moment of inertia by integration

(1) Moment of Inertia by Integration

تكامل

$$\int 4\,dx = 4\int dx = 4x$$

$$\int 4x\,dx = 4\int x\,dx = 4\,\frac{x^2}{2} = 2x^2$$

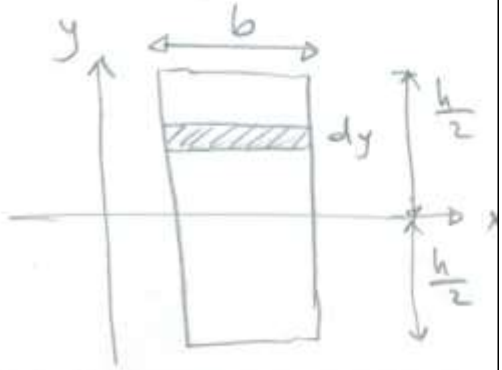
شكل طريقة المساحة لا تتغير باختلاف الأشكال

Example 1: find The shaded Area.

$dA = b * dy$

$\int dA = \int b * dy$

$A = b\,[\,y\,]_{-\frac{h}{2}}^{\frac{h}{2}}$



$A = b\left[\frac{h}{2} - \left(-\frac{h}{2}\right)\right]$

$A = b\left[\frac{h}{2} + \frac{h}{2}\right]$

$A = b\left[\frac{2h}{2}\right]$

$A = b * h$

$y = f(x)$ عام

$y = 3x^2$ خاص

لما $x = 1$ فإن y : ?

لما $x = 2$ فإن y : ?

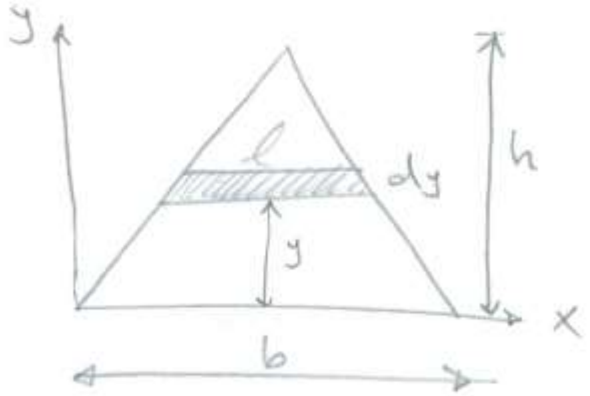
هو عكس التفاضل

$\int$ = opposite of differentiation

$\int$ = a Quick way of adding

$\int = \sum$ Capital Sigma = + + +

② find the shaded Area



$$\frac{l}{b} = \frac{h-y}{h}$$

$$l = \frac{b(h-y)}{h}$$

$$dA = l * dy$$

$$\int dA = \int_0^h \frac{b}{h}(h-y)\,dy$$

$$A = \frac{b}{h} * \left[hy - \frac{y^2}{2}\right]_0^h$$

$$A = \frac{b}{h} * \left[\left[h^2 - \frac{h^2}{2}\right] - (0)\right]$$

$$A = \frac{b}{h}\left[\frac{2h^2}{2} - \frac{1h^2}{2}\right]$$

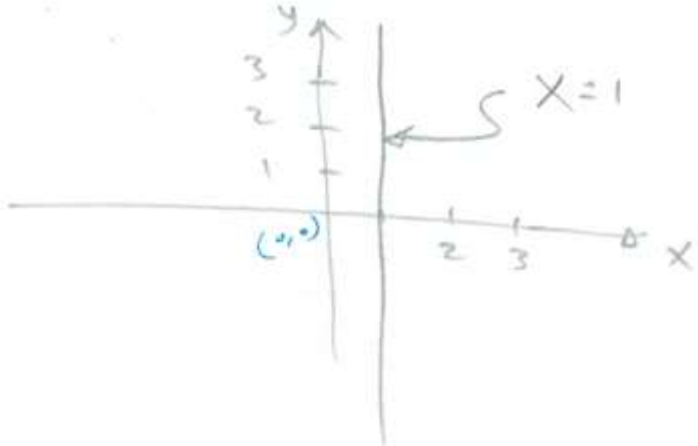
$$A = \frac{b}{h}\left[\frac{h^2}{2}\right]$$

$$A = \frac{bh^{\cancel{2}}}{\cancel{h} * 2}$$

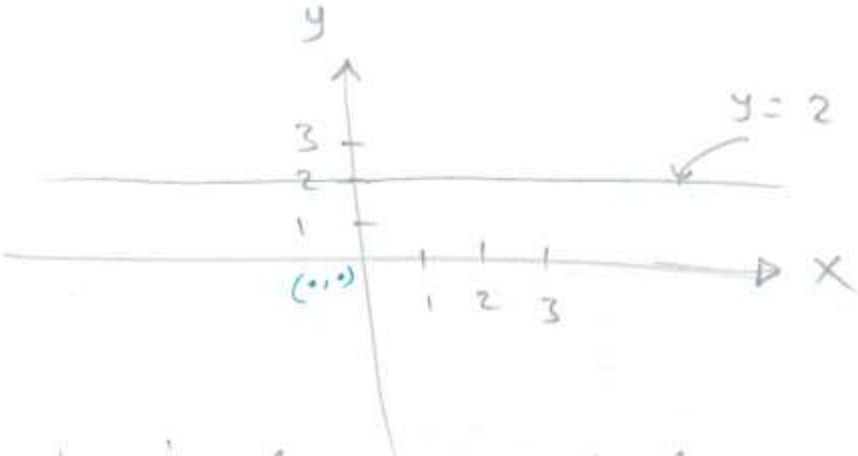
$$A = \frac{1}{2} * b * h$$

مساحة المثلث

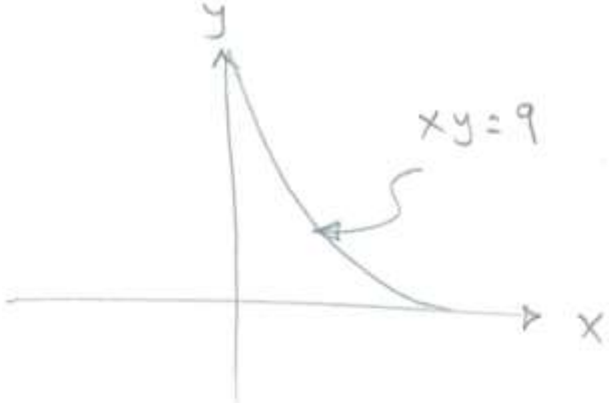
3 Example: Find the line $X = 1$



Example: find the line $y = 2$



Example: what is the equation of the curve



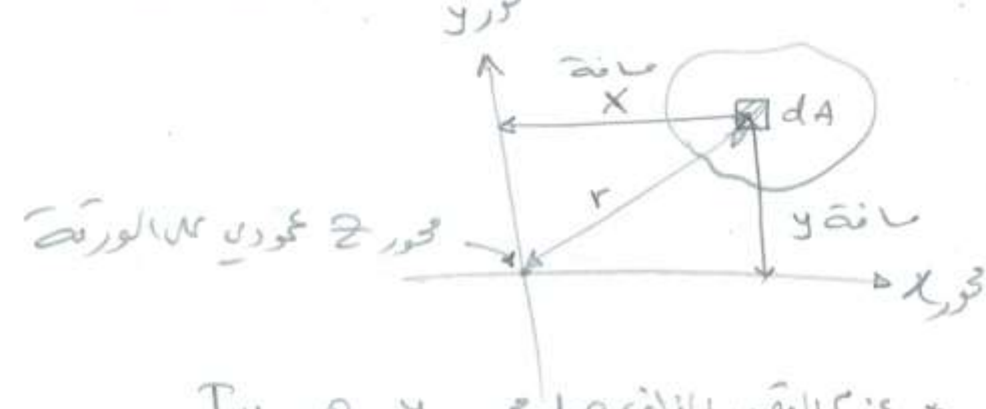
اما ان نقول

$$X = \frac{9}{y}$$

او ان نقول

$$y = \frac{9}{X}$$

Definitions تعاريف



* عزم القصور الذاتي حول محور y هو I_y

$$I_y = \int x^2 \cdot dA$$

* عزم القصور الذاتي حول محور x هو I_x

$$I_x = \int y^2 \cdot dA$$

* عزم القصور الذاتي حول محور z هو I_z ويسمى J ويسمى

Polar moment of Inertia

$$I_z = J = \int r^2 \cdot dA$$

$$J = \int (x^2 + y^2) \cdot dA$$

$$J = \int x^2 \cdot dA + \int y^2 \cdot dA$$

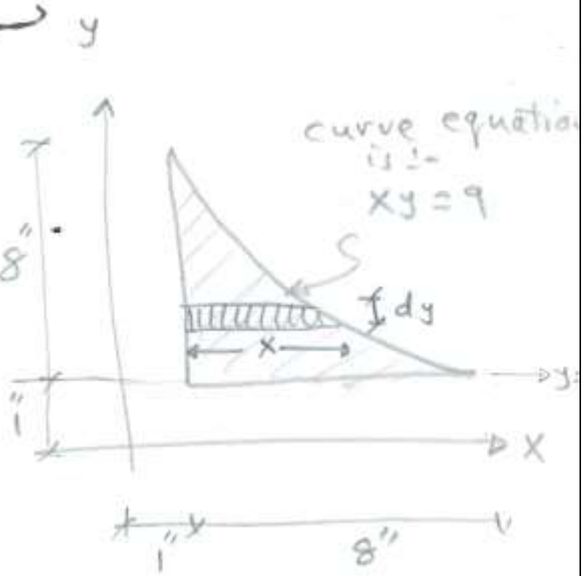
$$\therefore J = I_y + I_x$$

(5) سؤال

6-10 Page 293

Determin the moment of Inertia of the shaded Area with respect to the line $y = 1$ inch,

$$\boxed{I_x = \int y^2 \cdot dA}$$



$x = \frac{9}{y}$ معادلة

$$I @ y=1 = \int_1^9 (y-1)^2 * dA$$

$$= \int_1^9 (y-1)^2 * [(x-1) * dy]$$

$$\left(\frac{9}{y}-1\right)(y^2-2y+1) = \frac{9y^2}{y} - \frac{2y*9}{y} + \frac{9}{y} - (y^2-2y+1)$$

$$= 9y - 18 + \frac{9}{y} - y^2 + 2y - 1$$

$$= -y^2 + 11y - 19 + \frac{9}{y}$$

$$= \int_1^9 \left(-y^2 + 11y - 19 + \frac{9}{y}\right) dy$$

$$= \left[\frac{-y^3}{3} + \frac{11y^2}{2} - 19y + 9\ln y\right]_1^9$$

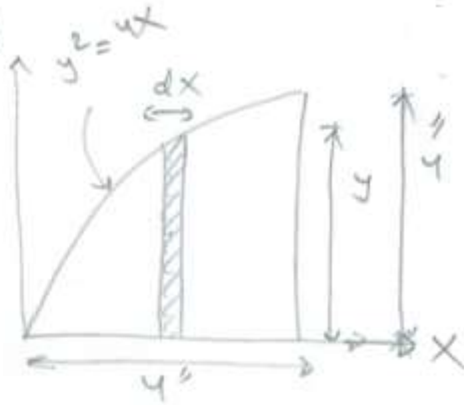
$$= \left[\frac{-9^3}{3} + \frac{11(9^2)}{2} - 19(9) + 9\ln 9\right] - \left[\frac{-1}{3} + \frac{11}{2} - 19 + 9\ln(1)\right]$$

$$= 51.27 - (-13.8)$$

$$= 65.1 \text{ in}^4$$

6

عزم القصور الذاتي حول محور X هو

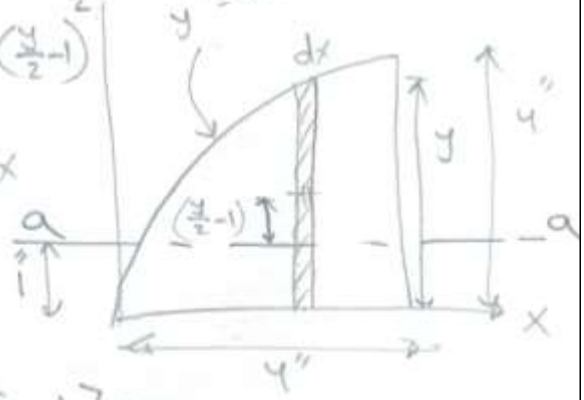


$$I_x = \int_0^4 \frac{dx * y^3}{12}$$

مثال

عزم القصور الذاتي حول محور a-a هو

$$y^2 = 4x \Rightarrow y = 2 * x^{\frac{1}{2}}$$



$$I_{a-a} = \int_0^4 \frac{dx * y^3}{12} + (dx * y) * \left(\frac{y}{2} - 1\right)^2$$

$$\int_0^4 \frac{(2 * x^{\frac{1}{2}})^3}{12} dx + \left[2x^{\frac{1}{2}} * \left(\frac{2x^{\frac{1}{2}}}{2} - 1\right)^2\right] dx$$

$$= \int_0^4 \frac{8}{12} x^{\frac{3}{2}} dx + \left[2x^{\frac{1}{2}} * (x - 2x^{\frac{1}{2}} + 1)\right] dx$$

ملاحظة:

$(x-1)^2$

$= x^2 - 2x + 1$

$$= \int_0^4 \frac{2}{3} x^{1.5} dx + \left(2x^{\frac{3}{2}} - 4x + 2x^{\frac{1}{2}}\right) dx$$

$$= \int_0^4 \frac{2}{3} x^{1.5} + \frac{2x^{3/2}}{3} - 4x + 2x^{0.5}$$

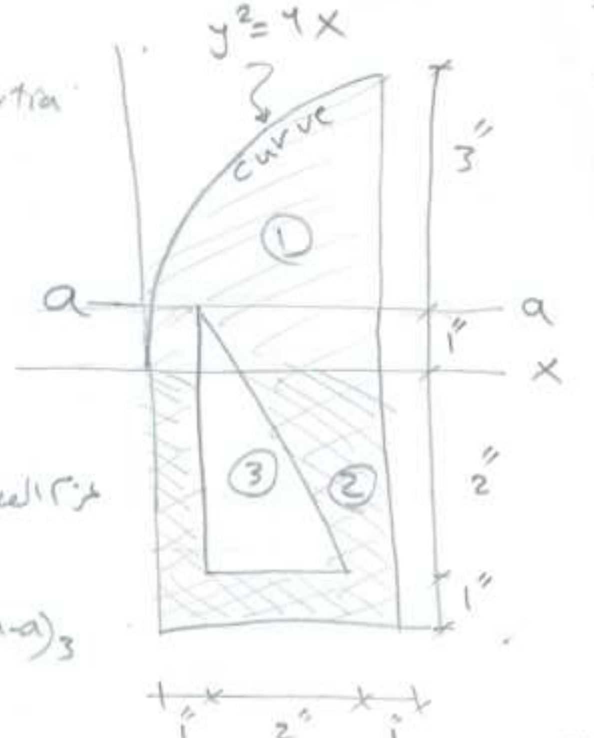
$$= \int_0^4 \left(\frac{8}{3} x^{1.5} - 4x + 2x^{0.5}\right) dx$$

$$= \left[\frac{8}{3} * \frac{x^{2.5}}{2.5} - \frac{4 * x^2}{2} + 2 * \frac{x^{1.5}}{1.5}\right]_0^4$$

$$= 12.8 \ in^4$$

7 6-44 Page 310

Determin moment of Inertia of The Shaded Area with respect to The a-axis



1 المساحة تحت القوس هي

2 المساحة تحت المستطيل هي

3 المساحة تحت المثلث هي

I_{a-a} هو a-a عزم القصور الذاتي حول

$$I_{a-a\,Total} = I_{a-a)_1} + I_{a-a)_2} - I_{a-a)_3}$$

last Example Result

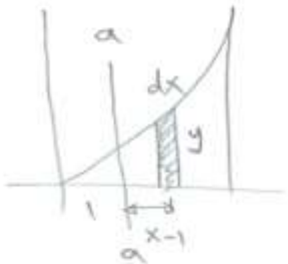
$$= 12.8 + \left[\frac{4 \times 3^3}{12} + (4\times3)(2.5)^2\right] - \left\{\frac{2\times3^3}{36} + \left(\frac{1}{2}\times2\times3\right)\times(2^2)\right\}$$

$$= 12.8 + 9 + 75 - (1.5 + 12)$$

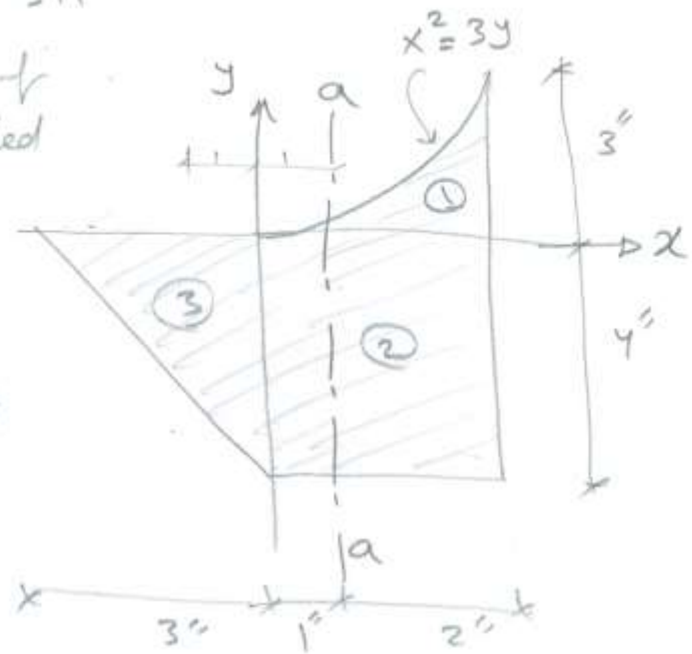
$$= 83.3 \text{ in}^4$$

(8) 6-48 Page 311

Determin the moment of Inertia of the shaded Area with respect to the a-axis



$y = \frac{x^2}{3}$



$$I_{a-a)_{Total}} = I_{a-a)_1} + I_{a-a)_2} + I_{a-a)_3}$$

$$= I_{a-a)_1} + \left[\frac{4 \times 3^3}{12} + (4\times3)(0.5)^2\right] + \left[\frac{4\times3^3}{36} + \left(\tfrac{1}{2}\times4\times3\right)(1+1)^2\right]$$

$$I_{a-a)_1} = \int_0^3 y \times dx \times (x-1)^2 = \int_0^3 \frac{x^2}{3}(x^2-2x+1)\,dx$$

$$= \frac{1}{3}\int_0^3 (x^4 - 2x^3 + x^2)\,dx = \frac{1}{3}\left[\frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3}\right]_0^3$$

$$= \frac{1}{3}\left[\left[\frac{3^5}{5} - \frac{2(3)^4}{4} + \frac{(3)^3}{3}\right] - (0)\right]$$

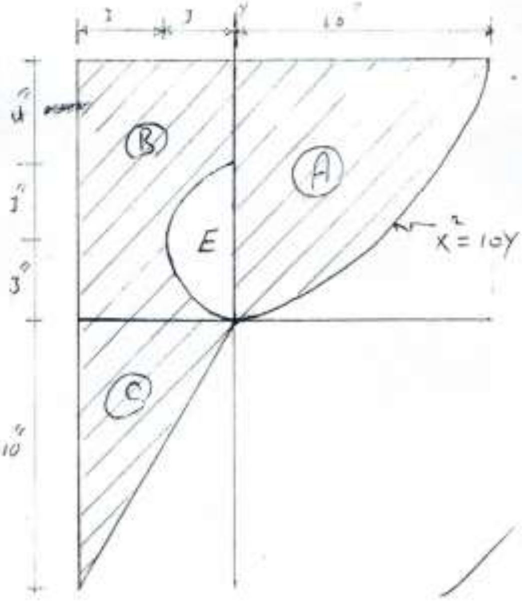
$$I_{a-a)_1} = 5.7 \text{ in}^4$$

$$\therefore I_{a-a)_{Total}} = 5.7 + 12 + 27$$
$$= 44.7 \text{ in}^4$$

6.52 $I_y = ?$

Solution

I_y for Area A



$$dI_y = \frac{dy\,(x)^3}{12} + x\,dy\left(\frac{x}{2}\right)^2$$

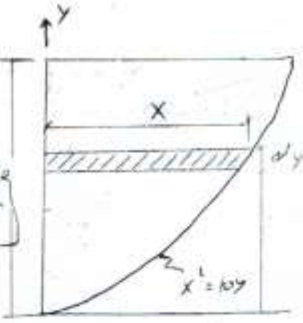
$$dI_y = \frac{x^3\,dy}{3} = \frac{(10)^{1.5}(y)^{1.5}}{3}$$

$$= 10.541\,y^{1.5}$$

$$I_y = \int_0^{10} 10.541\,y^{1.5}\,dy$$

$$= 10.541\left[\frac{y^{2.5}}{2.5}\right]_0^{10}$$

$$= 1333.34 \text{ in}^4$$



$$I_y = \overset{A}{1333.34} + \overset{B}{\left[\frac{10\times 6^3}{12} + (6\times 10)(3)^2\right]} + \overset{C}{\left[\frac{10\times(6)^3}{36} + \frac{6\times 10}{2}(4)^2\right]} - \overset{D}{\left[\frac{\pi(3)^4}{8}\right]} + \cancel{Ad^2}$$

$$= 1333.34 + 720 + 540 - 31.81$$

$$= 2561.5 \text{ in}^4$$

$y = \frac{x^2}{10}$

$x = (10^{1/2})\,y^{1/2}$