5.7.2 Longitudinal temperature distribution (along riser)

The flow of the working fluid in risers absorbs the accumulated heat in the absorber plate. As a result, the temperature of the working fluid increases as it flows towards riser exit. To find the equation describing the temperature distribution of the working fluid inside the riser an incremental element of length Δy is taken at the location y along the riser where energy balance is applied as follows (Fig. 5.7):

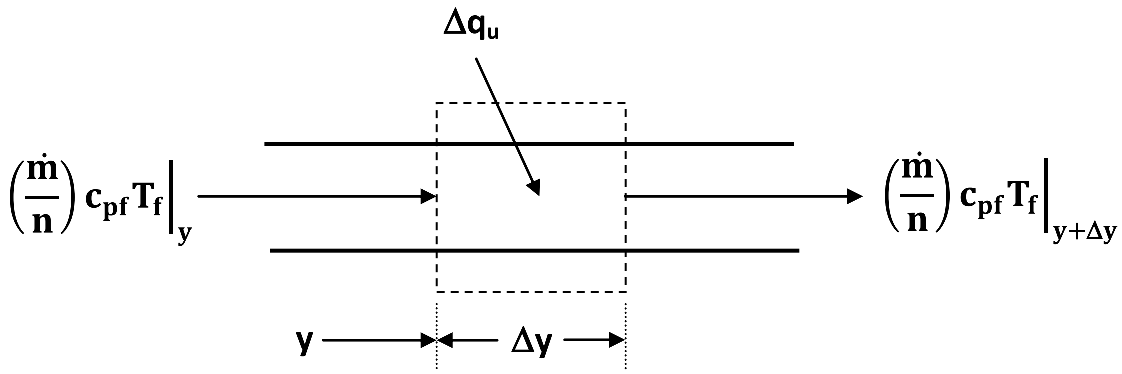

Fig. (5.7): Energy balance on the incremental length Δy of the riser.

$$\begin{bmatrix}\text{Heat Transferred}\\ \text{from Absorber}\end{bmatrix} = \begin{bmatrix}\text{Heat Gained by}\\ \text{Working Fluid}\end{bmatrix}$$

$$q_u \Delta y = \left(\frac{\dot{m}}{n}\right) c_{pf} T_f\Big|_{y+\Delta y} - \left(\frac{\dot{m}}{n}\right) c_{pf} T_f\Big|_y \qquad 5.38$$

Where $\dot{m}$ is the total mass flow rate in the collector and n is the number of risers.

Dividing both sides of eq. 5.38 by Δy and using eq. 5.36 for q_u to get:

$$\frac{nWF}{\dot{m}c_{pf}}\left[I_T \tau\alpha - U_L(T_f - T_a)\right] = \frac{T_f|_{y+\Delta y} - T_f|_y}{\Delta y}$$

Taking the limits of Δy approaching zero, it follows that:

$$\frac{nWF}{\dot{m}c_{pf}}\left[I_T \tau\alpha - U_L(T_f - T_a)\right] = \frac{dT_f}{dy}$$

Rearranging the equation to put each variable with its derivative:

$$\frac{dT_f}{I_T\tau\alpha - U_L(T_f - T_a)} = \frac{nWF}{\dot{m}c_{pf}}dy$$

The equation is multiplied by $-U_L$ to get the derivative of the left side denominator:

$$\int_{T_{fi}}^{T_f} \frac{-U_L dT_f}{I_T\tau\alpha - U_L(T_f - T_a)} = \int_0^y \frac{-nWU_L F}{\dot{m}c_{pf}}dy$$

Performing the integration between riser inlet and any location y along the riser:

$$\ln\frac{I_T\tau\alpha - U_L(T_f - T_a)}{I_T\tau\alpha - U_L(T_{fi} - T_a)} = \frac{-nWU_L F}{\dot{m}c_{pf}}y$$

or:

$$\frac{I_T\tau\alpha - U_L(T_f - T_a)}{I_T\tau\alpha - U_L(T_{fi} - T_a)} = e^{\frac{-nWU_L F}{\dot{m}c_{pf}}y} \qquad 5.39$$

Equation 5.39 represents the working fluid longitudinal temperature distribution T_f at any location y along the riser. Accordingly if y=L at the riser exit then $T_f=T_{fo}$ or the collector fluid exit temperature. The term nWL is the collector area A_c :

$$\frac{I_T\tau\alpha - U_L(T_{fo} - T_a)}{I_T\tau\alpha - U_L(T_{fi} - T_a)} = e^{\frac{-A_c U_L F}{\dot{m}c_{pf}}} \qquad 5.40$$

5.8 Heat removal factor F_R

It is defined as the ratio of the actual heat gain Q_u by the working fluid in the collector to the hypothetical amount of heat that would be gained if the whole collector is at the fluid inlet temperature T_{fi} :

$$F_R = \frac{Q_u}{A_c[I_T\tau\alpha - U_L(T_{fi} - T_a)]} \qquad 5.41$$

Another expression for the heat removal factor can be found by from equation 5.3:

$$Q_u = \dot{m}c_{pf}(T_{fo} - T_{fi}) \qquad 5.3$$

$$F_R = \frac{\dot{m}c_{pf}(T_{fo} - T_{fi})}{A_c[I_T\tau\alpha - U_L(T_{fi} - T_a)]} = \frac{\dot{m}c_{pf}}{A_c U_L}\left[\frac{U_L(T_{fo} - T_{fi})}{I_T\tau\alpha - U_L(T_{fi} - T_a)}\right]$$

Adding and subtracting the terms T_aU_L and $I_T\tau\alpha$ and rearranging the terms to get:

$$F_R = \frac{\dot{m}c_{pf}}{A_c U_L}\left(\frac{[I_T\tau\alpha - U_L(T_{fi} - T_a)] - [I_T\tau\alpha - U_L(T_{fo} - T_a)]}{[I_T\tau\alpha - U_L(T_{fi} - T_a)]}\right)$$

$$F_R = \frac{\dot{m}c_{pf}}{A_c U_L}\left(1 - \frac{[I_T\tau\alpha - U_L(T_{fo} - T_a)]}{[I_T\tau\alpha - U_L(T_{fi} - T_a)]}\right)$$

Equation 5.40 can be used to substitute instead of the fraction in the parentheses:

$$F_R = \frac{\dot{m}c_{pf}}{A_c U_L}\left(1 - e^{\frac{-A_c U_L F}{\dot{m}c_{pf}}}\right) \qquad 5.42$$

Another parameter called Collector Flow Factor (FF) can be derived by dividing the heat removal factor (F_R) by the collector efficiency factor (F):

$$FF = \frac{F_R}{F} = \frac{\dot{m}c_{pf}}{A_c U_L F}\left(1 - e^{\frac{-A_c U_L F}{\dot{m}c_{pf}}}\right) = CR\left(1 - e^{\frac{-1}{CR}}\right) \qquad 5.43$$

The dimensionless parameter $CR = \frac{\dot{m}c_{pf}}{A_c U_L F}$ is called collector capacity rate. It is worth noting that all the parameters F, F_R , FF and CR are dimensionless numbers.

The heat removal factor F_R is equivalent to the heat exchanger effectiveness (ε) well known in heat exchanger theory which is defined as the ratio of the actual heat transferred by the heat exchanger to the maximum amount that would be transferred if the whole heat exchanger is at the working fluid inlet temperature.

5.9 Absorber plate mean temperature T_{pm}

It is important to evaluate a single temperature representative of both transverse and longitudinal temperature distributions so that the top heat loss coefficient can be estimated based on it. This temperature is termed as the mean plate temperature T_{pm}. It can be evaluated by equating the useful heat gain Q_u from equations 5.1 and 5.41 as follows:

$$Q_u = A_c\left[\tau\alpha I_T - U_L\left(T_{pm} - T_a\right)\right] \qquad 5.1$$

$$Q_u = A_c F_R[\tau\alpha I_T - U_L(T_{fi} - T_a)] \qquad 5.41$$

The above two equations are solved together to get the mean plate temperature:

$$T_{pm} = T_{fi} + \frac{Q_u}{A_c U_L F_R}(1 - F_R) \qquad 5.45$$

5.9 Working fluid mean temperature T_{fm}

It is also important to determine the mean temperature of the working fluid inside the riser since it is used to evaluate the properties of the working fluid and the heat transfer coefficient inside the riser. To find T_{fm} the working fluid temperature distribution from eq. 5.39 is integrated along the riser length as follows:

$$T_{fm} = \frac{1}{L}\int_0^L T_f dy \qquad 5.46$$

Substituting for T_f from eq. 5.39 and performing the integration to get:

$$T_{fm} = T_{fi} + \frac{Q_u}{A_c U_L F_R}\left(1 - \frac{F_R}{F}\right) \qquad 5.47$$

Ex. 5.3: A flat−plate solar collector of 4 m^2 area operates with an efficiency factor of 0.9, a transmittance−absorptance product of 0.8 and an overall heat loss coefficient of 8 W/(m^2 °C). Water enters the collector at 20 °C with a total mass flow rate of 0.05 kg/s. If the incident normal irradiance is 1000 W/m^2 and the ambient temperature is 10 °C then find:

a) Water exit temperature. b) Mean plate temperature. c) Mean fluid temperature. d) Collector efficiency. Take water specific heat as 4180 J/(kg °C).

Sol. $$F_R = \frac{\dot{m} c_{pf}}{A_c U_L}\left(1 - e^{\frac{-A_c U_L F}{\dot{m} c_{pf}}}\right) = \frac{0.05\times4180}{4\times8}\left(1 - \exp\left(\frac{-4\times8\times0.9}{0.05\times4180}\right)\right) = 0.84$$

$$Q_u = A_c F_R[\tau\alpha I_T - U_L(T_{fi} - T_a)] = 4 \times 0.84[0.8 \times 1000 - 8 \times 10] = 2419.2\ \text{W}$$

$$T_{fo} = T_{fi} + \frac{Q_u}{\dot{m} c_{pf}} = 20 + \frac{2419.2}{0.05 \times 4180} = 31.57\ ℃$$

$$T_{pm} = T_{fi} + \frac{Q_u}{A_c U_L F_R}(1 - F_R) = 20 + \frac{2419.2}{4 \times 8 \times 0.84}(1 - 0.84) = 34.4\ ℃$$

$$T_{fm} = T_{fi} + \frac{Q_u}{A_c U_L F_R}\left(1 - \frac{F_R}{F}\right) = 20 + \frac{2419.2}{4 \times 8 \times 0.84}\left(1 - \frac{0.84}{0.9}\right) = 26\ ℃$$

$$\eta_c = \frac{Q_u}{A_c I_T} = \frac{2419.2}{4 \times 1000} = 0.6048 = 60.48\%$$