

Semiconductor Optoelectronics

Lecture 4: Nonequilibrium Excess Carriers in Semiconductors

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Text book

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Basic Principles

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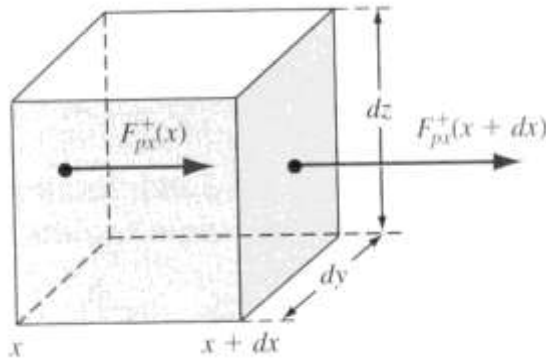
Chapter 6: Nonequilibrium Excess Carriers in Semiconductors



Continuity Equations

Let us consider the flux of particles into and out of a small box.

Assume the flow of particles only occurs in 1-D along the x-axis and the box is a differential volume with dimension dx by dy by dz .



Assume our particles are holes. Then using a first-order Taylor expansion we can relate the flux of particles into the box to the flux of particles out of the box as

$$F_{px}^+(x + dx) = F_{px}^+(x) + \frac{\partial F_{px}^+}{\partial x} dx$$

Continuity Equations

The net increase in particles within the volume would be the difference of the two fluxes (multiplied by the surface area)

$$\left[F_{px}^+(x) - F_{px}^+(x + dx) \right] \underbrace{dydz}_{\text{Area}} = \left[\frac{-\partial F_{px}^+}{\partial x} dx \right] dydz$$

But the net increase in particles inside the box could also be written

$$\frac{\partial p}{\partial t} \underbrace{dxdydz}_{\text{Volume}}$$

So

$$\frac{\partial p}{\partial t} dxdydz = - \frac{\partial F_{px}^+}{\partial x} dxdydz$$

This only represents the buildup or decrease of particles in the box due to different flow rates however.

Continuity Equations

We also need to account for the effects of generation and recombination that may be occurring within the box. These two phenomena would also contribute to an increase or decrease of the particle concentration within the box. Including these terms we have:

$$\frac{\partial p}{\partial t} dx dy dz = -\frac{\partial F_{px}^+}{\partial x} dx dy dz + g_p dx dy dz - \frac{p}{\tau_{pt}} dx dy dz$$

p is the density of holes and τ_{pt} is the combined hole lifetime (it includes both the thermal equilibrium carrier lifetime and the excess carrier lifetime).

Dividing through by $dx \cdot dy \cdot dz$ we reach the Continuity Equations.

$$\frac{\partial p}{\partial t} = -\frac{\partial F_p^+}{\partial x} + g_p - \frac{p}{\tau_{pt}} \quad (\text{Holes})$$
$$\frac{\partial n}{\partial t} = -\frac{\partial F_n^-}{\partial x} + g_n - \frac{n}{\tau_{nt}} \quad (\text{Electrons})$$

Time-Dependent Diffusion Equations

Our current densities are given by

$$J_p = e\mu_p pE - eD_p \frac{\partial p}{\partial x}$$

$$J_n = e\mu_n nE + eD_n \frac{\partial n}{\partial x}$$

By dividing the current density by the unit of charge we obtain particle flux:

$$\frac{J_p}{e} = F_p^+ = \mu_p pE - D_p \frac{\partial p}{\partial x}$$

$$\frac{J_n}{(-e)} = F_n^- = -\mu_n nE - D_n \frac{\partial n}{\partial x}$$

Time-Dependent Diffusion Equations

By substituting these expressions into the Continuity Equations we get:

$$\frac{\partial p}{\partial t} = -\frac{\partial F_p^+}{\partial x} + g_p - \frac{p}{\tau_{pt}} = -\mu_p \frac{\partial(pE)}{\partial x} + D_p \frac{\partial^2 p}{\partial x^2} + g_p - \frac{p}{\tau_{pt}}$$

$$\frac{\partial n}{\partial t} = -\frac{\partial F_n^-}{\partial x} + g_n - \frac{n}{\tau_{nt}} = +\mu_n \frac{\partial(nE)}{\partial x} + D_n \frac{\partial^2 n}{\partial x^2} + g_n - \frac{n}{\tau_{nt}}$$

Both p (or n) and E can be functions of position so we need to use the chain rule

$$\frac{\partial(pE)}{\partial x} = E \frac{\partial p}{\partial x} + p \frac{\partial E}{\partial x}$$

$$\frac{\partial(nE)}{\partial x} = E \frac{\partial n}{\partial x} + n \frac{\partial E}{\partial x}$$

Time-Dependent Diffusion Equations

Thus

$$\frac{\partial p}{\partial t} = -\mu_p \left(E \frac{\partial p}{\partial x} + p \frac{\partial E}{\partial x} \right) + D_p \frac{\partial^2 p}{\partial x^2} + g_p - \frac{p}{\tau_{pt}}$$

$$\frac{\partial n}{\partial t} = +\mu_n \left(E \frac{\partial n}{\partial x} + n \frac{\partial E}{\partial x} \right) + D_n \frac{\partial^2 n}{\partial x^2} + g_n - \frac{n}{\tau_{nt}}$$

If we assume we have a homogeneous semiconductor (the doping concentration of electrons and holes is uniform throughout the semiconductor), then

$$p(x) = p_0 + \delta p(x)$$

$$n(x) = n_0 + \delta n(x)$$

and our partial derivatives of $p(x)$ and $n(x)$ just become partial derivatives of $\delta p(x)$ and $\delta n(x)$.

Time-Dependent Diffusion Equations

Thus we get the time-dependent diffusion equations for electrons and holes in a homogeneous semiconductor:

$$\frac{\partial(\delta p)}{\partial t} = D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p \left(E \frac{\partial(\delta p)}{\partial x} + p \frac{\partial E}{\partial x} \right) + g_p - \frac{p}{\tau_{pt}}$$
$$\frac{\partial(\delta n)}{\partial t} = D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \left(E \frac{\partial(\delta n)}{\partial x} + n \frac{\partial E}{\partial x} \right) + g_n - \frac{n}{\tau_{nt}}$$

Example 4

The concentration of holes in a semiconductor is given by $p(x) = 5 \times 10^{15} e^{-x/L_p}$ cm^{-3} for $x \geq 0$. Determine the hole diffusion current density at

- a) $x=0$
- b) $x=L_p$

if the material is silicon with $D_p = 10 \text{ cm}^2/\text{s}$ and $L_p = 50 \mu\text{m}$

$$J_p = -eD_p \frac{dp}{dx} = -eD_p \frac{d}{dx} \left[5 \times 10^{15} e^{-x/L_p} \right]$$
$$= \frac{eD_p (5 \times 10^{15}) e^{-x/L_p}}{L_p}$$

a) $x=0$

(i) $J_p = \frac{(1.6 \times 10^{-19})(10)(5 \times 10^{15})}{50 \times 10^{-4}}$
 $= 1.6 \text{ A/cm}^2$

b) $x=L_p$

(i) $J_p = \frac{(1.6 \times 10^{-19})(10)(5 \times 10^{15}) e^{-1}}{50 \times 10^{-4}}$
 $= 0.589 \text{ A/cm}^2$

Example 5

The steady-state electron distribution in silicon can be approximated by a linear function of x . The maximum electron concentration occurs at $x=0$ and is $n_{(0)}=2 \times 10^{16} \text{ cm}^{-3}$.

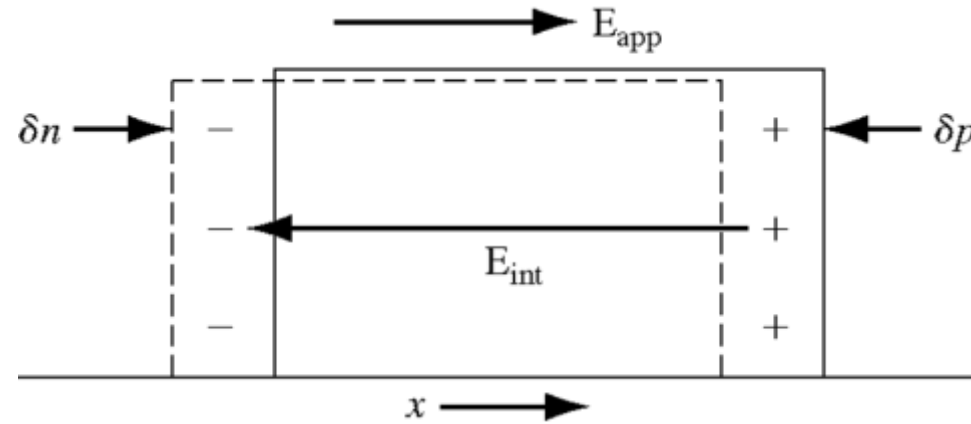
At $x=0.012 \text{ cm}$, the electron concentration is $5 \times 10^{15} \text{ cm}^{-3}$. If the electron diffusion coefficient is $D_n=27 \text{ cm}^2/\text{s}$, determine the electron diffusion current density

$$J_n = eD_n \frac{dn}{dx} = eD_n \frac{\Delta n}{\Delta x} \quad \text{I}$$

$$J_n = (1.6 \times 10^{-19})(27) \left[\frac{2 \times 10^{16} - 5 \times 10^{15}}{0 - 0.012} \right]$$

$$J_n = -5.4 \text{ A/cm}^2$$

Ambipolar Transport



- Since the internal E-field creates a force attracting the electrons and holes, this E-field will hold the pulses of excess electrons and holes together.
- The negatively charged electrons and positively charged holes then will drift or diffuse together with a single effective mobility or diffusion coefficient.
- This phenomenon is called **Ambipolar diffusion** or **Ambipolar transport**.

Ambipolar Transport Equations

Since electrons and holes are generated in pairs

$$\text{Similarly, } R_n = \frac{n}{\tau_{nt}} = R_p = \frac{p}{\tau_{pt}} \equiv R$$

And since they are generated in pairs (assuming charge neutrality), $\delta n = \delta p$.

Thus we can write the time-dependent diffusion equations as:

$$\frac{\partial(\delta p)}{\partial t} = \frac{\partial(\delta n)}{\partial t} = D_p \frac{\partial^2(\delta n)}{\partial x^2} - \mu_p \left(E \frac{\partial(\delta n)}{\partial x} + p \frac{\partial E}{\partial x} \right) + g - R$$

$$\frac{\partial(\delta n)}{\partial t} = D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \left(E \frac{\partial(\delta n)}{\partial x} + n \frac{\partial E}{\partial x} \right) + g - R$$

Multiplying the first equation by $\mu_n n$ and the second equation by $\mu_p p$ we get:

Ambipolar Transport Equations

$$\mu_n n \frac{\partial(\delta n)}{\partial t} = \mu_n n D_p \frac{\partial^2(\delta n)}{\partial x^2} - \mu_n \mu_p n \left[E \frac{\partial(\delta n)}{\partial x} + p \frac{\partial E}{\partial x} \right] + \mu_n n (g - R)$$

$$\mu_p p \frac{\partial(\delta n)}{\partial t} = \mu_p p D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \mu_p p \left[E \frac{\partial(\delta n)}{\partial x} + n \frac{\partial E}{\partial x} \right] + \mu_p p (g - R)$$

Adding these we eliminate the $\frac{\partial E}{\partial x}$ term

$$(\mu_n n + \mu_p p) \frac{\partial(\delta n)}{\partial t} = (\mu_n n D_p + \mu_p p D_n) \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \mu_p (p - n) E \frac{\partial(\delta n)}{\partial x} + (\mu_n n + \mu_p p) (g - R)$$

We can simplify/rewrite this by dividing through by $(\mu_n n + \mu_p p)$.

Ambipolar Transport Equations

The Ambipolar Transport Equation

$$\frac{\partial(\delta n)}{\partial t} = D' \frac{\partial^2(\delta n)}{\partial x^2} + \mu' E \frac{\partial(\delta n)}{\partial x} + g - R$$

$$D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} \quad \mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$$

However, since $\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q}$

HW

$$D' = \frac{D_n D_p (n + p)}{D_n n + D_p p}$$

Ambipolar Transport Equations

$$D' = \frac{D_n D_p (n + p)}{D_n n + D_p p}$$

$$\mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$$

For “strongly” p-type (or n-type) material under low level injection this reduces considerably.

For p-type, low-level injection

$$p_0 \gg n_0$$

$$p_0 \gg \delta n$$

$$D' \cong D_n$$

$$\mu' \cong \mu_n$$

For n-type, low-level injection

$$n_0 \gg p_0$$

$$n_0 \gg \delta n$$

$$D' \cong D_p$$

$$\mu' \cong -\mu_p$$

Ambipolar Transport Equations

Also for low-level injection (small # of excess carriers) the probability of recombining will be almost constant (the chance of hitting a majority carrier won't change much), so $\tau_{nt} = \tau_n$ and $\tau_{pt} = \tau_p$.

For generation and recombination we have a combination of thermal-equilibrium generation/recombination and excess carrier generation/recombination. Looking at just electrons we have:

$$g - R = g_n - R_n = (G_{n0} + g'_n) - (R_{n0} + R'_n)$$

However, for thermal equilibrium we know $G_{n0} = R_{n0}$ so

$$g - R = g'_n - R'_n = g'_n - \frac{\delta n}{\tau_{n0}}$$

and we would have a similar expression for the holes.

Ambipolar Transport Equations

Combining all this gives our final Ambipolar Transport Equations:

$$\begin{aligned} D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n E \frac{\partial(\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}} &= \frac{\partial(\delta n)}{\partial t} & p\text{-type} \\ D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p E \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}} &= \frac{\partial(\delta p)}{\partial t} & n\text{-type} \end{aligned}$$

These equations simply say that there can be a change in our excess carrier concentration over time because:

1. They diffuse away.
2. They drift away.
3. More are generated.
4. They recombine.

Quasi-Fermi Energy Levels

The thermal-equilibrium electron and hole concentrations are functions of the Fermi energy level. We can write

$$n_o = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right] = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

$$p_o = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right] = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]$$

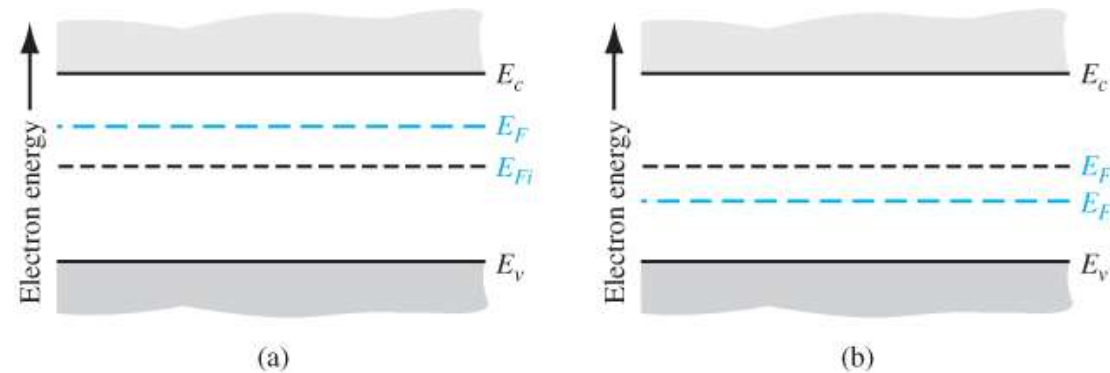


Figure 6.14 | Thermal-equilibrium energy-band diagrams for (a) n-type semiconductor and (b) p-type semiconductor.

Quasi-Fermi Levels

The Fermi Level is constant in a sample under equilibrium. However, with generation or current flowing (even in steady-state) we are in nonequilibrium so E_F is no longer a constant.

However, we still want to know (or relate) total electron and hole concentrations. This can be done by considering a “shift” in E_F that represent the “More” or “Less” p or n nature of the semiconductor. These new levels are called E_{Fn} and E_{Fp} , the Quasi-Fermi Levels.

$$n = n_0 + \delta n = n_i e^{(E_{Fn} - E_{Fi})/kT}$$
$$p = p_0 + \delta p = n_i e^{(E_{Fi} - E_{Fp})/kT}$$

Quasi-Fermi Energy Levels

If excess carriers are created in a semiconductor, we are no longer in thermal equilibrium and the Fermi energy is strictly no longer defined. If δn and δp are excess electron and hole concentrations respectively, we write

$$n_o + \delta n = n_i \exp\left[\frac{E_{Fn} - E_{Fi}}{kT}\right]$$

$$p_o + \delta p = n_i \exp\left[\frac{-(E_{Fp} - E_{Fi})}{kT}\right]$$

Quasi-Fermi Energy Levels

Question: Consider an n-type semiconductor at $T=300\text{K}$ with carrier concentration of $n_0=10^{15}\text{ cm}^{-3}$, $n_i=10^{10}\text{ cm}^{-3}$, and $p_0=10^5\text{ cm}^{-3}$. In nonequilibrium, assume that the excess carrier concentrations are $\delta n=\delta p= 10^{13}\text{ cm}^{-3}$

The Fermi level for thermal equilibrium can be determined

$$E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right) = 0.2982\text{ eV}$$

In nonequilibrium, quasi Fermi level for electrons and holes becomes

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_0 + \delta n}{n_i}\right) = 0.2984\text{ eV}$$

Example 6

Question: Consider n-type silicon doped at $N_d = 5 \times 10^{15} \text{ cm}^{-3}$. It is found that $E_{Fn} - E_F = 1.02 \times 10^{-3} \text{ eV}$. The value of $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$. Determine

a) What is the excess carrier concentration

b) Determine $E_{Fn} - E_{Fi}$ and $E_{Fi} - E_{fp}$. Assume $\delta n = \delta p$

For n-type $E_{Fn} - E_F = (E_{Fn} - E_{Fi}) - (E_F - E_{Fi})$

$$= kT \ln \left(\frac{n_o + \delta n}{n_i} \right) - kT \ln \left(\frac{n_o}{n_i} \right) = kT \ln \left(\frac{n_o + \delta n}{n_o} \times \frac{n_i}{n_i} \right)$$

$$= kT \ln \left(\frac{n_o + \delta n}{n_o} \right)$$

$$0.00102 = (0.0259) \ln \left(\frac{5 \times 10^{15} + \delta n}{5 \times 10^{15}} \right)$$

$$5 \times 10^{15} + \delta n = 5 \times 10^{15} \exp \left(\frac{0.00102}{0.0259} \right) \Rightarrow \delta n \cong 2 \times 10^{14} \text{ cm}^{-3}$$

Example 6

Question: Consider n-type silicon doped at $N_d = 5 \times 10^{15} \text{ cm}^{-3}$. It is found that $E_{Fn} - E_F = 1.02 \times 10^{-3} \text{ eV}$. The value of $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$. Determine
a) What is the excess carrier concentration
b) Determine $E_{Fn} - E_{Fi}$ and $E_{Fi} - E_{Fp}$. Assume $\delta n = \delta p$

$$\begin{aligned} E_{Fn} - E_{Fi} &= kT \ln \left(\frac{n_o + \delta n}{n_i} \right) \\ &= (0.0259) \ln \left(\frac{5 \times 10^{15} + 2 \times 10^{14}}{1.5 \times 10^{10}} \right) \\ &= 0.33038 \text{ eV} \end{aligned}$$

$$\begin{aligned} E_{Fi} - E_{Fp} &\cong kT \ln \left(\frac{\delta p}{n_i} \right) \\ &= (0.0259) \ln \left(\frac{2 \times 10^{14}}{1.5 \times 10^{10}} \right) \\ &= 0.2460 \text{ eV} \end{aligned}$$