

## Worked Examples in Boiling and Condensation

**Ex. 1** Water is to be boiled in a cylindrical stainless steel container of 20 cm diameter. Heat is furnished from the bottom side at a rate of 5000 W. If the water volume is 10 L and its initial temperature is 20 °C then find the mass of the water left in the container after one hour of exposure to heat and find the excess temperature. The boiling occurs at atmospheric pressure. Use Rohsenow formula with  $C_{sf} = 0.0133$  and  $n = 1$ . Neglect any thermal losses to the surroundings.

**Sol.** Thermophysical properties of water at 1 atm are:

$$T_{sat} = 373.15 \text{ K}, P_s = 1.0133 \text{ bar}, v_f 10^3 = 1.044 \text{ m}^3/\text{kg}, v_g = 1.679 \text{ m}^3/\text{kg}, h_{fg} = 2257 \text{ kJ/kg},$$

$$c_{pf} = 4.217 \text{ kJ}/(\text{kg.K}), c_{pg} = 2.029 \text{ kJ}/(\text{kg.K}), \mu_f 10^6 = 279 \text{ (N.s)}/\text{m}^2, \mu_g 10^6 = 12.02 \text{ (N.s)}/\text{m}^2,$$

$$k_f 10^3 = 680 \text{ W}/(\text{m.K}), k_g 10^3 = 24.8 \text{ W}/(\text{m.K}), Pr_f = 1.76, Pr_g = 0.984, \sigma_f 10^3 = 58.9 \text{ N/m},$$

$$\beta_f 10^6 = 750.1 \text{ 1/K}$$

**For the single phase heating from 20 °C to 100 °C :**

$$Q_i \text{ (J)} = Q_{\text{heater}} \text{ time} = M_w c_{pl} \Delta T$$

$$5000 \times \text{time1} = 0.01 \times 10^3 / 1.044 \times 4217 \times 80$$

$$\text{time1} = 646 \text{ s} \quad \text{so time2} = 3600 - 646 = 2954 \text{ s}$$

**For the boiling phase:**

$$Q_{\text{heater}} = \text{boiling rate} \times h_{fg} \quad 5000 = \text{boiling rate} \times 2257000 \quad \text{boiling rate} = 0.0022 \text{ kg/s}$$

$$\text{Boiled mass} = \text{boiling rate} \times \text{time2} = 0.0022 \times 2954 = 6.5 \text{ kg}$$

$$\text{So the water left mass} = 10 - 6.5 = 3.5 \text{ kg}$$

$$q'' = Q_{\text{heater}} / A = Q_{\text{heater}} / (\pi D^2 / 4) = 5000 / (3.14159 \times 0.2^2 / 4) = 159150 \text{ W/m}^2$$

$$q'' = \mu_l h_{fg} \left( \frac{g(\rho_l - \rho_v)}{\sigma} \right)^{1/2} \left( \frac{c_{pl}(T_s - T_{sat})}{C_{sf} h_{fg} Pr_l^n} \right)^3$$

$$159150 = 279 \times 10^{-6} \times 2257 \times 10^3 \left( \frac{9.81(10^3 / 1.044 - 1 / 1.679)}{58.9 \times 10^{-3}} \right)^{1/2} \left( \frac{4217(T_s - T_{sat})}{0.0133 \times 2257 \times 10^3 \times 1.76} \right)^3$$

So the Excess Temperature =  $T_s - T_{sat} = 10.75 \text{ °C}$  (Nucleate Boiling)

**Ex. 2** Water at 1 atm is to be condensed on a vertical surface of 100 cm height and 50 cm width. If the surface is kept at 80 °C by cooling it from one side while the condensation takes place on the other side then find the mass of the condensate after 30 minutes. If the cooling water enters at 40 °C and a flow rate of 0.5 kg/s then what will be its temperature after cooling the vertical surface? Find also the thickness of the condensate film at the midway and the bottom of the vertical surface.

**Sol.** Water thermophysical properties are the same as in Ex. 1

The effective latent heat of condensation is evaluated first:

$$h'_{fg} = h_{fg} + 0.68c_{p,l}(T_{sat} - T_s) \quad h'_{fg} = 2257000 + 0.68 \times 4217(100 - 80) = 2314400 \text{ J/kg}$$

Nusselt formula is used to find the condensation heat transfer coefficient:

$$\bar{h}_L = 0.943 \left[ \frac{g\rho_l(\rho_l - \rho_v)k_l^3 h'_{fg}}{\mu_l(T_{sat} - T_s)L} \right]^{1/4}$$

$$\bar{h}_L = 0.943 \left[ \frac{9.81 \times 1000 / 1.044 \left( \frac{1000}{1.044} - \frac{1}{1.679} \right) 0.68^3 \times 2314400}{279 \times 10^{-6} (100 - 80) \times 1} \right]^{0.25}$$

$$\bar{h}_L = 5852.4 \text{ W/(m}^2 \text{ °C)}$$

The heat released by the condensate to the vertical surface is:

$$Q_c = \bar{h}_L A (T_{sat} - T_s) = 5852.4 \times 1 \times 0.5 (100 - 80) \quad Q_c = 58.524 \text{ kW}$$

$$Q_c = \dot{m}_w c_{pw} (T_{wo} - T_{wi}) \quad 58524 = 0.5 \times 4217 (T_{wo} - 40) \quad T_{wo} = 67.75 \text{ °C}$$

The thickness of the condensate film at two points:

$$\delta(x) = \left[ \frac{4k_l \mu_l (T_{sat} - T_s) x}{g\rho_l(\rho_l - \rho_v) h'_{fg}} \right]^{1/4}$$

$$\delta_{x=0.5 \text{ m}} = \left[ \frac{4 \times 0.68 \times 279 \times 10^{-6} (100 - 80) \times 0.5}{9.81 \times 1000 / 1.044 \left( \frac{1000}{1.044} - \frac{1}{1.679} \right) 2314400} \right]^{0.25} = 0.138 \text{ mm}$$

$$\delta_{x=1 \text{ m}} = \left[ \frac{4 \times 0.68 \times 279 \times 10^{-6} (100 - 80) \times 1}{9.81 \times 1000 / 1.044 \left( \frac{1000}{1.044} - \frac{1}{1.679} \right) 2314400} \right]^{0.25} = 0.164 \text{ mm}$$