5.7 Temperature distributions in the absorber plate

The flow of the working fluid in the risers that are firmly welded to the absorber plate carries the accumulated heat outside the solar collector. The flow of heat between the absorber plate and the risers generates a transverse temperature distribution in the region of the plate between two adjacent risers (Fig. 5.4). Another longitudinal temperature distribution is generated by the working fluid along the flow direction in risers both in plate and fluid domains. The overall problem is a two dimensional transient heat transfer case where the three modes of conduction, convection and radiation are involved. The exact analytical solution is extremely complicated. To facilitate the solution and highlight the significant points of the thermal processes, the transverse and longitudinal temperature distributions are calculated separately where one dimensional heat transfer is assumed in each case. Further, steady state condition is assumed where constant values of irradiance and ambient temperature are applied. The method is well known in the literature as HWWB method after the initials of their founders (Hottel, Whillier, Woertz and Bliss) in the forties and fifties of the 20th century.

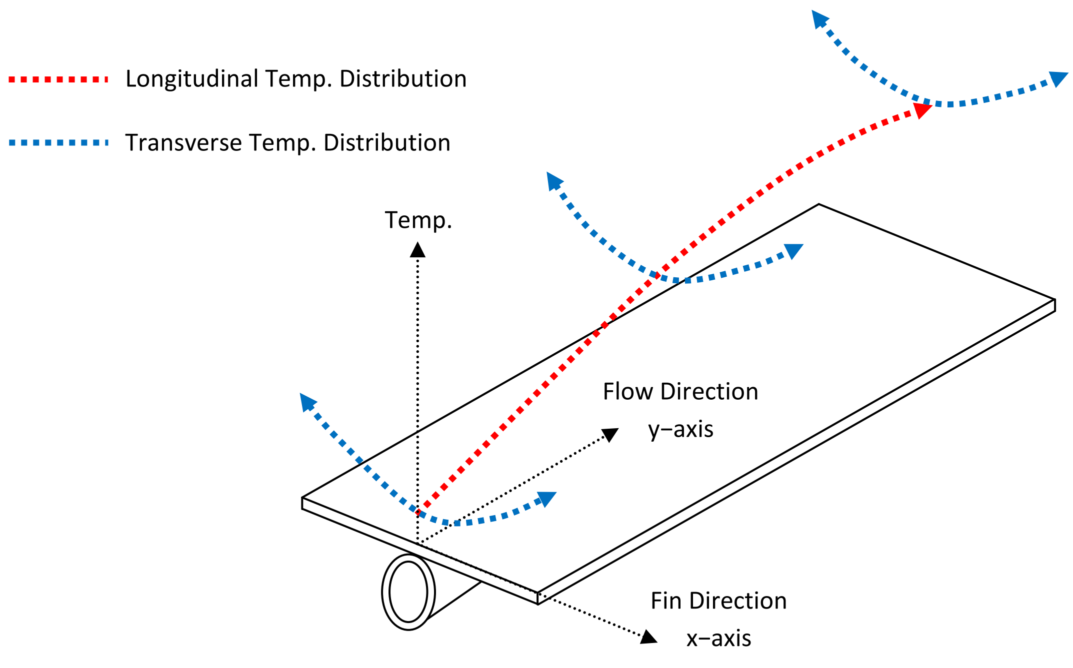

Fig. (5.4): Longitudinal (red line) and transverse (blue line) temperature distributions in the absorber plate.

5.7.1 Transverse temperature distribution (between two adjacent risers)

Figure 5.5 shows a cross sectional view of the absorber plate with two adjacent risers and the temperature profile in the plate. It can be seen that the temperature profile is symmetrical around the midpoint between the two risers where the temperature derivative dT/dx is zero and the plate temperature is maximum.

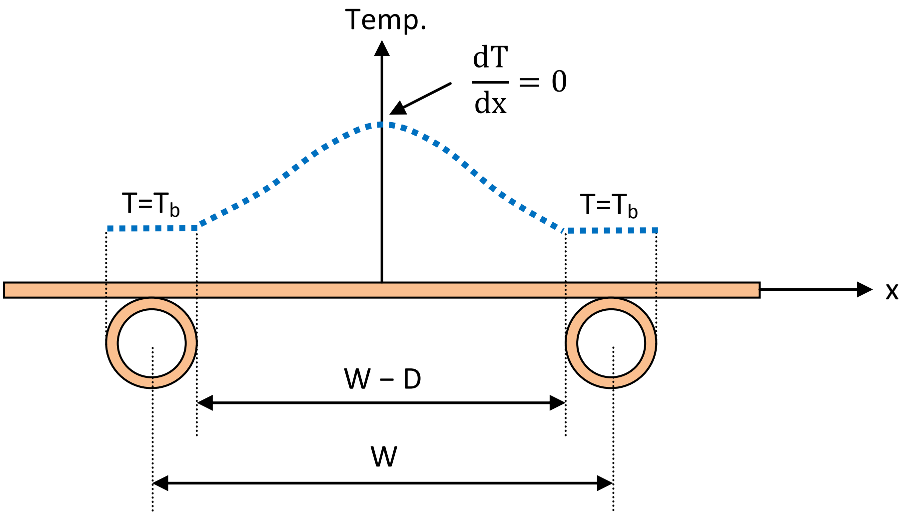

Fig. (5.5): Cross–sectional view of the absorber plate and two adjacent risers.

The analysis will consider only one half of the absorber–riser configuration to evaluate the temperature distribution between x = 0 at the midway between two risers and x = (W–D)/2 where the bonding material between the plate and riser exists. Energy balance will be carried out on the incremental length Δx of the fin–like region of length (W–D)/2 as shown in Fig. 5.6:

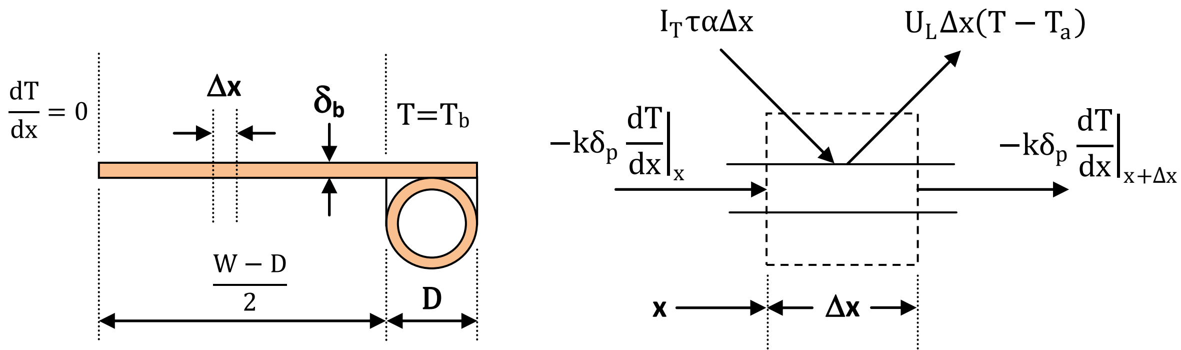

Fig. (5.6): The region under consideration of the absorber plate showing the incremental heat balance.

The following heat balance is the case of a rectangular fin with finite length of (W−D)/2 and a unit depth with one end insulated and the other at a constant temperature T_b which is the bond temperature:

$$\begin{pmatrix}\text{Heat input to}\\ \text{the element}\\ \text{by conduction}\end{pmatrix}+\begin{pmatrix}\text{Heat input to}\\ \text{the elemnt}\\ \text{by radiation}\end{pmatrix}=\begin{pmatrix}\text{Heat out of}\\ \text{the element}\\ \text{by conduction}\end{pmatrix}+\begin{pmatrix}\text{Heat out of}\\ \text{the element}\\ \text{by radiation}\end{pmatrix} \qquad 5.22$$

$$-k\delta_p \frac{dT}{dx}\bigg|_x + I_T\tau\alpha\Delta x = -k\delta_p \frac{dT}{dx}\bigg|_{x+\Delta x} + U_L\Delta x(T - T_a)$$

Dividing by Δx and rearranging:

$$\frac{k\delta_p \frac{dT}{dx}\Big|_{x+\Delta x} - k\delta_p \frac{dT}{dx}\Big|_x}{\Delta x} = U_L(T - T_a) - I_T\tau\alpha$$

When Δx approaches zero:

$$k\delta_p \frac{d^2T}{dx^2} = U_L(T - T_a) - I_T\tau\alpha$$

$$\frac{d^2T}{dx^2} = \frac{U_L}{k\delta_p}\left(T - T_a - \frac{I_T\tau\alpha}{U_L}\right) \qquad 5.23$$

To solve eq. 5.23 which is a second order ordinary differential equation, two boundary conditions are required:

$$\text{at } x = 0 \qquad \frac{dT}{dx} = 0$$

$$\text{at } x = \frac{W - D}{2} \qquad T = T_b$$

Where T_b is the temperature of the absorber−riser bond. Eq. 5.23 with its boundary conditions can be simplified by defining the variable (ψ):

$$\Psi = T - T_a - \frac{I_T\tau\alpha}{U_L}$$

And the constant (m) as:

$$m^2 = \frac{U_L}{k\delta_p}$$

So, eq. 5.23 becomes:

$$\frac{d^2\psi}{dx^2} - m^2\psi = 0 \qquad 5.24$$

And the boundary conditions are also converted to:

$$\text{at } x = 0 \qquad \frac{d\psi}{dx} = 0$$

$$\text{at } x = \frac{W - D}{2} \qquad \psi = T_b - T_a - \frac{I_T\tau\alpha}{U_L}$$

The general solution of eq. 5.24 is:

$$\psi = C_1\sinh(mx) + C_2\cosh(mx) \qquad 5.25$$

The boundary conditions must be substituted in the general solution (eq. 5.25) to find the constants C_1 and C_2. The solution after evaluating the constants and re–substituting the value of ψ becomes:

$$\frac{T - T_a - \frac{I_T\tau\alpha}{U_L}}{T_b - T_a - \frac{I_T\tau\alpha}{U_L}} = \frac{\cosh(mx)}{\cosh\left(\frac{m(W-D)}{2}\right)} \qquad 5.26$$

With: $$m = \sqrt{\frac{U_L}{k\delta_p}}$$

Equation 5.26 represents the transverse temperature distribution between the absorber–riser bond and the mid–point between two adjacent risers.

To evaluate the heat transferred from the fin end to the riser per unit depth, Fourier's law of conduction is applied:

$$q_f' = -k\delta_p \left.\frac{dT}{dx}\right|_{x=\frac{W-D}{2}}$$

$$q_f' = \frac{k\delta_p m}{U_L}\left[I_T\tau\alpha - U_L(T_b - T_a)\right]\tanh\left[\frac{m(W-D)}{2}\right] \qquad 5.27$$

The quantity q_f' is the amount of heat transferred to the riser from one end. It must be multiplied by 2 to account for the other end. Multiplying and dividing eq. 5.27 by W−D and converting $\frac{k\delta_p}{U_L}$ to $\frac{1}{m^2}$ and rearranging to get:

$$q_f = (W - D)[I_T\tau\alpha - U_L(T_b - T_a)]\frac{\tanh\left[\frac{m(W-D)}{2}\right]}{\frac{m(W-D)}{2}} \qquad 5.28$$

The quantity $(W - D)[I_T\tau\alpha - U_L(T_b - T_a)]$ is the amount of useful heat gain received by the fin region between two adjacent risers considering the fin surface at the bond temperature T_b which the case of an ideal fin that delivers all the received heat to the riser. Accordingly the rest of eq. 5.28 represents the value that deviates the fin from ideal performance (or the fin efficiency F_f):

$$q_f = F_f(W - D)[I_T\tau\alpha - U_L(T_b - T_a)] \qquad 5.29$$

Where:

$$F_f = \frac{\tanh\left[\frac{m(W-D)}{2}\right]}{\frac{m(W-D)}{2}} \qquad 5.30$$

The quantity q_f is the amount of heat received by the riser from its two fin ends. The riser receives further heat from the part of absorber directly over it, namely q_{tube}:

$$q_{tube} = D[I_T\tau\alpha - U_L(T_b - T_a)] \qquad 5.31$$

So the total amount of useful heat gain received by the riser is q_u:

$$q_u = q_f + q_{tube} = [F_f(W - D) + D][I_T\tau\alpha - U_L(T_b - T_a)] \qquad 5.32$$

The amount of heat q_u undergoes two types of thermal resistances upon flowing from the absorber to the working fluid inside the risers. The first resistance is that of bonding material that welds the absorber to the riser, namely R_b:

$$R_b = \frac{t_b}{w_b k_b} \qquad 5.33$$

Where, t_b , w_b and k_b are the bond average thickness, width and thermal conductivity, respectively.

The second resistance is that between the inner surface of the riser and the working fluid, namely R_f:

$$R_f = \frac{1}{\pi D_i h_f} \qquad 5.34$$

Where D_i is the riser inner diameter and h_f is the convection heat transfer coefficient inside the riser.

The useful heat gain q_u can then be written in terms of resistances as follows:

$$q_u = \frac{T_b - T_f}{R_b + R_f} \qquad 5.35$$

Equations 5.32 and 5.35 can be combined to get:

$$q_u = WF[I_T \tau\alpha - U_L(T_f - T_a)] \qquad 5.36$$

Where:

$$F = \frac{\frac{1}{U_L}}{\frac{1}{[U_L[D + F_f(W - D)]]} + R_b + R_f} \qquad 5.37$$

Where F is the flat−plate solar collector efficiency factor which represents the ratio of the actual amount of heat per unit depth transferred from the absorber to the working fluid to the hypothetical amount that would be transferred if the absorber is at the working fluid temperature.