Semiconductor Optoelectronics

Lecture 3: Nonequilibrium Excess Carriers

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Text book

Semiconductor Physics and Devices

Basic Principles

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Chapter 6: Nonequilibrium Excess Carriers in Semiconductors

Excess Carriers

For the simple model of recombination we are using (direct band-to-band recombination) the probability of an electron-hole pair recombining is constant with time.

Moreover, the rate at which electrons recombine must be proportional to both the electron concentration and hole concentration. We can describe this mathematically with:

$$\frac{dn(t)}{dt} = \alpha_r \left[n_i^2 - n(t)p(t) \right]$$

where

$$n(t) = n_0 + \delta n(t)$$

$$p(t) = p_0 + \delta p(t)$$

The term $\alpha_r n_i^2$ is the thermal equilibrium generation rate.

Note then that the entire expression in the parentheses will be less than (or equal to) zero so that the derivative is negative. This should make common sense as the value of n(t) is decreasing due to recombination.

Excess Carriers

Since n_0 is constant with respect to time the derivative can be taken with respect to δn instead of n.

Also, since excess electrons and holes are generated and recombine in pairs we know that $\delta n(t) = \delta p(t)$.

Making these substitutions and expanding the terms out we find:

$$\frac{d(\delta n(t))}{dt} = \alpha_r \left[n_i^2 - (n_0 + \delta n(t))(p_o + \delta p(t)) \right]$$

$$\frac{d(\delta n(t))}{dt} = \alpha_r \left[n_i^2 - \{ n_0 p_0 + n_0 \delta p(t) + \delta n(t) p_0 + \delta n(t) \delta p(t) \} \right]$$

$$\frac{d(\delta n(t))}{dt} = -\alpha_r \left[n_0 \delta n(t) + \delta n(t) p_0 + \delta n(t) \delta n(t) \right]$$

Low-Level Injection

The differential equation we have derived up to this point isn't the easiest to solve at the moment. However, if we restrict ourselves to the case of "low-level injection" (a common situation) it becomes much simpler.

Low-level injection simply means that the number of excess carriers is much smaller than the thermal equilibrium values of the majority carrier concentration. That is (for p-type material), $\delta n(t) \ll p_0$.

For p-type material we also know that $n_0 \ll p_0$. Therefore, looking at our equation

$$\frac{d(\delta n(t))}{dt} = -\alpha_r\left[n_0 \delta n(t) + \delta n(t) p_0 + \delta n(t) \delta n(t)\right]$$

we can see that the $\delta n(t)p_0$ term will dominate the other two terms on the right-hand side of the equation.

Low-Level Injection

We can thus approximate this equation as

$$\frac{d(\delta n(t))}{dt} = -\alpha_r [\delta n(t) p_0]$$

This is a simple first-order differential equation with a solution of

$$\delta n(t) = \delta n(0) e^{-\alpha_r p_0 t} = \delta n(0) e^{-t/\tau_{n0}}$$

where τ_{n0}, the excess minority carrier lifetime, is given by $\tau_{n0} = (\alpha_r p_0)^{-1}$. Note that the excess minority carrier lifetime depends on the majority carrier concentration.

The excess carrier recombination rate, R_n', is the change in the number of excess carriers, $\delta n(t)$, so we can write

$$R_n' = \frac{-d(\delta n(t))}{dt} = \alpha_r p_0 \delta n(t) = \frac{\delta n(t)}{\tau_{n0}}$$

Low-Level Injection

For direct band-to-band recombination, the excess majority carrier holes recombine at the same rate (if an electron has recombined, it obviously must have recombined with a hole therefore subtract BOTH one free electron and one free hole).

Since the two rates are equal we can write, for p-type material,

$$R_n' = R_p' = \frac{\delta n(t)}{\tau_{n0}}$$

A similar derivation can be done for low-level injection in n-type material to yield

$$R_n' = R_p' = \frac{\delta n(t)}{\tau_{p0}}$$

Example 1

Consider silicon at T=300 that is doped with donor impurity atoms to a concentration of $N_d = 5 \times 10^5\ cm^{-3}$. The excess carrier lifetime time is 2×10^{-7} s.

a) Determine the thermal equilibrium recombination rate of holes
b) Excess carriers are generated such that $\delta_n = \delta_p = 10^{14}\ cm^{-3}$. What is the excess holes recombination rate for this condition?

Example 1

Consider silicon at T=300 that is doped with donor impurity atoms to a concentration of N_d = 5 x 10^5 cm^{-3} . The excess carrier lifetime time is 2 x 10^{-7} s.

a) Determine the thermal equilibrium recombination rate of holes

$$n_o = N_d = 5\times10^{15}\,\text{cm}^{-3} \rightarrow p_o = \frac{n_i^2}{N_d} = \frac{\left(1.5\times10^{10}\right)^2}{5\times10^{15}} = 4.5\times10^4\ \text{cm}^{-3}$$

$$\text{(a)}\ \ \text{Minority carrier hole lifetime is a constant where } \tau_{pt} = \tau_{p0} = 2\times10^{-7}\,\text{s}$$

$$R_{po} = \frac{p_o}{\tau_{p0}} = \frac{4.5\times10^4}{2\times10^{-7}} = 2.25\times10^{11}\,\text{cm}^{-3}\text{s}^{-1}$$

Example 1

Consider silicon at T=300 that is doped with donor impurity atoms to a concentration of N_d = 5 x 10^5 cm^{-3} . The excess carrier lifetime time is 2 x 10^{-7} s.

b) Excess carriers are generated such that δn= δp=10^{14} $cm^{-3.}$ What is the excess holes recombination rate this condition?

$$R'_{po} = \frac{\delta p}{\tau_{p0}} = \frac{10^{14}}{2 \times 10^{-7}} \text{ cm}^{-3}$$

$$= 5 \times 10^{20} \text{cm}^{-3}\text{s}^{-1}$$

Example 2

GaAs at T=300K is uniformly doped with acceptor impurity atoms to a concentration of $N_a = 2 \times 10^{16}$ cm^{-3}. Assume an excess carrier lifetime of 5×10^{-7} s.

a) Determine the excess electron-hole recombination rate if the excess electron concentration is $\delta n = 5 \times 10^{14}$ cm^{-3}
b) Using the result of (a), what is the lifetime of holes?

Example 2

GaAs at T=300K is uniformly doped with acceptor impurity atoms to a concentration of $N_a = 2 \times 10^{16}$ cm^{-3}. Assume an excess carrier lifetime of 5×10^{-7} s.

a) Determine the excess electron-hole recombination rate if the excess electron concentration is $\delta n = 5 \times 10^{14}$ cm^{-3}

$$R' = \frac{\delta n}{\tau_{n0}} = \frac{5\times 10^{14}}{5\times 10^{-7}} = 10^{21}\,\text{cm}^{-3}\text{s}^{-1}$$

Example 2

GaAs at T=300K is uniformly doped with acceptor impurity atoms to a concentration of N_a = 2 x 10^{16} cm^{-3}. Assume an excess carrier lifetime of 5 x 10^{-7} s.

b) Using the result of (a), what is the lifetime of holes?

$$p_o = N_a = 2\times10^{16}\,\text{cm}^{-3} \rightarrow n_o = \frac{n_i^2}{p_o} = \frac{\left(1.8\times10^{6}\right)^2}{2\times10^{16}} = 1.62\times10^{-4}\ \text{cm}^{-3}$$

$$R_p = \frac{p_o}{\tau_{pt}} = \frac{n_o}{\tau_{n0}}$$

$$\tau_{pt} = \frac{p_o}{n_o}\cdot\tau_{n0} = \frac{\left(2\times10^{16}\right)}{\left(1.62\times10^{-4}\right)}\cdot\left(5\times10^{-7}\right) = 6.17\times10^{13}\text{s}$$

Example 3

An n-type silicon sample contains a donor concentration of $N_d = 10^{16}$ cm^{-3}. The minority carrier hole lifetime is found to be $\tau_{p0} = 20\ \mu s$

a) What is the lifetime of the majority carrier electrons
b) Determine the thermal-equilibrium generation rate for electrons and holes in this material.
c) Determine the thermal-equilibrium recombination rate for electrons and holes in this material

Example 3

An n-type silicon sample contains a donor concentration of $N_d = 10^{16}$ cm^{-3}. The minority carrier hole lifetime is found to be $\tau_{p0} = 20$ μs

a) What is the lifetime of the majority carrier electrons

(a) Recombination rates are equal

$$\frac{n_o}{\tau_{nO}} = \frac{p_o}{\tau_{pO}}$$

$$n_o = N_d = 10^{16} \text{ cm}^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5\times 10^{10})^2}{10^{16}} = 2.25\times 10^4 \text{ cm}^{-3}$$

Then

$$\frac{10^{16}}{\tau_{nO}} = \frac{2.25\times 10^4}{20\times 10^{-6}}$$

which yields

$$\tau_{nO} = 8.89\times 10^{+6} \text{ s}$$

Example 3

An n-type silicon sample contains a donor concentration of $N_d = 10^{16}\ cm^{-3}$. The minority carrier hole lifetime is found to be $\tau_{p0} = 20\ \mu s$

b) Determine the thermal-equilibrium generation rate for electrons and holes in this material.

(b) Generation rate = recombination rate

Then

$$G = \frac{2.25\times10^{4}}{20\times10^{-6}} = 1.125\times10^{9}\ \text{cm}^{-3}\ \text{s}^{-1}$$

Example 3

An n-type silicon sample contains a donor concentration of $N_d = 10^{16}$ cm^{-3}. The minority carrier hole lifetime is found to be $\tau_{p0} = 20\ \mu s$

c) Determine the thermal-equilibrium recombination rate for electrons and holes in this material

(c)

$$R = G = 1.125 \times 10^9 \text{ cm}^{-3}\text{ s}^{-1}$$