Trip Distribution Model

Introduction

The decision to travel for a given purpose is called trip generation. These generated trips from each zone is then distributed to all other zones based on the choice of destination. This is called trip distribution which forms the second stage of travel demand modeling. There are a number of methods to distribute trips among destinations; and two such methods are growth factor model and gravity model. Growth factor model is a method which respond only to relative growth rates at origins and destinations and this is suitable for short-term trend extrapolation. In gravity model, we start from assumptions about trip making behavior and the way it is influenced by external factors. An important aspect of the use of gravity models is their calibration that is the task of fixing their parameters so that the base year travel pattern is well represented by the model.

Trip distribution is a process by which the trips generated in one zone are allocated to other zones in the study area. These trips may be within the study area (internal-internal) or between the study area and areas outside the study area (internal-external).
For example, if the trip generation analysis results in an estimate of 200 HBW trips in zone 10, then the trip distribution analysis would determine how many of these trips would be made between zone 10 and all the other internal zones.
In addition, the trip distribution process considers internal-external trips (or vice versa) where one end of the trip is within the study area and the other end is outside the study area.

Several basic methods are used for trip distribution, among these are:

- The gravity Model,
- Growth factor models, and intervening opportunities.

The gravity model is preferred because it uses the attributes of the transportation system and land-use characteristics and has been calibrated extensively for many urban areas. The gravity model has achieved virtually universal use because of its simplicity, its accuracy. Growth factor models, which were used more widely in the

1950s and 1960s, require that the origin-destination matrix be known for the base (or current) year, as well as an estimate of the number of future trip ends in each zone. The intervening opportunities model and other models are available but not widely used in practice.

Gravity Model

The most widely used and documented trip distribution model is the gravity model, which states that the number of trips between two zones is directly proportional to the number of trip attractions generated by the zone of destination and inversely proportional to a function of time of travel between the two zones. Mathematically, the gravity model is expressed as:

$$T_{ij} = P_i \left[\frac{A_j F_{ij} K_{ij}}{\sum_j A_j F_{ij} K_{ij}} \right]$$

where

T_{ij} = number of trips that are produced in zone i and attracted to zone j
P_i = total number of trips produced in zone i
A_j = number of trips attracted to zone j
F_{ij} = a value which is an inverse function of travel time
K_{ij} = socioeconomic adjustment factor for interchange ij

The values of P_i and A_j have been determined in the trip generation process. The sum of P_i for all zones must equal the sum of A_j for all zones. K_{ij} values are used when the estimated trip interchange must be adjusted to ensure that it agrees with the observed trip interchange. The values for F_{ij} are determined by a calibrating process in which trip generation values as measured in the O-D survey are distributed using the gravity model. After each distribution process is completed, the percentage of trips in each trip length category produced by the gravity model is compared with the percentage of trips recorded in the O-D survey. If the percentages do not agree, then the F_{ij} factors that were used in the distribution process are adjusted and another gravity model trip distribution is performed. The calibration process is continued until the trip length percentages are in agreement.
Figure 1 illustrates F values for calibrations of a gravity model. (Normally this curve is a semilog plot.) F values can also be determined using travel time values and an inverse relationship between F and t. For example, the relationship for F might be in

the form t^{-1}, t^{-2}, e^{-t}, and so forth, since F values decrease as travel time increases. The friction factor can be expressed as:

$F = ab^t e^{-ct}$,

where parameters a, b, and c are based on national data sources, such as NCHRP Report 365, or the formula may be calibrated using local data.

The socioeconomic factor is used to make adjustments of trip distribution K_{ij} values between zones where differences between estimated and actual values are significant. The K value is referred to as the "socioeconomic factor" since it accounts for variables other than travel time. The values for K are determined in the calibration process, but it is used judiciously when a zone is considered to possess unique characteristics.

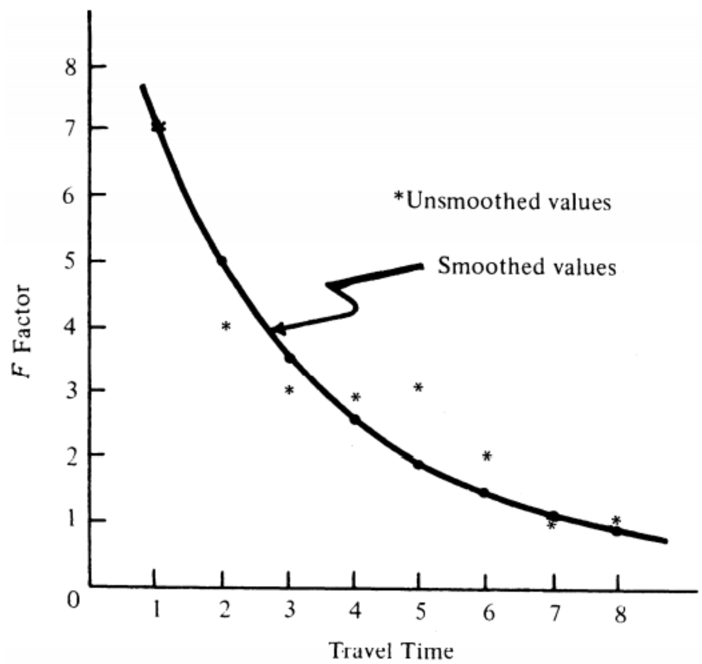

Figure 1 Calibration of F Factors.

<u>Example</u> Use of Calibrated F Values and Iteration

To illustrate the application of the gravity model, consider a study area consisting of three zones. The data have been determined as follows: the number of productions and attractions has been computed for each zone by methods described in the section on trip generation, and the average travel times between each zone have been determined. Both are shown in Tables 1 and 2. Assume K_{ij} is the same unit value for all zones. Finally, the F values have been calibrated as previously described and are shown in Table 12.11 for each travel time increment. Note that the intra-zonal travel time for zone 1 is larger than those of most other inter-zone times because of the

geographical characteristics of the zone and lack of access within the area. This zone could represent conditions in a congested downtown area.
Determine the number of zone-to-zone trips through two iterations.

Solution:
The number of trips between each zone is computed using the gravity model and the given data. (Note: F_{ij} is obtained by using the travel times in Table 2 and selecting the correct F value from Table 3. For example, travel time is 2 min between zones 1 and 2. The corresponding F value is 52.).

Table 1 Trip Productions and Attractions for a Three-Zone Study Area.

Zone	*1*	*2*	*3*	*Total*
Trip productions	140	330	280	750
Trip attractions	300	270	180	750

Table 2 Travel Time between Zones (min)

Zone	*1*	*2*	*3*
1	5	2	3
2	2	6	6
3	3	6	5

Table 3 Travel Time versus Friction Factor

Time (min)	*F*
1	82
2	52
3	50
4	41
5	39
6	26
7	20
8	13

Note: *F* values were obtained from the calibration process.

$$T_{ij} = P_i \left[\frac{A_j F_{ij} K_{ij}}{\sum_{j=1}^{n} A_j F_{ij} K_{ij}} \right] \qquad K_{ij} = 1 \text{ for all zones}$$

$$T_{1-1} = 140 \times \frac{300 \times 39}{(300 \times 39) + (270 \times 52) + (180 \times 50)} = 47$$

$$T_{1-2} = 140 \times \frac{270 \times 52}{(300 \times 39) + (270 \times 52) + (180 \times 50)} = 57$$

$$T_{1-3} = 140 \times \frac{180 \times 50}{(300 \times 39) + (270 \times 52) + (180 \times 50)} = 36$$

$$P_1 = 140$$

Make similar calculations for zones 2 and 3

$T_{2-1} = 188$	$T_{2-2} = 85$	$T_{2-3} = 57$	$P_2 = 330$
$T_{3-1} = 144$	$T_{3-2} = 68$	$T_{3-3} = 68$	$P_3 = 280$

The results summarized in Table 4 represent a singly constrained gravity model. This constraint is that the sum of the productions in each zone is equal to the number of productions given in the problem statement. However, the number of attractions estimated in the trip distribution phase differs from the number of attractions given. For zone 1, the correct number is 300, whereas the computed value is 379. Values for zone 2 are 270 versus 210, and for zone 3, they are 180 versus 161. To create a doubly constrained gravity model where the computed attractions equal the given attractions, calculate the adjusted attraction factors according to the formula

$$A_{jk} = \frac{A_j}{C_{j(k-1)}} A_{j(k-1)}$$

where

A_{jk} = adjusted attraction factor for attraction zone (column) j, iteration k
$A_{jk} = A_j$ when $k = 1$
C_{jk} = actual attraction (column) total for zone j, iteration k
A_j = desired attraction total for attraction zone (column) j
j = attraction zone number, $j = 1, 2, \ldots, n$
n = number of zones
k = iteration number, $k = 1, 2, \ldots, m$
m = number of iterations

To produce a doubly constrained gravity model, repeat the trip distribution computations using modified attraction values so that the numbers attracted will be increased or reduced as required. For zone 1, for example, the estimated attractions were too great. Therefore, the new attraction factors are adjusted downward by multiplying the original attraction value by the ratio of the original to estimated attraction values.

$$\text{Zone 1:} A_{12} = 300 \times \frac{300}{379} = 237$$

$$\text{Zone 2:} A_{22} = 270 \times \frac{270}{210} = 347$$

$$\text{Zone 3:} A_{32} = 180 \times \frac{180}{161} = 201$$

Apply the gravity model for all iterations to calculate zonal trip interchanges using the adjusted attraction factors obtained from the preceding iteration. In practice, the gravity model becomes

$$T_{ij} = P_i \left[\frac{A_j F_{ij} K_{ij}}{\sum_j A_j F_{ij} K_{ij}} \right]$$

Where T_{ijk} is the trip interchange between i and j for iteration k, and $A_{jk} = A_j$ when k = 1. Subscript j goes through one complete cycle every time k changes, and i goes through one complete cycle every time j changes. This formula is enclosed in parentheses and subscripted to indicate that the complete process is performed for each trip purpose.

Perform a second iteration using the adjusted attraction values.

$$T_{1-1} = 140 \times \frac{237 \times 39}{(237 \times 39) + (347 \times 52) + (201 \times 50)} = 34$$

$$T_{1-2} = 140 \times \frac{347 \times 52}{(237 \times 39) + (347 \times 52) + (201 \times 50)} = 68$$

$$T_{1-3} = 140 \times \frac{201 \times 50}{(237 \times 39) + (347 \times 52) + (201 \times 50)} = 37$$

$$P_1 = 140$$

Make similar calculations for zones 2 and 3.

$T_{2-1} = 153 \quad T_{2-2} = 112 \quad T_{2-3} = 65 \quad P_2 = 330$

$T_{3-1} = 116 \quad T_{3-2} = 88 \quad T_{3-3} = 76 \quad P_3 = 280$

The results are summarized in Table 4. Note that, in each case, the sum of the attractions is now much closer to the given value. The process will be continued until there is a reasonable agreement (within 5%) between the A that is estimated using the gravity model and the values that are furnished in the trip generation phase.

Table 4 Zone-to-Zone Trips: Second Iteration, Doubly Constrained.

Zone	*1*	*2*	*3*	*Computed P*	*Given P*
1	34	68	38	140	140
2	153	112	65	330	330
3	116	88	76	280	280
Computed *A*	303	268	179	750	750
Given *A*	300	270	180	750	

When should a singly constrained gravity model or the doubly constrained gravity model be used? The singly constrained gravity model may be preferred if the friction factors are more reliable than the attraction values. The doubly constrained gravity model is appropriate if the attraction values are more reliable than friction factors. To illustrate either choice, consider the following example:

Example: Selecting Singly or Doubly Constrained Gravity Model Results

A three-zone system with 900 home-based shopping productions is shown in Table 5. Zones 1 and 2 each generate 400 productions, while zone 3 generates 100 productions. Each zone contains a shopping mall with 300 attractions. The shopping mall in zone 1 can be easily reached due to the parking availability and transit service. Thus, F11, F21, and F31 = 1.0. Parking costs at the shopping mall in zone 2 are moderate with some transit service. Thus, F12, F22, and F32 = 0.5. Parking costs at the mall in zone 3 is high and transit service is unavailable. Thus, F13, F23, and F33 = 0.2.

Application of the singly constrained gravity model yields the results shown in Table 6 and application of the doubly constrained gravity model yields the results shown in Table 7.

Table 5 Home-Based Shopping Productions and Attractions.

Zone	*Productions*	*Attractions*
1	400	300
2	400	300
3	100	300
Total	900	900

Table 6 Zone-to-Zone Trips: Singly Constrained Gravity Model.

Zone	*1*	*2*	*3*	*Computed P*	*Given P*
1	235	118	47	400	400
2	235	118	47	400	400
3	59	29	12	100	100
Computed *A*	529	265	106	900	900
Given *A*	300	300	300	900	

Table 7 Zone-to-Zone Trips: Doubly Constrained Gravity Model.

Zone	*1*	*2*	*3*	*Computed P*	*Given P*
1	133	133	133	400	400
2	133	133	133	400	400
3	33	33	33	100	100
Computed *A*	300	300	300	900	900
Given *A*	300	300	300	900	

Which of the results shown for the singly constrained gravity model and for the doubly constrained gravity model are more likely to be the most accurate?

Solution: Table 6 is more likely to be accurate if engineering judgment suggests the occurrence of travel impedances and thus the friction factors are more accurate than trip attractions. Table 7 is more likely to be accurate if the attractions are more accurate than the friction factors. In practice, these judgments must be made based on the quality of the data set. For example, if local land-use data had been recently used to develop trip attraction rates whereas friction factors had been borrowed from another area, then the selection of the doubly constrained gravity model results in Table 7 is recommended.

Growth Factor Models

Trip distribution can also be computed when the only data available are the origins and destinations between each zone for the current or base year and the trip generation values for each zone for the future year. This method was widely used when O-D data were available but the gravity model and calibrations for F factors had not yet become operational. Growth factor models are used primarily to distribute trips between zones in the study area and zones in cities external to the study area. Since they rely upon an existing O-D matrix, they cannot be used to forecast traffic between zones where no traffic currently exists. Further, the only measure of travel friction is the amount of current travel. Thus, the growth factor method cannot reflect changes in travel time between zones, as does the gravity model.

The most popular growth factor model is the Fratar method, which is a mathematical formula that proportions future trip generation estimates to each zone as a function of the product of the current trips between the two zones T_{ij} and the growth factor of the attracting zone G_j. Thus,

$$T_{ij} = (t_i G_i) \frac{t_{ij} G_j}{\sum_x t_{ix} G_x}$$

where

T_{ij} = number of trips estimated from zone i to zone j
t_i = present trip generation in zone i
G_x = growth factor of zone x
$T_i = t_i G_i$ = future trip generation in zone i
t_{ix} = number of trips between zone i and other zones x
t_{ij} = present trips between zone i and zone j
G_j = growth factor of zone j

The following example illustrates the application of the growth factor method.

Example: Forecasting Trips Using the Fratar Model

A study area consists of four zones (A, B, C, and D). An O-D survey indicates that the number of trips between each zone is as shown in Table 8. Planning estimates for the area indicate that in five years the number of trips in each zone will increase by the growth factor shown in Table 9 and that trip generation will be increased to the amounts shown in the last column of the table.

Table 8 Present Trips between Zones.

Zone	*A*	*B*	*C*	*D*
A	—	400	100	100
B	400	—	300	—
C	100	300	—	300
D	100	—	300	—
Total	600	700	700	400

Table 9 Present Trip Generation and Growth Factors.

Zone	*Present Trip Generation (trips/day)*	*Growth Factor*	*Trip Generation in Five Years*
A	600	1.2	720
B	700	1.1	770
C	700	1.4	980
D	400	1.3	520

Table 10 First Estimate of Trips between Zones.

Zone	*A*	*B*	*C*	*D*	*Estimated Total Trip Generation*	*Actual Trip Generation*
A	—	428	141	124	693	720
B	428	—	372	—	800	770
C	141	372	—	430	943	980
D	124	—	430	—	554	520
Totals	693	800	943	554		

Generation in zone A is estimated as 693 trips, whereas the actual value is 720 trips. Similarly, the estimate for zone B is 800 trips, whereas the actual value is 770 trips. Proceed with a second iteration in which the input data are the numbers of trips between zones as previously calculated. Also, new growth factors are computed as the ratio of the trip generation expected to occur in five years and the trip generation estimated in the preceding calculation. The values are given in Table 11.

Table 11 Growth Factors for Second Iteration.

Zone	*Estimated Trip Generation*	*Actual Trip Generation*	*Growth Factor*
A	693	720	1.04
B	800	770	0.96
C	943	980	1.04
D	554	520	0.94

The calculations for the second iteration are left to the reader to complete and the process can be repeated as many times as needed until the estimate and actual trip generation values are close in agreement.

A more general form of growth factor model than the Fratar method is the average growth factor model. Rather than weighting the growth of trips between zones i and j by the growth across all zones, as is done in the Fratar method, the growth rate of trips between any zones i and j is simply the average of the growth rates of these zones.

$$T'_{ij} = T_{ij}\left(\frac{G_i + G_j}{2}\right)$$

Application of the average growth factor method proceeds similarly to that of the Fratar method. As iterations continue, the growth factors converge toward unity. Iterations can cease when an acceptable degree of convergence in the values is reached; one such practice is to continue until all growth factors are within 5 percent of unity (i.e., between 0.95 and 1.05).