Transportation Problems

Introduction to transportation problem

The transportation problem is to transport various amounts of a single homogeneous commodity that are initially stored at various origins, to different destinations in such a way that the total transportation cost is a minimum. It can also be defined as to ship goods from various origins to various destinations in such a manner that the transportation cost is a minimum. The availability as well as the requirements is finite. It is assumed that the cost of shipping is linear.

Mathematical Formulation

Let there be m origins, i^{th} origin possessing a_i units of certain product. Let there be n destinations, with destination j requiring b_j units of a certain product.

Let c_{ij} be the cost of shipping one unit from i^{th} source to j^{th} destination let x_{ij}be the amount to be shipped from i^{th} source to j^{th} destination it is assumed that the total availabilities Σai satisfy the total requirements Σbj i.e. Σai= Σbj(i=1,2,3…m and j=1,2,3..n).

The problem now, is to determine non-negative of x_{ij}satisfying both the availability constraints. $\sum_j^n = a_i$ for i=1,2,….,m

As well as requirement constraints

$\sum_j^m = b_i$ for i=1,2,….,n

And the minimizing cost of transportation (shipping)

Z=$\sum_{i=1}^m \sum_{j=1}^m xijcij$ (objective function)

This special type of LPP is called as transportation problem.

Tabular Representation

Let 'm' denote number of factories (f1,f2…fm)

Let 'n' denote number of warehouse (w1,w2….wn)

w→ f	W1	W2	..	Wn	Capacities (availability)
F1 F2 . . Fm	C11 C21 . . Cm1	C12 C22 . . Cm2	.. ..	C1n C2n . . Cmn	A1 A2 .. .. Am
Required	B1	B2	..	Bn	$\sum ai = \sum bj$

w→ f	W1	W2	..	Wn	Capacities (availability)
F1 F2 . . Fm	X11 X21 . . Xm1	X12 X22 . . Xm2	.. ..	W1n X2n . . Xmn	A1 A2 .. .. Am
Required	B1	B2	..	Bn	$\sum ai = \sum bj$

In general these two tables are combined by inserting each unit cost c_{ij} with the corresponding amount x_{ij} in the cell (I,j). the product $c_{ij}\, x_{ij}$ gives the net cost of shipping units from the factory fi to warehouse w_j.

Some Basic Definitions.

- Feasible solution
- A set of non-negative individual allocations (x_{ij} ≥0) which simultaneously removes deficiencies is called as feasible solution.
- Basic feasible solution
- A feasible solution to 'm' origin, 'n' destination problem is said to be basic if the number of positive allocations are m+n-1. If the number of allocations is less than m+n-1 then it is called as degenerate basic feasible solution. Otherwise it is called as non-degenerate basic feasible solution.
- Optimum solution a feasible solution is said to be optimal if it minimizes the total transportation cost.

  Methods for initial basic feasible solution

  Some simple methods to obtain the initial basic feasible solution are

  1- North – west corner rule

  2- Row minima method

  3- Column minima method

  4- Lowest cost entry method (matrix minima method)

  5- Vogel's approximation method (unit cost penalty method)

1- North –west corner rule

Step 1

- The first assignment is made in the cell occupying the upper left-hand (north-west) corner of the table.
- The maximum possible amount is allocated here i.e. x11=min (a1,b1). This value of x11 is then entered in the cell (1,1) of the transportation table.

Step 2

i. If b1 > a1, move vertically downwards to the second row and make the second allocation of amount x21=min(a2,b1-x11) in the cell (2,1).

ii. If b1 < a1, move horizontally right side to the second column and make the second allocation of amount x12=min(a1-x11,b2) in the cell(1,2).

iii. If b1=a1, there is tie for the second allocation. One can make a second allocation of magnitude x12=min(a1-a1,b2) in the cell(1,2) or x21=min(a2,b1-b1)in the cell(2,1).

Step 3

Start from the new north-west corner of the transportation table and repeat steps 1 and step 2 until all the requirements are satisfied.

Examples:

Find the initial basic feasible solution by using north-west corner rule

1-

w→ f	W1	W2	W3	W4	Factory capacity
F1	19	30	50	10	7
F2	70	30	40	60	9
F3	40	8	70	20	18
Warehouse requirement	5	8	7	14	34

Solution :

	W1	W2	W3	W4	Availability		
F1	5 (19)	2 (30)			7	2	0
F2		6 (30)	3 (40)		9	3	0
F3			4 (70)	14 (20)	18	14	0
Requirements	5	8	7	14			
	0	6	4	0			
		0	0				

Initial basic feasible solution

X11-5, X12=2, X22-6, X23=3, X33=4, X34=14

The transportation cost is

5(19)+2(30)+6(30)+3(40)+4(70)+14(20)=$1015

2-

	D1	D2	D3	D4	Supply
O1	1	5	3	3	34
O2	3	3	1	2	15
O3	0	2	2	3	12
O4	2	7	2	4	19
Demand	21	25	17	17	80

Solution

	D1	D2	D3	D4	Supply		
O 1	21 (1)	13 (5)			34	13	0
O 2		12 (2)	3 (1)		15	3	0
O 3			12 (3)		12	0	0
O4			2 (2)	17 (4)	19	17	0
Demand	21	25	17	17			
	0	12	14	0			
		0	2				

<u>Initial Basic Feasible Solution</u>

X11=21,X12=13,X22=12,X23=3,X33=12,X43=2,X44=17

The transportation cost is

21(1)+13(5)+12(3)+3(1)+12(2)+2(2)+17(4)=$221

	D1	D2	D3	D4	D5	Supply
O1	2	11	0	3	7	4
O2	1	4	7	2	1	8
O3	3	1	4	8	12	9
Demand	3	3	4	5	6	

<table>
<tr><td></td><td>D1</td><td>D2</td><td>D3</td><td>D4</td><td>D5</td><td colspan="3">Supply</td></tr>
<tr><td>O 1</td><td>3
(2)</td><td>1
(11)</td><td></td><td></td><td></td><td>4</td><td>1</td><td>0</td></tr>
<tr><td>O 2</td><td></td><td>2
(4)</td><td>4
(7)</td><td>2
(2)</td><td></td><td>8</td><td>6</td><td>2</td></tr>
<tr><td>O 3</td><td></td><td></td><td></td><td>3
(8)</td><td>6
(12)</td><td>9</td><td>6</td><td>0</td></tr>
<tr><td rowspan="3">Demand</td><td>3</td><td>3</td><td>4</td><td>5</td><td>6</td><td colspan="3" rowspan="3"></td></tr>
<tr><td>0</td><td>2</td><td>0</td><td>3</td><td>0</td></tr>
<tr><td></td><td>0</td><td></td><td>0</td><td></td></tr>
</table>

$X11 = 3, X12 = 1, X22 = 2, X23 = 4, X24 = 2, X34 = 3, X35 = 6$

<u>The Transportation Cost is</u>

3(2)+1(11)+2(4)+4(7)+2(2)+3(8)+6(12)=$153

2-Row Minima Method

Step 1

- The smallest cost in the first row of the transportation table is determine.
- Allocate as much as possible amount xij=min(a1,bj) in the cell (1,j) so that the capacity of the origin or the destination is satisfied.

Step 2

- If x1j=a1, so that the availability at origin o1 is completely exhausted, cross out the first row of the table and move to second row.
- If X1j=bj, so that the requirement at determine Dj is satisfied , cross out the j^{th} column and reconsider the first row with the remaining availability of origin O1.
- If x1j=a1=bj, the origin capacity a1 is completely exhausted as well as the requirement at destination Dj is satisfied. An arbitrary tie-breaking choice is made. Cross out the j^{th} column and make the second allocation X1k=0 in the cell(1,k) with c1k being the new minimum cost in the first row. Cross out the first row and move to second row.

Step 3

Repeat steps 1 and 2 for the reduced transportation table until all the requirements are satisfied.

Examples:

Determine the initial basic feasible solution using Row minima method.

1-

	W1	W2	W3	W4	availability
F1	19	30	50	10	7
F2	70	30	40	60	9
F3	40	80	70	20	18
Requirements	5	8	7	14	

Solution :

	W1	W2	W3	W4	
F1	(19)	(30)	(50)	7 (10)	X
F2	(70)	(30)	(40)	(60)	9
F3	(40)	(80)	(70)	(20)	18
	5	8	7	7	

	W1	W2	W3	W4	
F1	(19)	(30)	(50)	7 (10)	X
F2	(70)	(30)	8 (40)	(60)	1
F3	(40)	(80)	(70)	(20)	18
	5	x	7	7	

	W1	W2	W3	W4	
F1	(19)	(30)	(50)	7 (10)	X
F2	(70)	8 (30)	1 (40)	(60)	x
F3	(40)	(80)	(70)	(20)	18
	5	x	6	7	

	W1	W2	W3	W4	
F1	(19)	(30)	(50)	7 (10)	X
F2	(70)	8 (30)	1 (40)	(60)	9
F3	5 (40)	(80)	6 (70)	7 (20)	18
	x	x	x	x	

Initial basic feasible solution

X14=7, X22=8, X23=1, X31=5, X33=6, X34=7

THE TRANSPORTATION COST IS

7(10)+8(30)+1(40)+5(40)+6(70)+7(20)=$1110

2-

	A	B	C	Availability
I	50	30	220	1
II	90	45	170	4
II	250	200	50	4
Requirements	4	2	3	

	A	B	C	Availability		
I		1 30		1	0	
II	3 90	1 45		4	3	0
II	1 250		3 50	4	1	0
Requirements	4	2	3			
	1	1	1			
	0	0				

Initial basic feasible solution

$X_{12}=1$, $X_{21}=3$, $X_{22}=1$, $X_{31}=1$, $X_{33}=3$

The transportation cost is

$1(30)+3(90)+1(45)+1(250)+3(50)=\$745$

3-Column minima method

Step 1

Determine the smallest cost in the first column of the transportation table. Allocate

$X_{i1}=\min(a_i,b_1)$ in the cell(I,1).

Step 2

- If $X_{i1}=b_1$, cross out the first column of the table and move towards right to the second column.
- If $X_{i1}=a_i$, cross out the i^{th} row of the table and reconsider the first column with the remaining demand.
- If $X_{i1}=b_1=a_i$, cross out the i^{th} row and make the second allocation $x_{k1}=0$ in the cell(k,1) with c_{k1} being the new minimum cost in the first column, cross out the column and move towards right to the second column.

Step 3

Repeat steps 1 and 2 for the reduced transportation table until all the requirements are satisfied.

Examples :

Use column minima method to determine an initial basic feasible solution :

1-

	W1	W2	W3	W4	Availability
F1	19	30	50	10	7
F2	70	30	40	60	9
F3	40	80	70	20	18
Requirements	5	8	7	14	

	W1	W2	W3	W4	
F1	5 (19)	(30)	(50)	(10)	2
F2	(70)	(30)	(40)	(60)	9
F3	(40)	(80)	(70)	(20)	18
	x	8	7	14	

	W1	W2	W3	W4	
F1	5 (19)	2 (30)	(50)	(10)	X
F2	(70)	(30)	1 (40)	(60)	9
F3	(40)	(80)	(70)	(20)	18
	x	6	7	14	

	W1	W2	W3	W4	
F1	5 (19)	2 (30)	(50)	(10)	X
F2	(70)	6 (30)	(40)	(60)	3
F3	(40)	(80)	(70)	(20)	18
	x	x	7	14	

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	W1	W2	W3	W4	
F1	5 (19)	2 (30)	(50)	(10)	X
F2	(70)	6 (30)	3 (40)	(60)	X
F3	(40)	(80)	(70)	(20)	18
	x	x	4	14	

	W1	W2	W3	W4	
F1	5 (19)	2 (30)	(50)	(10)	X
F2	(70)	6 (30)	3 (40)	(60)	9
F3	5 (40)	(80)	4 (70)	(20)	14
	x	x	x	14	

	W1	W2	W3	W4	
F1	5 (19)	2 (30)	(50)	(10)	X
F2	(70)	6 (30)	3 (40)	(60)	9
F3	(40)	(80)	4 (70)	14 (20)	X
	x	x	x	x	

Initial basic feasible solution

X11=5, X12=2, X22=6,X23=3,X33=4,X34=14

The transportation cost is

5(19)+2(30)+6(30)+3(40)+4(70)+14(20)=$1015

2-

	D1	D2	D3	D4	Availability
S1	11	13	17	14	250
S2	16	18	14	10	300
S3	21	24	13	10	400
Requirements	200	225	275	250	

	D1	D2	D3	D4			
S1	200 (11)	50 (13)			250	50	0
S2		175 (18)		125 (10)	300	125	0
S3			275 (13)	125 (10)	400	125	0
	200	225	275	250			
	0	175	0	0			
		0					

Initial basic feasible solution

X11=200,X12=50,X22=175,X24=125,X33=275,X34=125

The transportation cost is

200(11)+50(13)+175(18)+125(10)+275(13)+125(10)=$12075

4- Lowest cost entry method (matrix minima method)

Step 1

Determine the smallest cost in the matrix of the transportation table. Allocate XIJ=min(ai,bj) in the cell(I,j)

Step 2

- If Xij=ai, cross out the i^{th} row of the table and decrease bj by ai. Go to step 3.
- If Xij=bj, cross out the j^{th} column of the table and decrease ai by bj. Go to step 3.
- If Xij=ai=bj, cross out the i^{th} row or j^{th} column but not both.

Step 3

Repeat steps 1 and 2 for the resulting reduced transportation table until all the requirements are satisfied. Whenever the minimum cost is not unique, make an arbitrary choice among the minima.

Examples:

Find the initial basic feasible solution using matrix minima method

1-

	W1	W2	W3	W4	Availability
F1	19	30	50	10	7
F2	70	30	40	60	9
F3	40	8	70	20	18
Requirements	5	8	7	14	

	W1	W2	W3	W4	
F1	(19)	(30)	(50)	(10)	7
F2	(70)	(30)	(40)	(60)	9
F3	(40)	8 (8)	(70)	(20)	10
	5	X	7	14	

	W1	W2	W3	W4	
F1	(19)	(30)	(50)	7 (10)	X
F2	(70)	(30)	(40)	(60)	9
F3	(40)	8 (8)	(70)	(20)	10
	5	X	7	7	

	W1	W2	W3	W4	
F1	(19)	(30)	(50)	7 (10)	X
F2	(70)	(30)	(40)	(60)	9
F3	(40)	8 (8)	(70)	7 (20)	3
	5	X	7	14	

	W1	W2	W3	W4	
F1	(19)	(30)	(50)	7 (10)	X
F2	(70)	(30)	(40)	(60)	9
F3	3 (40)	8 (8)	(70)	7 (20)	X
	2	X	7	X	

	W1	W2	W3	W4	
F1	(19)	(30)	(50)	7 (10)	X
F2	2 (70)	(30)	7 (40)	(60)	X
F3	3 (40)	8 (8)	(70)	7 (20)	X
	X	X	X	X	

Initial basic feasible solution

X14=7,X21=2,X23=7,X31=3,X32=8,X34=7

The transportation cost is

2-

W1	W2	W3	W4	W5	Availability
2	11	10	3	7	4
1	4	7	2	1	8
3	9	4	8	12	9
3	3	4	5	6	
W1	W2	W3	W4	W5	

			4 (3)		4	0			
3 (1)				5 (1)	8	5	0		
	3 (9)	4 (4)	1 (8)	1 (12)	9	5	4	1	0
3	3	4	5	6					
0	0	0	1	1					
			0	0					

Initial basic feasible solution

X14=4,X21=3,X25=5,X32=3,X33=4,X34=1,X35=1

The transportation cost is

4(3)+3(1)+5(1)+3(9)+4(4)+1(8)+1(12)=$78

5-Vogel's approximation method (unit cost penalty method)

Step 1

For each row of the table, identify the smallest and the next to smallest cost. Determine the different between them for each row. These are called penalties. Put them aside by enclosing them in the parenthesis against the respective rows. Similarly compute penalties for each column.

Step 2

Identify the row or column with the largest penalty. If a tie occurs then use an arbitrary choice. Let the largest penalty corresponding to the i^{th} row have the cost cij allocate the largest possible amount xij=min(ai,bj) in the cell(I,j) and cross out either i^{th} row or j^{th} column in the usual manner.

Step 3

Again compute the row and column penalties for the reduced table and then go to step 2. Repeat the procedure until all the requirements are satisfied.

<u>Examples:</u> find the initial basic feasible solution using vogel's approximation method.

1-

	W1	W2	W3	W4	Availability
F1	19	30	50	10	7
F2	70	30	40	60	9
F3	40	80	70	20	18
Requirements	5	8	7	14	

	W1	W2	W3	W4	Availability	
F1	19	30	50	10	7	19-10=9
F2	70	30	40	60	9	40-30=10
F3	40	80	70	20	18	20-8=12
	5	8	7	14		
	40-19=21	30-8=22	50-40=10	20-10=10		

	W1	W2	W3	W4	Availability	
F1	19	30	50	10	7	9
F2	70	30	40	60	9	10
F3	40	8 8	70	20	10	12
	5	0	7	14		
	21	x	10	10		

	W1	W2	W3	W4	Availability	
F1	19 5	30	50	10	2	9
F2	70	30	40	60	9	20
F3	40	8 8	70	20	10	20
	0	0	7	14		
	X	X	10	10		

	W1	W2	W3	W4	Availability	
F1	19 5	30	50	10	2	40
F2	70	30	40	60	9	20
F3	40	8 8	70	20 10	0	x
	0	0	7	4		
	X	X	10	50		

	W1	W2	W3	W4	Availability	
F1	19 5	30	50	10 2	0	X
F2	70	30	40	60	9	20
F3	40	8 8	70	20 10	0	X
	0	0	7	2		
	X	X	10	50		

	W1	W2	W3	W4	Availability	
F1	19 5	30	50	10 2	0	X
F2	70	30	40 7	60 2	0	X
F3	40	8 8	70	20 10	0	X
	0	0	0	0		
	X	X	X	X		

Initial basic feasible solution

XX11=5,X14=2,X23=7,X24=2,X32=8,X34=10

The transportation cost is 5(19)+2(10)+7(40)+2(60)+8(8)+10(20)=$779

2-

Stores Warehouse	I	II	III	IV	Availability
A	21	16	15	13	11
B	17	18	14	23	13
C	32	27	18	41	19
Requirements	6	10	12	15	

	I	II	III	IV	Availability	
A	21	16	15	13	11	2
B	17	18	14	23	13	3
C	32	27	18	41	19	9
	6	10	12	15		
	4	2	1	10		

	I	II	III	IV	Availability	
A	21	16	15	13 11	0	X

B	17	18	14	23	13	3
C	32	27	18	41	19	9
	6	10	12	4		
	15	9	4	18		

	I	II	III	IV	Availability	
A	21	16	15	13 1 1	0	X
B	17 6	18 3	14	23 4	0	X
C	32	27	18	41	19	9
	0	10	12	0		
	X	9	4	X		

	I	II	III	IV	Availability	
A	21	16	15	13 11	0	X
B	17 6	18 3	14	23 4	0	X
C	32	27 7	18 12	41	0	X
	0	0	0	0		
	X	X	X	X		

Initial basic feasible solution

$X14=11, X21=6, X22=3, X24=4, X32=7, X33=12$, and the transportation cost is:

$11(13)+6(17)+3(18)+4(23)+7(27)+12(18)=\$796$