10.Friction:-

When two surfaces are in contact, these surfaces were either **frictionless** or **rough.**

A. If they were **frictionless**, the force each surface exerted on the other was normal to the surfaces and the two surfaces could move freely with respect to each other.

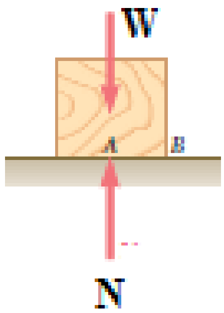
B. If they were **rough**, it was assumed that tangential forces **(Friction force)** could develop to prevent the motion of one surface with respect to the other.

Friction is the force that resists the movement of two contacting surfaces that slide relative to one another. This force always acts tangent to the surface at the point of contact and is directed so as to oppose the possible or existing motion between the surfaces.

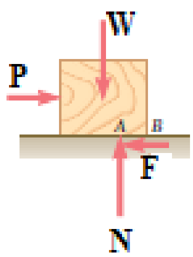
10.1 Law of Friction

The laws of friction are exemplified by the following experiment.

1. A block of weight W is placed on a horizontal plane surface.



2. A horizontal force **P** is applied to the block. If **P** is small, the block will not move; some other horizontal force must therefore exist, which balances **P**.This other force is **the static friction force F**.



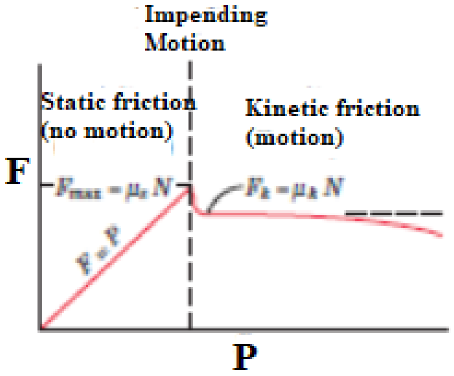
3. If the force **P** is increased, the friction force **F** also increases, continuing to oppose **P**, until its magnitude reaches a certain maximum value $\mathbf{F_m}$.

4.If **P** is further increased, the friction force cannot balance it anymore and the block start sliding.

NOTE:

If N reach the point B before F reaches its maximum value, the block will tip about B before it can start sliding.

5.As soon as the block has been set in motion, the magnitude of **F** drops from $\mathbf{F_m}$ to a lower value $\mathbf{F_k}$(kinetic friction force).



Then from the previous experiment;

The value $\mathbf{F_m}$ of the static friction force is proportional to the normal component **N**

$$\boldsymbol{F_m = \mu_s N}$$

Where

μ_s is the coefficient of static friction

The magnitude $\mathbf{F_k}$ of the kinetic friction force may be put in the form

$$\mathbf{F_k = \mu_k N}$$

Where

μ_k is the coefficient of kinetic friction

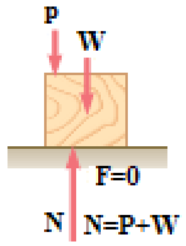
NOTES

1. The maximum frictional force F is proportional to the normal force N.

2. The limiting static friction force is greater than the kinetic frictional force.

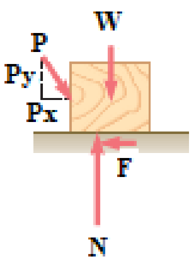
From above, there are four different situations can occur when a rigid body is in contact with a horizontal surface:

1. The forces applied to the body do not tend to move it along the surface of contact; there is no friction force.



a)No friction

2. The applied forces tend to move the body along the surface of contact but are not large enough to set it in motion. The friction force F which has developed can be found by solving equation of equilibrium for the body. Since there is no evidence that **F** has reached its maximum value, the equation $\mathbf{F_m = \mu_s N}$ cannot be used to determine the friction force.



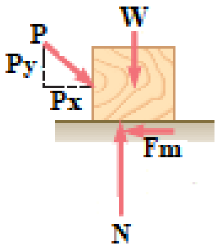
b)No Motion

$$F = P_x$$

$$F < \mu_s N$$

$$N = P_y + W$$

3.The applied forces are such that the body is just about to slide. We say the motion is impending. The friction force **F** has reached its maximum value $\mathbf{F_m}$ and, together with the normal force N, balances the applied force. Both the equations of equilibrium and the equation $\mathbf{F_m = \mu_s N}$ can be used.



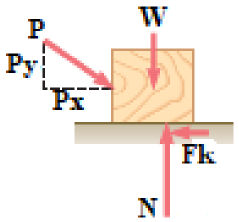
c)Motion impending

$$F_m = P_x$$

$$F_m = \mu_s N$$

$$N = P_y + W$$

4. The body is sliding under the action of the applied forces; and the equations of equilibrium do not apply any more. However, **F** is now equal to $\mathbf{F_k}$.



d) Motion

$$F_k < P_x$$

$$F_k = \mu_k N$$

$$N = P_y + W$$

10.2 Coefficient of Friction.

The coefficient of static friction μ_s, is defined as the ratio of the magnitude of the maximum static frictional force F, to the magnitude of the normal force N, between the two surfaces. It is depend on the nature of the surfaces in contact.

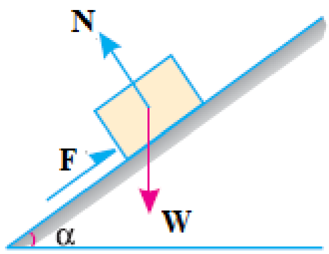
$$\mu_s = \frac{F_m}{\mathbf{N}}$$

Approximate values of coefficient of static friction for various dry surfaces are given in the following table. The corresponding values of the coefficient of kinetic friction would be about 25 percent smaller.

Surface in contact	μ_s
Steel on steel	0.4-0.8
Wood on wood	0.2-0.5
Metal on stone	0.3-0.7
Rubber on concrete	0.6-0.8

10.3 Angle of Friction

Consider a body of weight **W** resting on an inclined plane.



Let the angle of inclination (**α**) be gradually increased, till the body just start sliding down the plane. This angle of inclined plane (**φ**), at which a body just begins to slide down the plane, is called the angle of friction. This is also equal to the angle, which the normal reaction makes with the vertical.

$$\tan\varphi = \frac{F}{N} = \mu_s$$

10.4 Types of Friction Problems

Type 1. No apparent impending motion.

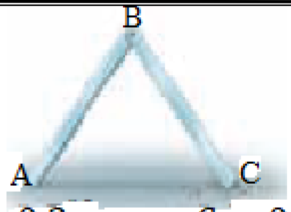
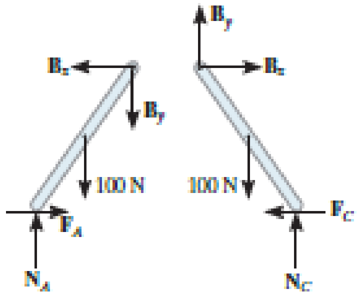
Type 2. Impending motion at all points of contact.

Type 3. Impending motion at some points of contact.

Type 1

Problems in this category are equilibrium problems, which require the number of unknowns to be equal to the number of available equilibrium equations. Once the frictional forces are determined from the solution, their values must be checked to be sure they satisfy $F \leq \mu N$

; Otherwise, slipping will occur and the body will not remain in equilibrium.

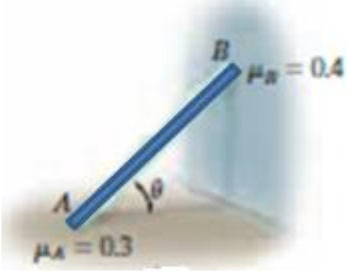
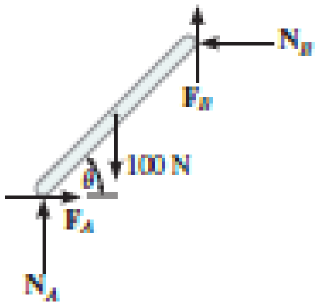
 $\mu A = 0.3$ $\mu C = 0.5$	

Type 2

In this case the total number of unknowns will equal the total number of available equilibrium equations plus the total number of available frictional equations $F = \mu N$.

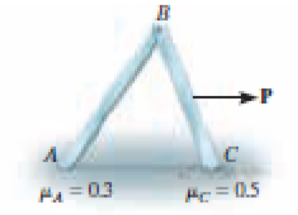
When motion is impending at the points of contact, then $F = \mu N$;

whereas if the body is slipping, then $F_k = \mu_k$

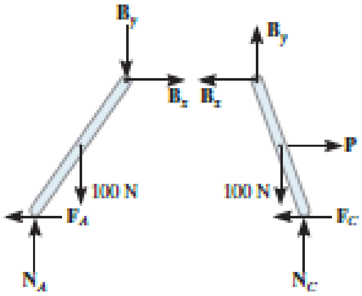
 Finding the small angle at which the 100Nbar can be placed against the wall without slipping.	

Type 3

In this type, the number of unknowns will be less than the number of available equilibrium equations plus the number of available frictional equations or conditional equations for tipping. As a result, several possibilities for motion or impending motion will exist and the problem will involve a determination of the kind of motion which actually occurs.



Determine the horizontal force P needed to cause movement.



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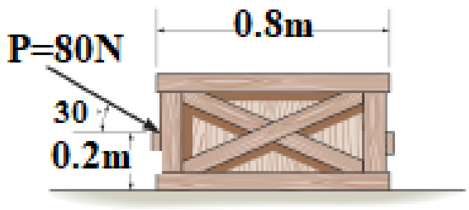
Examples

Example(1):-

The uniform crate shown in figure has a mass of 20kg. If a force P=80N is applied to the crate, determine if it remains in equilibrium. The coefficient of friction is 0.3.



Solution:-

$\rightarrow \sum F_x = 0$

$80 \cos 30 - F = 0 \quad F = 69.3\text{N} \leftarrow$

$\uparrow \sum F_y = 0$

$-80 \sin 30 + Nc - 196.2 = 0 \quad Nc = 236.2\text{N} \uparrow$

$\circlearrowleft \sum M_O = 0$

$80 \sin 30(0.4) - 80 \cos 30(0.2) + Nc(x) = 0$

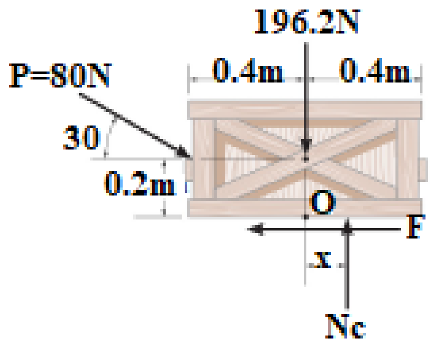
$x = -0.00908\text{m} = -9.08\text{mm}$

No tipping will occur since:-

1.$\mathbf{x < 0.4\ m}$

2.$\mathbf{F = 69.3 < F_{max}} = \mu Nc = 0.3(236.2) = \mathbf{70.9N}$

$\therefore$The crate was still in equilibrium.



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Example(2):-

It is observed that when the bed of the dump truck is raised to an angle of θ=25 the vending machines will begin to slide off the bed, determine the static coefficient of friction between a vending machine and the surface of the truck bed.

Solution:-

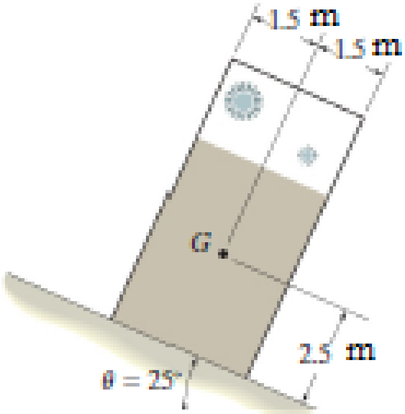
From the F.B.D

$\rightarrow \sum F_x = 0$

$W \sin 25 - F = 0 \qquad (1)$

$\uparrow \sum F_y = 0$

$N - W\cos 25 = 0 \qquad (2)$

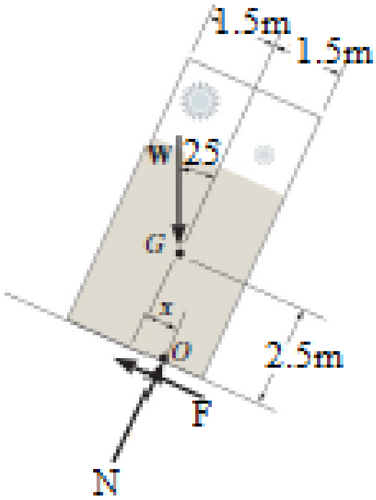


From Eqs (1) and (2)

$F = \mu N \qquad W\sin 25 = \mu(W\cos 25)$

$\mu = \tan 25 = 0.466$

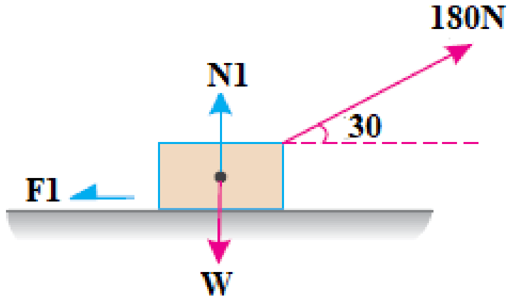
or from angle of friction p153.

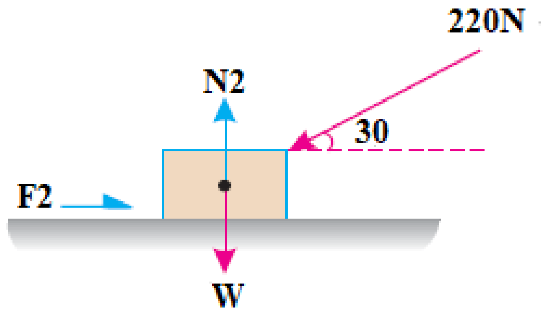


Example(3):-

A body, resting on a rough horizontal plane, required a pull of **180N** inclined at **30º** to the plane just to move it. It was found that a push of **220N** inclined at **30º** to the plane just moved the body. Determine the weight of the body and the coefficient of friction.

Solution:-





From a pull of 180N;

$\uparrow \sum F_y = 0$

N1-W+180 sin 30º=0

N1=W-180 sin 30º=W-90

$\rightarrow \sum F_x = 0$

180 cos 30º-F1=0

F1=180 cos 30º=180x0.866=155.9N

$F1=F_{m1}=\mu_s\, N1=\mu_s(W-90)$

$155.9= \mu_s(W-90)$ (1)

From a push of 220N

$\uparrow\sum F_y = 0$

N2-W-220 sin 30º=0

N2=W+220 sin 30º=W+110

$\rightarrow\sum F_x = 0$

F2-220 cos 30º=0

F2=220 cos 30º=220x0.866=190.5N

$F2=F_{m2}=\mu_s\, N2=\mu_s(W+110)$

$190.5= \mu_s(W+110)$ (2)

Dividing (1) by (2)

$$\frac{155.9}{190.5} = \frac{\mu_s(W-90)}{\mu_s(W+110)} = \frac{W-90}{W+110}$$

155.9W+17149=190.5W-17145

34.6W=34294

W=991.2N

Now substituting in (1)

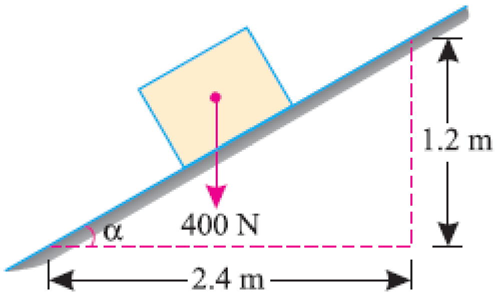
$155.9=\mu_s(991.2-90)=901.2\mu_s$

$\mathbf{\mu_s=0.173}$

Example(4):-

A n inclined plane is used to unload slowly a body weighing **400N** from a truck **1.2m** high into the ground. The coefficient of friction between the underside of the body and the plank is **0.3**. State whether it is necessary to push the body down the plane or hold it back from sliding down. What minimum force is required parallel to the plane for this purpose.

Solution:-



$$\tan\alpha = \frac{1.2}{2.4} = 0.5 \quad \alpha = 26.5°$$

1.N=W cos α=400 cos 26.5°=357.9N

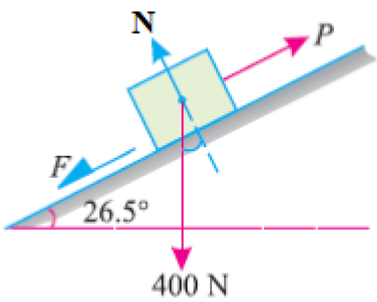
$F_m = \mu_s N = 0.3 \times 357.9 = \mathbf{107.3N}$

Resolving the 400 N along the plane.

=400 sin α=400×sin 26.5°=**178.5N**

The force along the plane (which is responsible for sliding the body) is more than the force of friction; therefore, the body will slide down.

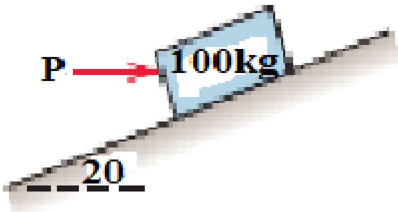
It is not necessary to push the body down the plane; rather it is necessary to hold it back from sliding down.



2.**P**=178.5-107.3=**71.2N**

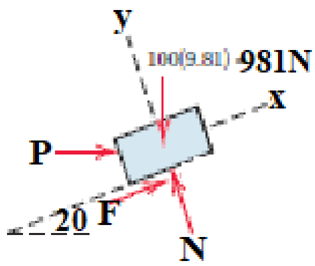
Example(5):-

Determine the magnitude and direction of the friction force acting on the 100kg block shown if, first,P=500N and, second, P=100N.The coefficient of static friction is 0.2,and the coefficient of kinetic friction is 0.17. The force is applied with the block initially at rest.



Solution:-

There is no way of telling from the statement of the problem whether the block will remain in equilibrium or whether it will begin to slip following the application of P, therefore assume the block is in equilibrium;



F.B.D.

$\rightarrow \sum F_x = 0$

$P\cos 20 + F - 981\sin 20 = 0$

$\uparrow \sum F_y = 0$

$N - P\sin 20 - 981\cos 20 = 0$

$1.P = 500\text{ N subs in above eqs}\quad F = -134.3N\;\; N = 1093N$

$F_{max} = \mu N = 0.2(1093) = 219N$

$F < F_{max}$ then the assumption was correct $F = 134.3N$ down the plane

$2.P = 100\text{ N subs in above eqs}\quad F = 242N\;\; N = 956N$

$F_{max} = \mu N = 0.2(956) = 191.2N$

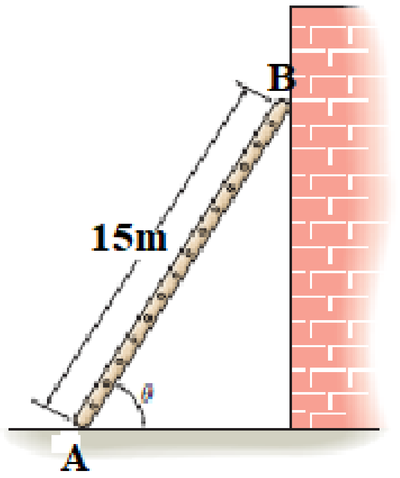
$F > F_{max}$ then the assumption was incorrect

$F_k = \mu_k N = 0.17(956) = 162.5N$ up the plane

Example(6):-

The ladder has a uniform weight of 80N and rests against the wall at B.If the coefficient of friction at A and B is 0.4, determine the smallest angle θ at which the ladder will not slip.

Solution:-



Since the ladder is required to be on the verge to slide down, then:-

$F_A = \mu N_A = 0.4N_A$

$F_B = \mu N_B = 0.4N_B$

From the F.B.D.

$\rightarrow \sum F_x = 0$

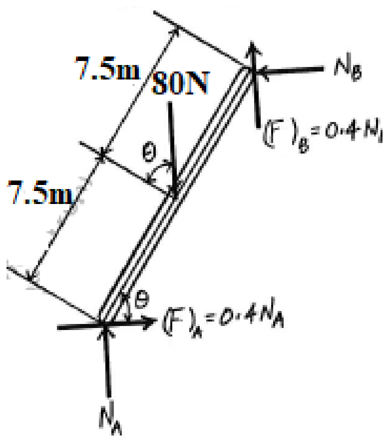
$0.4N_A - N_B = 0 \quad N_B = 0.4N_A \qquad (1)$

$\uparrow \sum F_y = 0$

$N_A + 0.4N_B - 80 = 0 \qquad (2)$

Solving eqs,(1) and (2)

$N_A = 68.97N \qquad N_B = 27.59N$



F.B.D

$\circlearrowleft \sum M_A = 0$

$$0.4(27.59)(15\cos\theta) + 27.59(15\sin\theta) - 80\cos\theta(7.5) = 0$$

$413.79\sin\theta - 434.48\cos\theta = 0$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{434.48}{413.79} = 1.05 \qquad \theta = 46.4$

Example(7):-

Two blocks A=100N and B=150N are resting on ground. Coefficient of friction between ground and block B is 0.1 and that between block B and A is 0.3. find the minimum value of weight P in that pan so that motion starts. Find whether B is stationary with respect to ground and A moves or B is stationary with respect to A.

Solution:-

1.B is stationary and A moves

F.B.D of block A

$$\uparrow \sum F_y = 0$$

$$N_1 + P\sin 30 - 100 = 0$$

$$N_1 = 100 - P\sin 30$$

$$\rightarrow \sum F_x = 0$$

$$P\cos 30 - 0.3N_1 = 0$$

$$P\cos 30 - 0.3(100 - P\sin 30) = 0$$

$$P = 29.53N$$

2.A and B are moving together

F.B.D of block A and B

$$\uparrow \sum F_y = 0$$

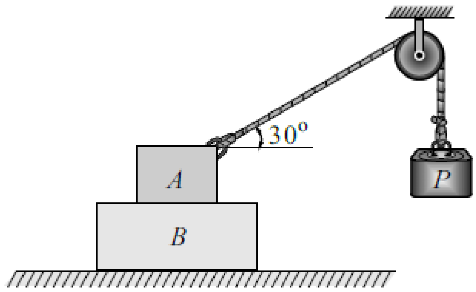
$$N_2 - 250 + P\sin 30 = 0$$

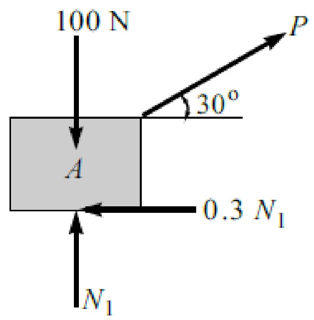
$$N_2 = 250 - P\sin 30$$

$$\rightarrow \sum F_x = 0$$

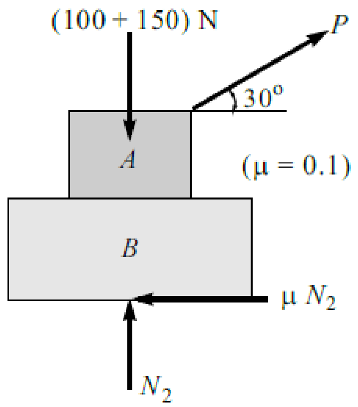
$$P\cos 30 - 0.1N_2 = 0$$

P=27.29N $\quad \therefore P = 27.29N$





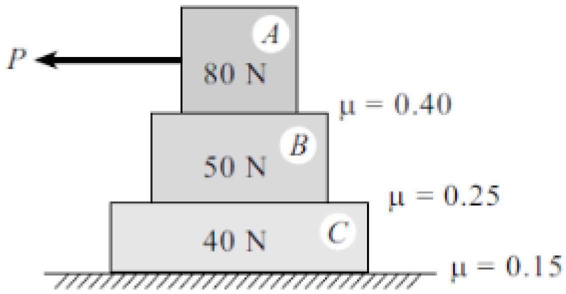
FBD of Block *A*



FBD of Block *A* and *B* Together

Example(8):-

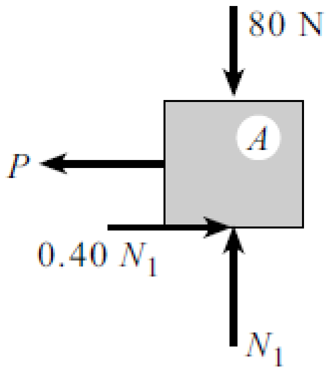
Three blocks are placed on the surface one above the other. The static coefficient of friction between the blocks and block C and surface is shown in the figure. Determine the maximum value of P that can be applied before any slipping take place.



Solution:-

1.Block A has impending motion and blocks B and C remain intact

F.B.D of block A



FBD of Block *A*

$$\uparrow \sum F_y = 0$$

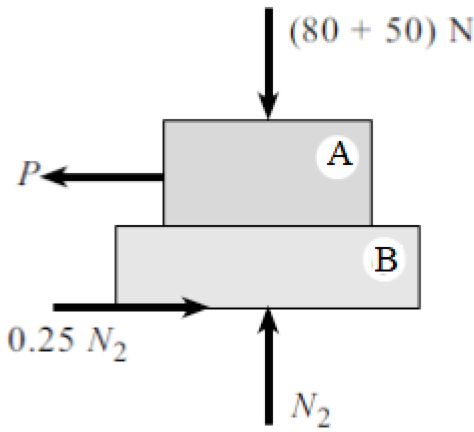
$$N_1 - 80 = 0 \quad \therefore N_1 = 80N$$

$$\rightarrow \sum F_x = 0$$

$$0.4N_1 - P = 0 \quad \therefore P = 32N \leftarrow$$

2.Blocks A and B together have impending motion and block C remains intact.

F.B.D of blocks A and B



FBD of Block *A* and Block *B* Together

$$\uparrow \sum F_y = 0$$

$$N_2 - (80 + 50) = 0 \quad \therefore N_2 = 130N$$

$$\rightarrow \sum F_x = 0$$

$$0.25N_2 - P = 0 \quad \therefore P = 32.5N \leftarrow$$

3. All the three blocks together have impending motion.

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F.B.D of blocks A, B and C

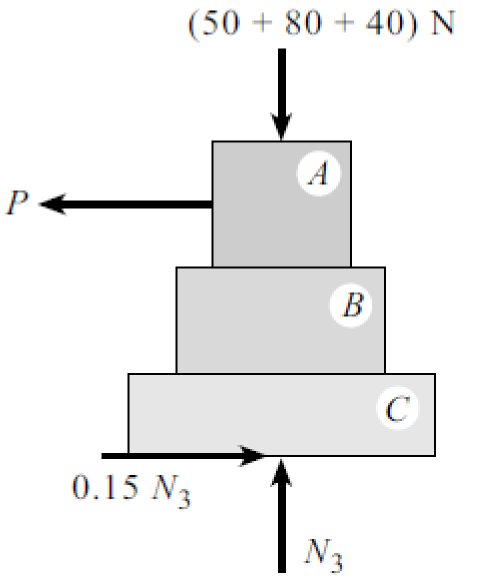
$\uparrow \sum \mathrm{F_y} = 0$

$N_3 - (50 + 80 + 40) = 0 \;\; N_3 = 170N$

$\rightarrow \sum F_x = 0$

$0.15N_3 - P = 0 \;\; \therefore N_3 = 25.5N$

Pmax=25.5N before any slipping take place

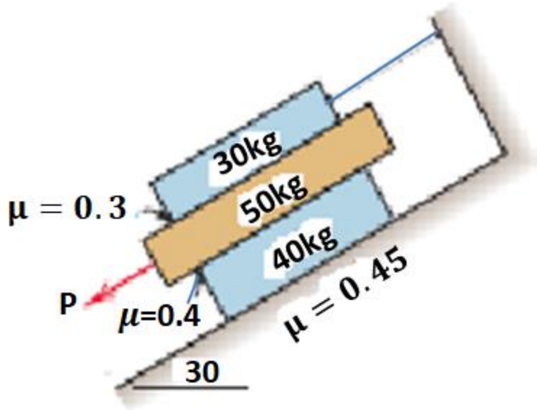


FBD of Block *A* and Block *B* and Block *C* Together

(C)

Example(9):-

The three flat blocks are positioned on the 30 incline as shown, and a force P parallel to the incline is applied to the middle block. Determine the maximum value which P may have before any slipping takes place.



Solution:-

There are two possible conditions for impending motion.

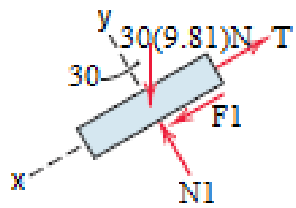
1. The 50kg block slips and the 40kg block remains in place.

F.B.D. (1) and (2)

$\uparrow \sum F_y = 0$

(30 kg)

$N_1 - 30(9.81)\cos 30 = 0 \quad N_1 = 255\text{N}$



F.B.D. 1

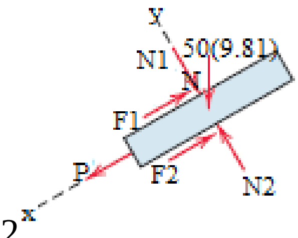
(50kg)

$N_2 - 50(9.81)\cos 30 - 255 = 0 \quad N_2 = 680\text{N}$

$\text{F1} = 0.3(255) = 76.5\text{N}$

$\text{F2} = 0.4(680) = 272\text{N}$

F.B.D.(2)



F.B.D.2

$\rightarrow \sum F_x = 0$

$\text{P} - 76.5 - 272 + 50(9.81)\text{sin}30 = 0 \quad \text{P} = 103.1\text{N}$

2 The 50kg and 40kg blocks move together with slipping occurring between the 40kg block and the incline.

$\uparrow \sum F_y = 0$

$N_3 - 255 - 90(9.81)\cos 30 = 0 \ \therefore N_3 = 1019N$

$\mathrm{F}_3 = 0.45(1019) = 459\mathrm{N}$

$\rightarrow \sum F_x = 0$

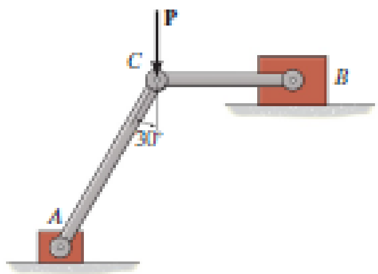
$76.5 + 459 - 90(9.81)\sin 30 - \mathrm{P} = 0 \quad \therefore \mathrm{P} = 94\mathrm{N}$

P_{max}=94N, motion impends for the 50kg and 40kg as a unit.

Example(10):-

Blocks A and B have a mass of 3kg and 9kg, respectively, and are connected to the weightless links. Determine the largest vertical force P that can be applied at the pin C without causing any movement. The coefficient of static friction between the blocks and the contacting surfaces is μ=0.3.(the links are two force members such us the truss member).

Solution:-



(a)

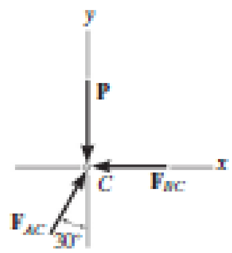
From F.B.D. of pin C

$\uparrow \sum F_y = 0$

$F_{AC} \cos 30 - P = 0 \qquad F_{AC} = 1.155P$

$\rightarrow \sum F_x = 0$

$1.155\ P \sin 30 - F_{BC} = 0 \qquad F_{BC} = 0.5774P$



Pin C

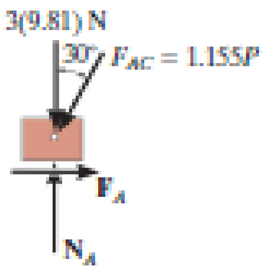
From F.B.D. of block A

$\rightarrow \sum F_x = 0$

$(1) F_A - 1.155\ P \sin 30 = 0 \qquad F_A = 0.5774P$

$\uparrow \sum F_y = 0$

$N_A - 1.155P \cos 30 - 3(9.81) = 0 \quad N_A = P + 29.43N \quad (2)$



F.B.D of block A

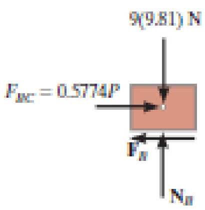
From F.B.D of block B

$\rightarrow \sum F_x = 0$

$(3) 0.5774P - F_B = 0 \qquad F_B = 0.5774P$

$\uparrow \sum F_y = 0$

$N_B - 9(9.81) = 0 \qquad N_B = 88.29N$



F.B.D of block B

If we assume block A slips first then

$F_A = \mu N_A = 0.3N_A$ (4)

Subs (1) and (2) into Eq(4)

0.5774P=0.3(P+29,43) P=31.8N

subs into Eq(3)

$F_B = 18.4$ N $< (F_B)\max = \mu N_B = 0.3(88.29) = 26.5$N

Block B will not slip, thus the above assumption is correct.

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Home Work

H.W(1)

Blocks A,B and C have weights of 50N,25N and 15N respectively. Determine the smallest horizontal force P that will cause impending motion. The coefficient of static friction between A and B is μ=0.3, between B and C is μ=0.4, and between block C and the ground is μ=0.35.(Ans: P=45N)