Extra Examples #2

<u>Conditional Probability</u>

Example 1: If six cards are selected at random (without replacement) from a standard deck of 52 cards, what is the probability there will be no pairs? (Two cards of the same denomination).

<u>Solution:</u>

Let Ei be the event that the first i cards have no pair among them. Then we want to compute P (E6), which is actually the same as P (E1. E2 · · · E6) =

$$\mathbf{P\ (E_1 \,.\, E_2 \ldots \ldots E_n) = P\ (E_1).\ P\ (E_2/E_1) - P\ (E_3/E_1\, E_2) \ldots.\ P\ (E_n/E_1\, E_2 \ldots E_{n-1}).}$$

$$= \frac{52}{52} \cdot \frac{48}{51} \cdot \frac{44}{50} \cdot \frac{40}{49} \cdot \frac{36}{48} \cdot \frac{32}{47} = ******$$

Example 2: The result of survey of group of 100 people who bought either a mobile phone or a tablet from any of two brands A and B is shown in the table below:

Brand	Mobile phone	Tablet	Total
A	20	10	30
B	30	40	70
Total	50	50	100

If a person is selected at random from the group, what is the probability that he:

1. Bought brand B?
2. Bought a mobile phone from brand B?
3. Bought a mobile phone given that he bought brand B?

<u>Solution:</u>

Let

EventM_p: bought a mobile phone

Event T: bought a tablet

Event A: bought brand B

Event B: bought brand B

1. A total of 70 people out of the total of 100 bought brand B, so that

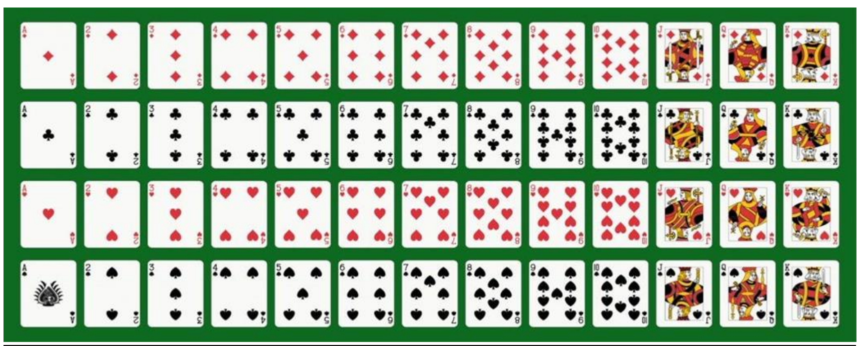
P (B) = $\frac{70}{100}$ = 0.7

2. A total of 30 people out of 100 bought a mobile from brand B, so that
   P (M_p *and* B) = $\frac{30}{100}$ = 0.3
3. P (M_p / B) = $\frac{(M_p\ .B)}{P\ (B)} = \frac{0.3}{0.7}$

Example 3: A single card is drawn from a deck. A card is selected at random, find the probability of selecting a:

a) King
b) Red card
c) King of red card
d) King given that it is red card
e) Red card given that it is red
f) Queen given that it is a Heart

Solution:



Note: Outcomes are all equally likely to occur:

a) Out of the sample space of 52 cards, there 4 kings, so
   P (King) = $\frac{4}{52} = \frac{1}{13}$
b) Out of the sample space of 52 cards, there are 26 red, so
   P (red) = $\frac{26}{52} = \frac{1}{2}$
c) 2 cards are King of red out of the 52 cards, so
   P (King and red) = $\frac{2}{52} = \frac{1}{26}$

d) Two methods to answer the question

1. P (King given that it is a red card) = P (king /red) = $\frac{P(king\ and\ red)}{P(red)}$ =

$= \frac{\frac{1}{26}}{\frac{1}{2}} = \frac{1}{13}$

2. Using the restricted sample Space

Out of the 26 red (card restricted sample space to the red only since this is the condition) there are 2 red, so

P (King/red) = $\frac{2}{26} = \frac{1}{13}$ same as was found above using the definition

e) Two methods to answer the question.

1. Using Definition of the conditional probability given above

P (red card given that it is a King) = P (red/king) = $\frac{P(red\ and\ king)}{P(king)}$

$= \frac{\frac{1}{26}}{\frac{1}{13}} = \frac{1}{2}$

2. Using the restricted sample space

Out of the 4 Kings cards (restricted sample space to the Kings) there are 2 red, so

P (red/King) = $\frac{2}{4} = \frac{1}{2}$ same as was found above using the definition

f) Two methods to answer the question.

1. Using Definition of the conditional probability given above

P (Queen given that it is Heart) = P (Queen/heart) = $\frac{\text{P(Queen and Heart)}}{\text{P(Heart)}}$

$= \frac{\frac{1}{52}}{\frac{13}{52}} = \frac{1}{13}$

2. Using the restricted sample space

Out of the 13 hearts (restricted sample space to the hearts) there is 1 Queen, so

P (Queen/heart) = $\frac{1}{13}$ same as was found above using the definition