system as:

1.Coplanar forces system

A.Concurrent coplanar forces system

متلاقية في مستو واحد

B.Non-concurrent coplanar forces system

غير متلاقية وفي مستو واحد

2.Non coplanar forces system

A.Concurrent non-coplanar forces system

متلاقية و ليست في مستو واحد

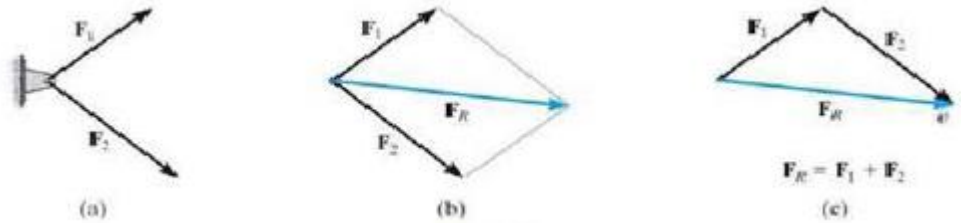
B.Non-concurrent non-coplanar forces system

غير متلاقية وليست في مستو واحد

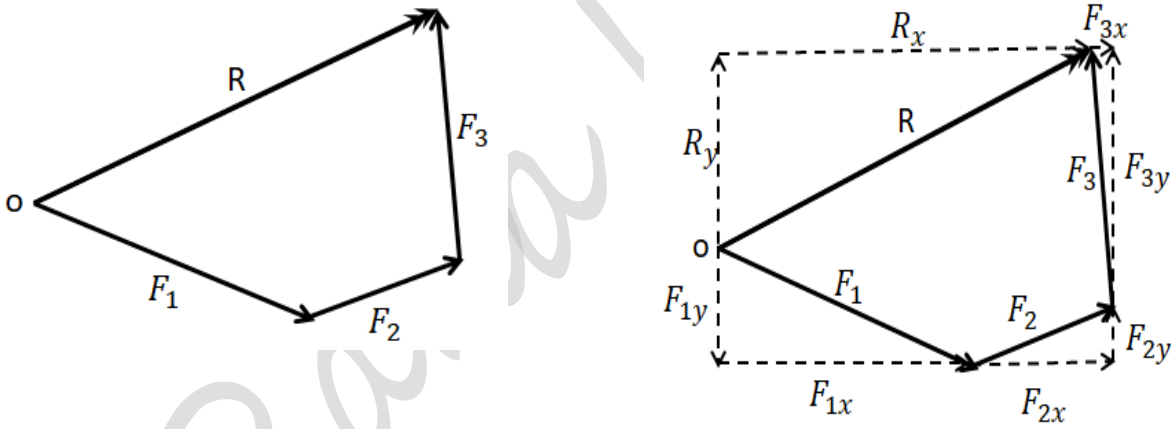
3.1Resultant of Concurrent Coplanar Forces System

Let the two forces F1 and F2 acting on the pin shown in Fig (a). The resultant of these two forces is the diagonal of the parallelogram as shown .These forces can be added together to form the resultant $F_R = F_1 + F_2$ Fig (b),From this construction, or using the triangle rule Fig (c), we can apply the low of cosine or the low of sines to the triangle in order to obtain the

magnitude of the resultant force and its direction. Note that the resultant must pass through the point of concurrence.



If there are more forces in the system, start from O as shown in figure below and draw F_1,then F_2 from the tip of F_1 and F_3 from the tip of F_2 .The resultant is represented by the vector from O to the tip of F_3 .



We will find out the resultant force for many forces acting on a rigid body by using the following equations:-

$$R_X = \sum F_X$$

$$R_Y = \sum F_Y$$

$$R = \sqrt{{R_X}^2 + {R_Y}^2}$$

The direction of resultant force may be determined as:

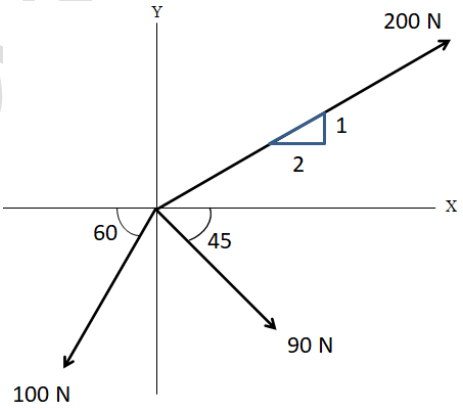
$$\theta = \tan^{-1}(\frac{R_Y}{R_X})$$

Where θ is the angle between the resultant and the x axis. The resultant passes through the point of concurrence of the forces of the system, and its sense can be detetmined from the components R_X and R_Y.

Examples

Example (1):-

Find the resultant force for the concurrent coplanar force system, shown in figure.



Solution:-

$$\rightarrow R_X = \sum F_X$$

$$= 200\ \frac{2}{\sqrt{5}} - 100\cos 60 + 90\cos 45 = +192.4\ \text{N} \rightarrow$$

$$\uparrow R_Y = \sum F_Y$$

$$= 200\ \frac{1}{\sqrt{5}} - 100\sin 60 - 90\sin 45 = -60.8\ \text{N}$$

$$= 60.8\text{N} \downarrow$$

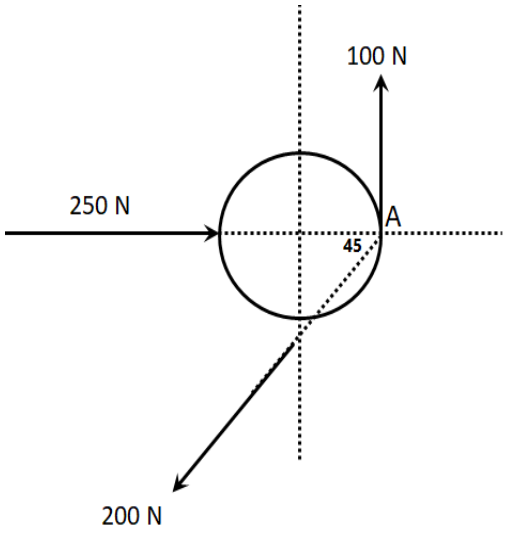
$$\text{NR} = \sqrt{(R_X)^2 + (R_Y)^2} = \sqrt{(192.4)^2 + (60.8)^2} = 202$$

$$\theta = \tan^{-1}(\frac{R_Y}{R_X})$$

$$\theta = \tan^{-1}(\frac{60.8}{192.4}) = 17.5$$

Example(2):

Determine the resultant force for the forces system shown in figure.



Solution:-

$$\rightarrow R_X = \sum F_X$$

$=250 - 200\cos 45 = 108.6\text{N} \rightarrow$

$$\uparrow R_Y = \sum F_Y$$

$=100 - 200 \sin 45 = -41.4\text{N} = 41.4\text{N} \downarrow$

$$R = \sqrt{(R_X)^2 + (R_Y)^2}$$

$$= \sqrt{(108.6)^2 + (41.4)^2}$$

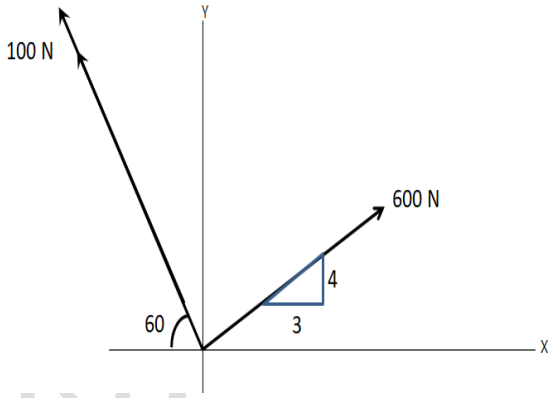
$=116.2$ N

$$\theta = \tan^{-1}\left(\frac{R_Y}{R_X}\right)$$

$$\theta = \tan^{-1}\left(\frac{41.4}{108.6}\right) = 20.8$$

Example (3):-

The 1000 N force is a resultant of two forces, one of which is 600 N, Determine the other force.



Solution:-

$$\rightarrow R_X = \sum F_X$$

$$-1000\cos 60 = 600\frac{3}{5} + F2_X$$

$$-1000 \times 0.5 = 360 + F2_X$$

$$F2_X = -860\ \text{N} = 860\text{N} \leftarrow$$

$$\uparrow R_Y = \sum F_Y$$

$$1000\sin 60 = 600\frac{4}{5} + F2_Y$$

$$F2_Y = 386.02\ N \uparrow$$

$$F = \sqrt{(860)^2 + (386.02)^2}$$

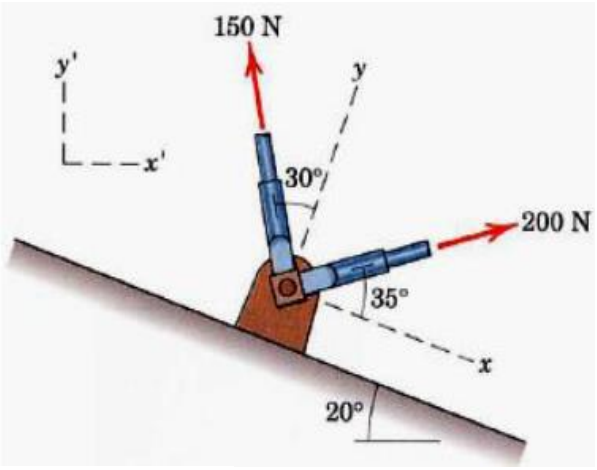
=942.62 N

$$= \tan^{-1}\left(\frac{R_Y}{R_X}\right)$$

$$= \tan^{-1}\left(\frac{386}{860}\right) = 24.17$$

Example(4):-

Find the resultant of the two force shown in figure.

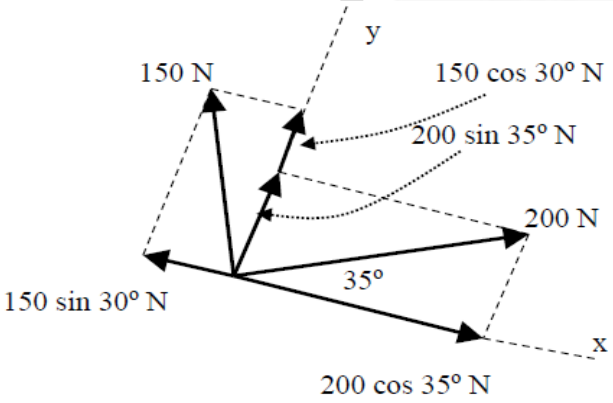


Solution:-

$$\rightarrow R_x = \sum F_x = 200\cos 35 - 150\sin 30 = 88.8\ N$$

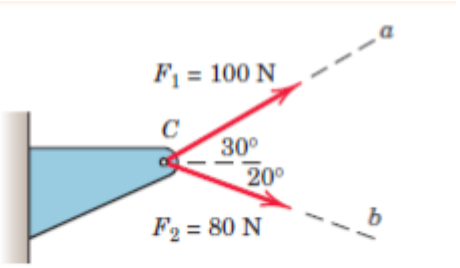
$$\uparrow R_y = \sum F_y = 200\sin 35 + 150\cos 30 = 245\ N$$

$$R = \sqrt{(R_X)^2 + (R_Y)^2} = \sqrt{88.8^2 + 245^2} = 260.6N$$



Example(5):-

Forces F1 and F2 acts on the bracket as shown in figure. Determine their resultant R.



Solution:-

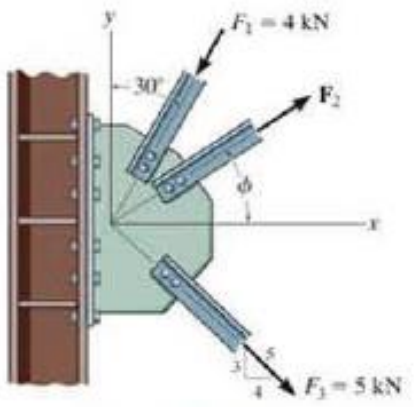
$$\rightarrow R_X = \sum F_X$$
$$= 100\cos 30 + 80\cos 20 = 161.77\text{N} \rightarrow$$

$$\uparrow R_Y = \sum F_Y = 100\sin 30 - 80\sin 20 = 22.63\text{N} \uparrow$$

$$R = \sqrt{(R_X)^2 + (R_Y)^2} = \sqrt{161.77^2 + 22.63^2}$$
$$= 163.4\text{N}$$

Example(6):-

If $\emptyset = 30$ and the resultant force acting on the plate is directed along positive x axis , determine the magnitude F_2 and the resultant force.



Solution:-

The resultant is horizontal

$$\therefore R_y = 0 = \sum F_y$$

$$R_x = R = \sum F_x$$

$$\uparrow R_y = 0 = -4\cos 30 + F_2 \sin 30 - 5\frac{3}{5}$$

$$F_2 = \frac{6.46}{\sin 30} = 12.9 \text{ kN}$$

$$\rightarrow R = R_x = \sum F_x$$

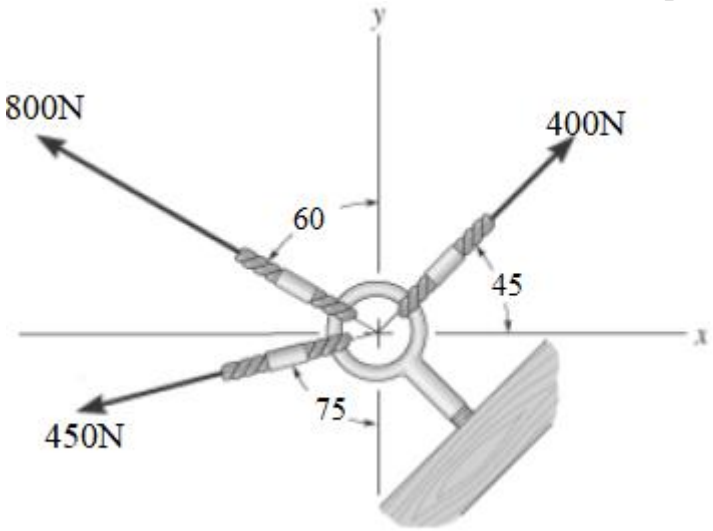
$$= -4\sin 30 + 12.9\cos 30 + 5\frac{4}{5}$$

$$R = 13.2\text{kN}$$

Home Work

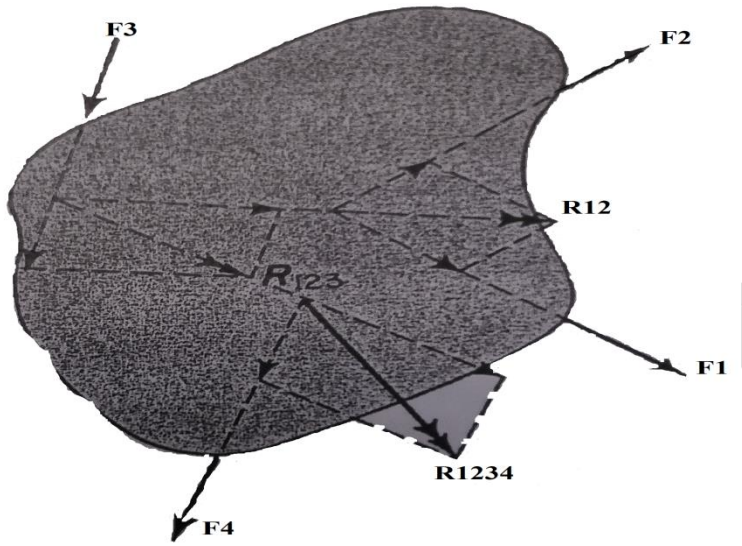
H.W 3

Determine the magnitude of the resultant Force and its direction, measured counterclockwise from the positive x axis.



3.2Resultant of a Non concurrent Coplanar Force System.

The resultant of this force system is either a single force or a couple.



The resultant is a couple when the resultant of all but one of the forces of the system and the remaining force form a couple.

To determine the resultant analytically, each force is resolved into rectangular components in the x and y directions.

1.When the algebraic sum of the components in either the x or y direction, or both, is different from zero, the resultant is a force, and its magnitude can be determined from the following equations:

$$R_X = \sum F_X$$

$$R_Y = \sum F_Y$$

$$R = \sqrt{(R_X)^2 + (R_Y)^2}$$

$$\tan^{-1}\theta_X = \frac{R_Y}{R_X}$$

The location of a point on the action line of the resultant is determined by the principle of moments.

$$Rq = \sum M_O$$

Where q is the perpendicular distance from the moment axis through o to the resultant force R.The direction from O to R is determined from the sense of R and $\sum M_O$

2. When the algebraic sum of the components of the forces is zero in two different direction, the resultant cannot be a single force but can be a couple in the plane of the forces. The magnitude of the moment of the couple and its sense of rotation can be determined as the algebraic sum of the moments of the forces of the system with respect to any point in the plane.

Note:-

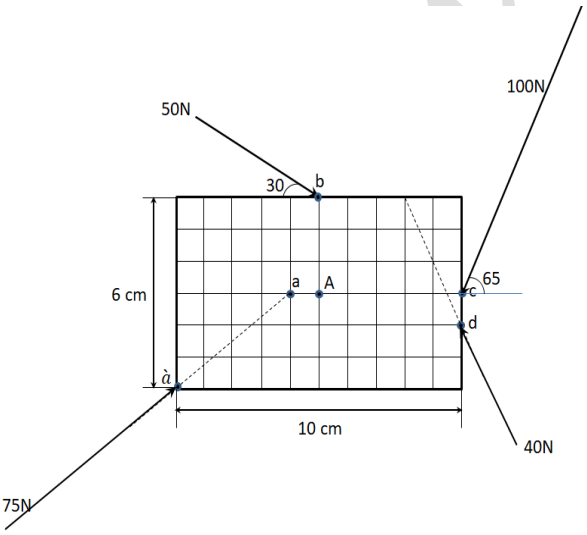
For the special case of a parallel coplanar force system with the x axis parallel to the forces of the system $R_y = 0$ and the resultant if a force, is parallel to the x axis.

Examples

Example (1):-

Determine the resultant of the force system and show it on a sketch located with respect to point A.

Solution:-



$$\rightarrow R_x = 75\left(\frac{4}{5}\right) + 50\cos 30 - 100\cos 65 - 40\left(\frac{1}{\sqrt{5}}\right)$$

$$= 43.1\text{N} \rightarrow$$

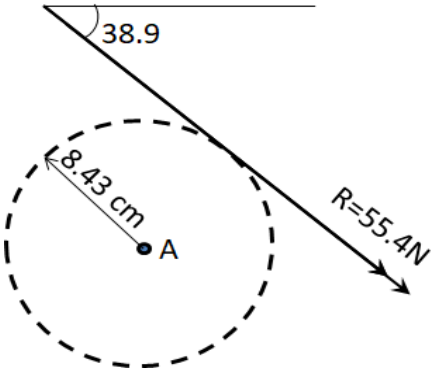
$$\uparrow R_y = 75\left(\frac{3}{5}\right) - 50\sin 30 - 100\sin 65 - 40\left(\frac{2}{\sqrt{5}}\right)$$

$$= -34.8\text{N} = 34.8\text{N} \downarrow$$

$$\text{R} = \sqrt{(\text{R}_x)^2 + (\text{R}_y)^2} = \sqrt{43.1^2 + 34.8^2}$$

$$= 55.4\text{N}$$

To locate the action line of the resultant force with respect to point A.



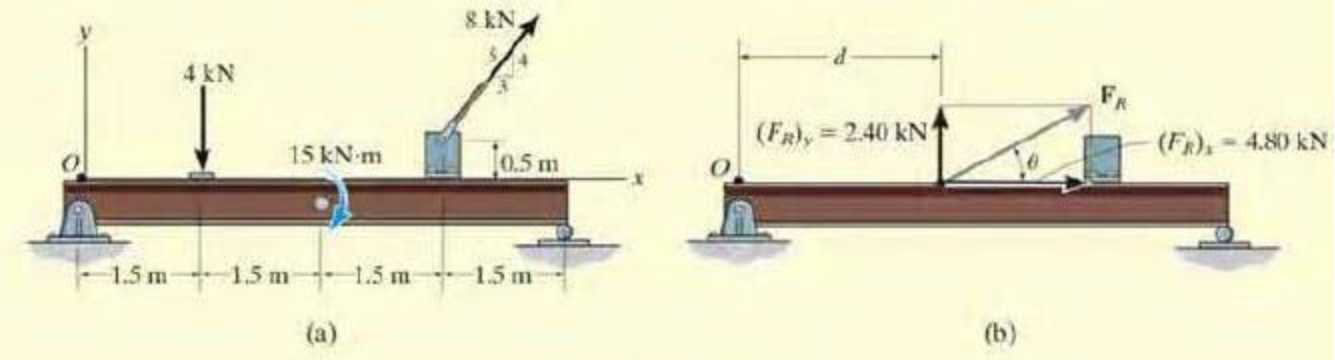
$$\sum M_A = 75\left(\frac{3}{5}\right)(1) + 50\cos 30(3) + 100\sin 65(5) + 40\left(\frac{1}{\sqrt{5}}\right)(1) - 40\left(\frac{2}{\sqrt{5}}\right)(5)$$

$$= 467\text{N.cm}$$

$$q = \frac{467}{55.4} = 8.43\ cm$$

Example(2):-

Find the resultant of the force and couple moment system acting on the beam shown in figure, and find where its line of action intersects the beam, measured from point O.



Solution:-

$$\rightarrow R_X = \sum F_x = 8\frac{3}{5} = 4.8\text{kN} \rightarrow$$

$$\uparrow R_y = \sum F_y = -4 + 8\frac{4}{5} = 2.4\text{kN} \uparrow$$

$$R = \sqrt{4.8^2 + 2.4^2} = 5.37 \text{ kN}$$

$$\theta = \tan^{-1}\frac{2.4}{4.8} = 26.6$$

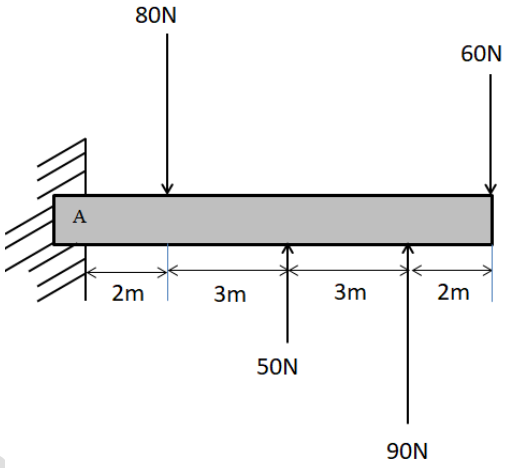
$$M_O = -4(1.5) - 15 - 8\left(\frac{3}{5}\right)0.5 + 8\left(\frac{4}{5}\right)4.5 = 5.4\text{kN.m}$$

Since the line of action of R_x acts through point O, only R_y produces a moment about point O.

$$q = \frac{5.4}{2.4} = 2.25 \text{ kN.m}$$

Example(3):-

Determine the resultant of the parallel forces shown in figure.



Solution:-

$R_x = 0$

$\uparrow R_y = -80 - 60 + 50 + 90 = 0\text{N}$

$\therefore$ the resultant is a couple

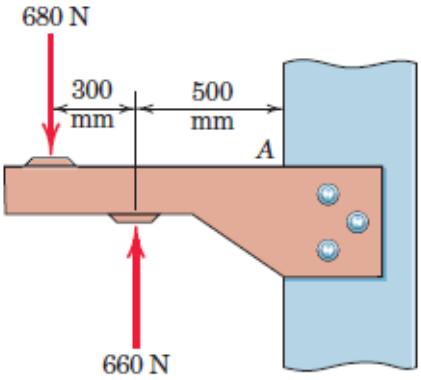
$$\sum \text{M}_\text{A} = 80 \times 2 + 60 \times 10 - 50 \times 5 - 90 \times 8$$

$$= -210\ \text{N.m} = 210\text{N.m}$$

The resultant is a couple=210 N.m

Example (4):-

Where does the resultant of the two forces act.



Solution:-

$R_x = 0$

$\uparrow R_y = -680 + 660 = -20\text{N} = 20\text{N} \downarrow$

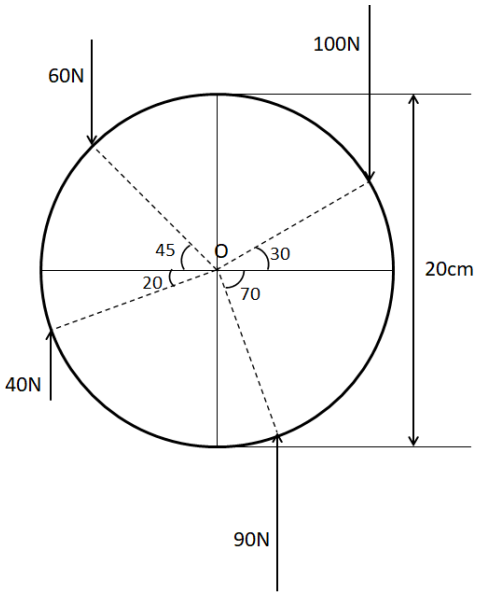
$\text{R} = \text{R}_\text{y} = 20\text{N} \downarrow$

$$\sum \text{M}_\text{A} = 680\text{N} \times 0.8\text{m} - 660\text{N} \times 0.5\text{m} = 214\text{N.m}$$

$$\text{q} = \frac{214}{20} = 10.7\text{m to the left of A}$$

Example(5):

Determine the resultant of the parallel force system and locate it with respect to O.



Solution:-

$$R_x = 0$$

$$R = R_y = \uparrow \sum F_y = -60 - 100 + 40 + 90$$

$$= -30\text{N} = 30\text{N} \downarrow$$

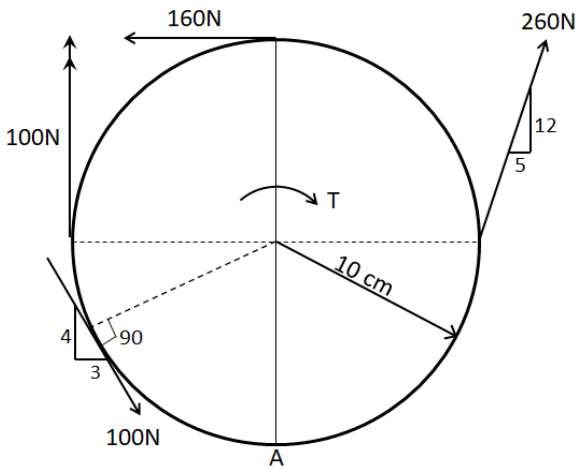
$$M_O = -60 \times 10\cos 45 + 100 \times 10\cos 30 - 90 \times 10\cos 70 + 40 \times 10\cos 20 = 509.8\ N.cm$$

$$q = \frac{509.8}{30} = 17 \text{ cm to the right of point O}$$

Example (6):-

The resultant of the three forces and the couple T in figure and an unknown force through A is the vertical 100N force through point B. Determine the unknown force through point A and the magnitude of the couple T.

Solution:-



Let the unknown force F_A act at point A

$$\rightarrow R_x = 0 = -160 + 260\frac{5}{13} + 100\frac{3}{5} + F_{Ax}$$

$$F_{Ax} = 0$$

$$\uparrow R_y = R = 100 = 260\frac{12}{13} - 100\frac{4}{5} + F_{Ay}$$

$$F_{Ay} = -60N = 60N \downarrow$$

To find the unknown couple

the moment of R about O

$= \sum$ the moment of the force about O

$$100 \times 10 = -160 \times 10 - 240 \times 10 - 100 \times 10 + T$$

T=6000N.cm

Home Work

H.W 4

Determine the equivalent resultant force of the system and specify its location measured from point A.