

Center of Gravity

(1) The center of gravity
The center of mass
The Centroid of the volume } The Same Point

centroid is always located on the axis of symmetry.



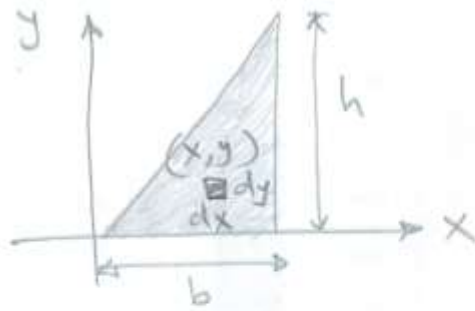
The line could be straight or curved

اذا كان الشكل Symmetry اي متناظر يكون من السهل
ايجاد ال Centroid ويكون فيه مركز هذه الاشكال
مثلا مركز المربع او مركز الدائرة او مركز المستطيل او
مركز الخط المستقيم
كن اذا كان الشكل غير متناظر كيف نعلم ايجاد مركزه ؟

(2)

$$\bar{x} = \frac{\int x \cdot dA}{\int dA}$$

$$\bar{y} = \frac{\int y \cdot dA}{\int dA}$$



- سوف نأخذ element صغيرة مربعة \square داخل هذا الشكل وسأخذ هذا element في $dx * dy$.
- معادلة الخط المائل والذي يمثل الوتر لهذا المثلث هي ميل الخط المثلثي المائل وهو $y = \frac{h}{b}x$.

- الآن باستخدام التكامل نجد قيمة المساحة لهذا الشكل وهي $\int dA$ ولتساوي $\int dx \cdot dy$ ويكون هناك تكاملين أحدهم باتجاه الـ x والآخر باتجاه الـ y . لذلك فإن الـ limit باتجاه الـ x يكون من صفر إلى b والـ limit باتجاه الـ y يكون من صفر إلى $\frac{h}{b}x$.

(3)

$$\int dA = \int_0^b \int_0^{\frac{h}{b}x} dy \cdot dx$$

$$= \int_0^b \left[y \right]_0^{\frac{h}{b}x} \cdot dx$$

$$= \int_0^b \left[\frac{h}{b}x - 0 \right] \cdot dx$$

$$= \int_0^b \frac{h}{b}x \cdot dx$$

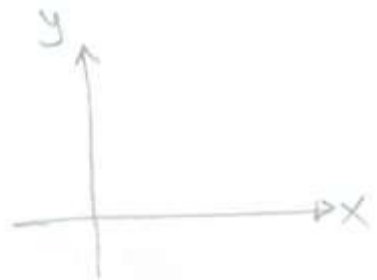
$$= \frac{h}{b} \int_0^b x \cdot dx$$

$$= \frac{h}{b} \left[\frac{x^2}{2} \right]_0^b$$

$$= \frac{h}{b} \left[\frac{b^2 - 0^2}{2} \right]$$

$$= \frac{h}{2b} \cdot b^2$$

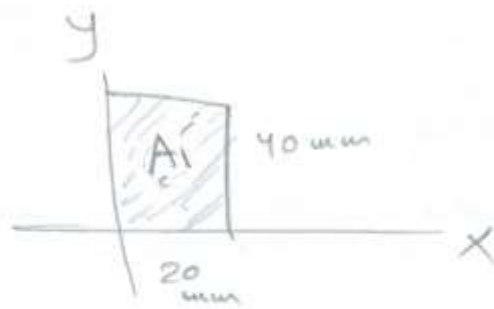
$$= \frac{1}{2} h \cdot b$$



④ Centroid of Composite or Complex Shape

Example

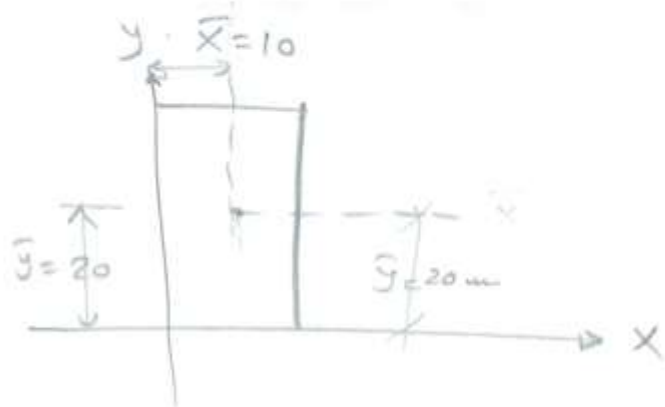
locate the Centroid of the Shape



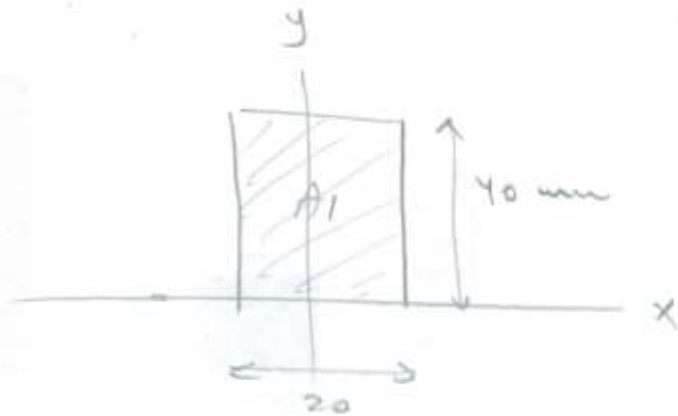
A_i	Area (mm^2)	X (mm)	$M_y = A_i \times X$	Y	M_x
①	$40 \times 20 = 800$	10	$800 \times 10 = 8000 \text{ mm}^3$	20	$800 \times 20 = 16000$

$$\bar{X} = \frac{\sum M_y}{\sum A} = \frac{8000}{800} = 10 \text{ mm}$$

$$\bar{Y} = \frac{\sum M_x}{\sum A} = \frac{16000}{800} = 20 \text{ mm}$$



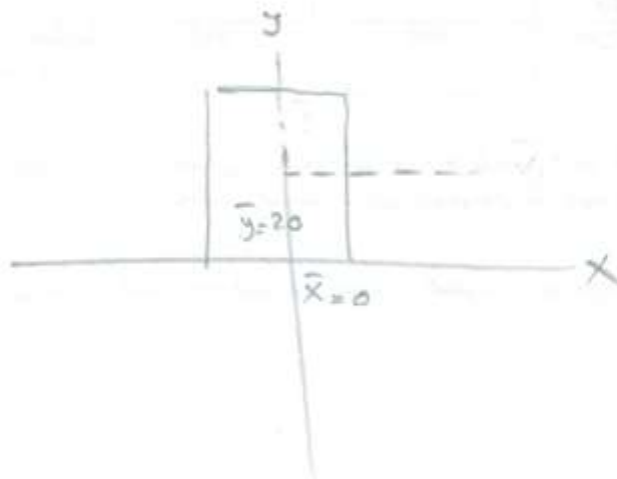
⑤ locate the centroid of the shape



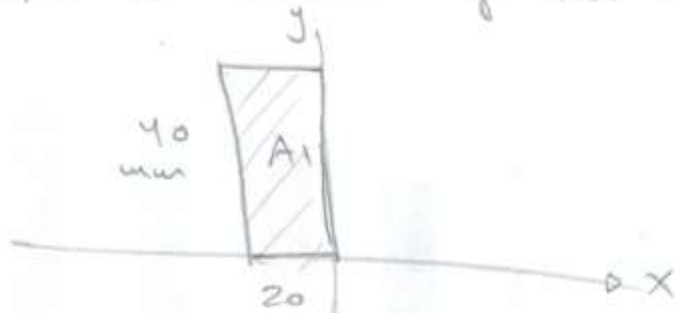
Area #	Area mm ²	(x) mm	My Area * X	(y) mm	Mx = Area * Y
A1	40 * 20 = 800	0	800 * 0 = 0	20	800 * 20 = 16000

$$\bar{x} = \frac{\sum My}{\sum A} = \frac{0}{800} = 0$$

$$\bar{y} = \frac{\sum Mx}{\sum A} = \frac{16000}{800} = 20 \text{ mm}$$



⑥ Locate the centroid of the shape



Area #	Area mm ²	(X) mm	My Area X	Y mm	Mx Area y
	20 × 40 = 800	-10	800 × -10 = -8000	20	800 × 20 = 16000

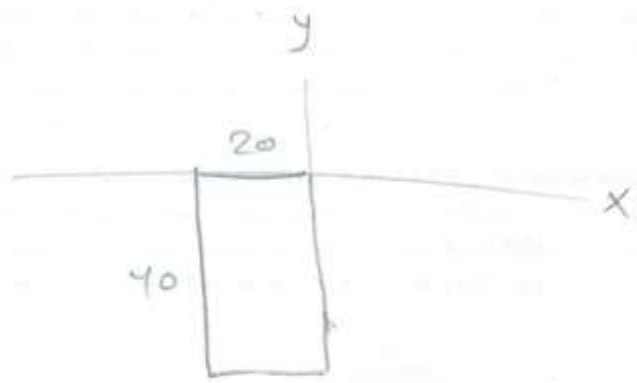
$$\bar{x} = \frac{\sum My}{\sum A} = \frac{-8000}{800} = -10 \text{ mm}$$

$$\bar{y} = \frac{\sum Mx}{\sum A} = \frac{16000}{800} = 20 \text{ mm}$$

MCC :- The Area of shape above is

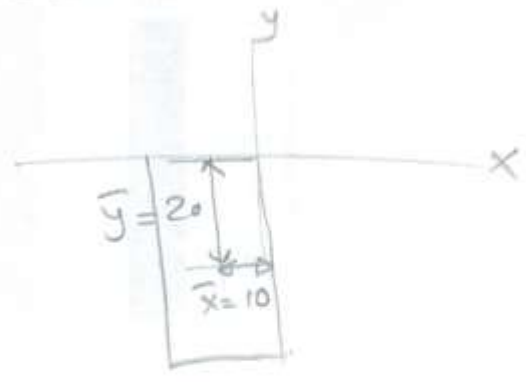
- ① 8000 m²
- ② -8000 mm²
- ③ 60 mm²
- ④ All of these result above
- ⑤ Not of these result above

7) locate the centroid of the shape?

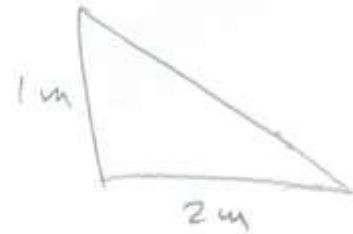


# Area	Area	x (mm)	My Area * x	y (mm)	Mx Area * y
	20 * 40 = 800	-10	800 * 10 = -8000	-20	800 * -20 = -16000
$\bar{x} =$	$\frac{\sum My}{\sum A}$	$\frac{-8000}{800} = -10 \text{ mm}$			

$$\bar{y} = \frac{\sum Mx}{\sum A} = \frac{-16000}{800} = -20 \text{ mm}$$



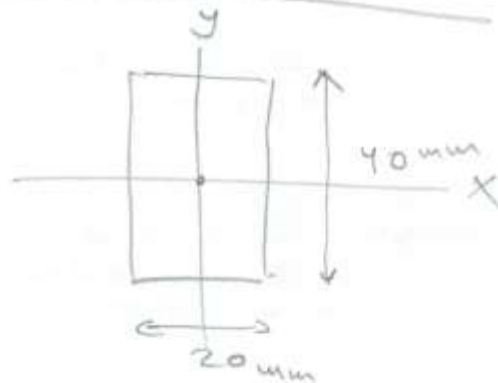
⑧ MCQ/ The Area of the Rectangle is more than the area of Triangle



- 1- True
- 2- False
- 3- Not Applicable
- 4- Not of Above Result

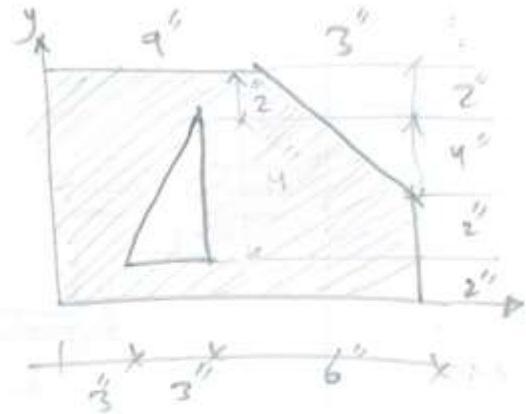
⑨ MCQ The \bar{y} for the

Figure shown is:-



- ① 20 mm
- ② 20 m
- ③ 10 mm
- ④ 10 m
- ⑤ 0
- ⑥ Not of the above Results

⑨ locate the centroid of the shaded Area



Area #	Area Sq. in.	x in.	My Area x X	y in.	Mx Area x Y
□	$12 \times 10 = 120$	6	$120 \times 6 = 720$	5	$120 \times 5 = 600$
▽	$\frac{1}{2} \times 3 \times 6 = 9$	11	-99	8	-72
△	$\frac{1}{2} \times 3 \times 6 = 9$	5	-45	4	-36
	102		576		492

$$\bar{X} = \frac{\sum My}{\sum A} = \frac{576}{102} = 5.65 \text{ in}$$

$$\bar{Y} = \frac{\sum Mx}{\sum A} = \frac{492}{102} = 4.82 \text{ in}$$

3-30 Locate the centroid of the shaded area

Fig.

Solve on



$$A_{\text{total}} = (12 \times 10) - \frac{3 \times 6}{2} - \frac{3 \times 6}{2}$$

$$A_{\text{total}} = 102 \text{ cm}^2$$

$$M_y = (12 \times 10)(6) - \frac{3 \times 6}{2}(5) - \frac{3 \times 6}{2}(11)$$

$$M_y = 576 \text{ cm}^3$$

$$\bar{x} = \frac{M_y}{A_{\text{total}}} = \frac{576}{102} = 5.65 \text{ in}$$

$$M_x = (10 \times 12)(5) - \frac{3 \times 6}{2}(4) - \frac{3 \times 6}{2}(8)$$

$$M_x = 492 \text{ in}^3$$

$$\bar{y} = \frac{M_x}{A_{\text{total}}} = 4.82 \text{ in} \quad (5.65, 4.82)$$

3:30 Determine the y coordinate of the centroid of the shaded area shown in fig. In the eq. $y^2 = x - 1$, x and y are in feet.

$$dA_1 = (2-x) dy$$

$$A_1 = \int_0^1 (2 - (y^2 + 1)) dy \Rightarrow A_1 = \int_0^1 (1 - y^2) dy$$

$$A_1 = \left[y - \frac{y^3}{3} \right]_0^1 \Rightarrow A_1 = \left[1 - \frac{1}{3} \right] = \frac{2}{3} \text{ ft}^2$$

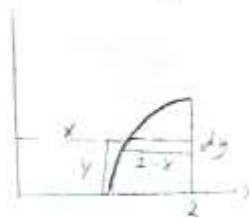
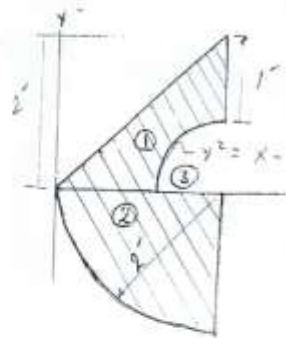
$$dM_{x_1} = y dA$$

$$M_{x_1} = \int_0^1 y (1 - y^2) dy \Rightarrow M_{x_1} = \int_0^1 (y - y^3) dy$$

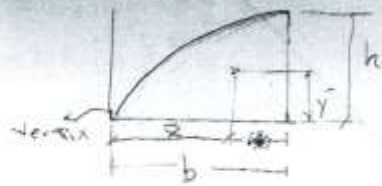
$$M_{x_1} = \left[\frac{y^2}{2} - \frac{y^4}{4} \right]_0^1 \Rightarrow M_{x_1} = \left(\frac{1}{2} - \frac{1}{4} \right) = \frac{1}{4} \text{ ft}^3$$

$$\bar{y}_1 = \frac{M_{x_1}}{A_1} = \frac{0.25}{0.667} = 0.375 \text{ ft}$$

$$A_{\text{total}} = \frac{2 \times 2}{2} + \frac{(2)^2 \pi}{4} - \frac{2}{3} = 4.475 \text{ ft}^2$$

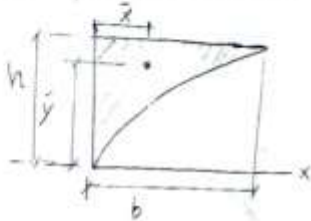


4- Quadrant of parabola Area \bar{x}



$$\frac{2bh}{3} \quad \frac{3b}{5} \quad \frac{3}{8}h$$

5- Parabolic spandrel

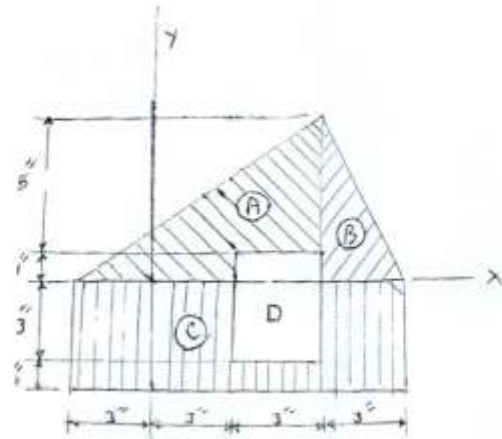


$$\frac{bh}{3} \quad \frac{3b}{10} \quad \frac{3h}{4}$$

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Ex Determine the Coordinates of the Centroid of the shaded area shown

	Area m ²	x cm	M _y cm ²	y cm	M _x cm ²
A	27	3	81	2	54
B	9	7	63	2	18
C	48	3	144	-2	-96
D	12	4.5	-54	-1	12
Σ	72		234		-12



$$\bar{x} = \frac{\Sigma M_y}{\Sigma A} = \frac{234}{72} = 3.25 \text{ cm}$$

$$\bar{y} = \frac{\Sigma M_x}{\Sigma A} = \frac{-12}{72} = -0.1667 \text{ cm}$$

3.27 locate the centroid of the shaded area shown.

$$A = \frac{6 \times 9}{2} + 9 \times 5 - (1.5)^2 \pi$$

$$A_{\text{total}} = 65 \text{ (in}^2\text{)}$$

$$M_y = \frac{6 \times 9}{2} (2) + 5 \times 9 (-2.5) +$$

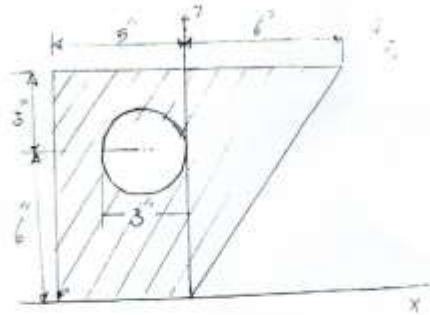
$$[-(1.5)^2 \pi (-1.5)] = -47.8 \text{ (in}^3\text{)}$$

$$\bar{x} = \frac{M_y}{A_{\text{total}}} = \frac{-47.8}{65} = -0.73 \text{ in}$$

$$M_x = \frac{6 \times 9}{2} (6) + 5 \times 9 (4.5) + [-(1.5)^2 \pi (6)] = 322.08 \text{ (in}^3\text{)}$$

$$\bar{y} = \frac{M_x}{A_{\text{total}}} = \frac{322.08}{65} = 4.95 \text{ (in)}$$

The centroid of the fig is (0.73, 4.95)

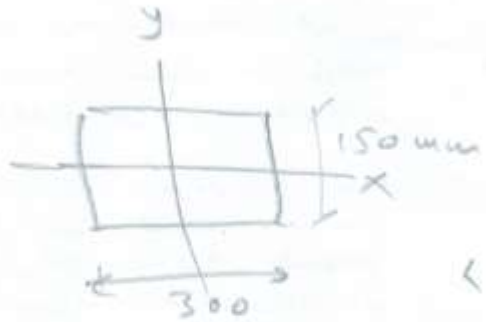


مكتبة الضيف
داخل كلية الهندسة

10) MCQ The moment Area of The Fig.

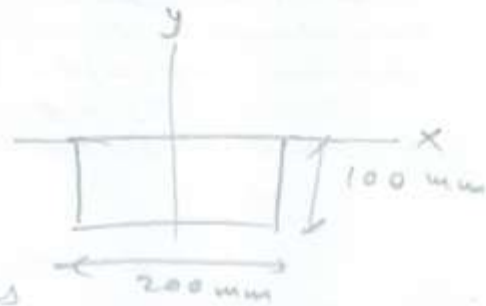
about X-AXIS is

- ① 45000 mm²
- ② 75 mm
- ③ 150 mm
- ④ No of The above answers



MCQ The \bar{y} of The fig shown is

- 1- 50 mm
- 2- 25 mm
- 3- 100 mm
- 4- No of The above Answers



MCQ The \bar{x} of The above fig is

- 1- 100 mm
- 2- 50 mm
- 3- 200 mm
- 4- - 50 mm
- 5- - 100 mm
- 6- Not of The above Answers