Mustansyriah University
College of Engineering
Civil Engineering Department
4th Stage


Reinforced Concrete-II

Design of Two-Way Slabs

Direct Design Method (DDM)

Prof. Dr. Ali Hameed Aziz

T.D Building and Construction (Bahsra Technical Institute) 1988
BSc. Civil Engineering (University of Baghdad) 1994
MSc. Civil Engineering-Structures (Jordan University) 2000
Ph.D. Civil Engineering-Structures (Al-Mustansiriyah University) 2006
Assist. Prof. 2010
Professor 2017

October 2021

1-Intruduction

RC Slabs: are constructed to provide flat surfaces, usually horizontal, in building floors, roofs, bridges, and other types of structures. The slab may be supported by walls, by RC beams usually cast monolithically with the slab, Masonry, by structural steel beams, directly by columns, or continuously by the ground. The depth of a slab is usually very small compared to its span.

Function: slab is the first part of the structure to be contact to live load and therefore carry this loading plus dead load and transmit to the supporting members. Slabs do this in bending and shear.

One-Way Slabs: If a slab is supported on two opposite sides only, it will bend or deflect in a direction perpendicular to the supported edges. The structural action is one way, and the loads are carried by the slab in the deflected short direction. If the slab is supported on four sides and the ratio of the long side to the short side is equal to or greater than two ($L/S \geq 2$), most of the load is carried in the short direction, and one-way action is considered for all practical purposes.

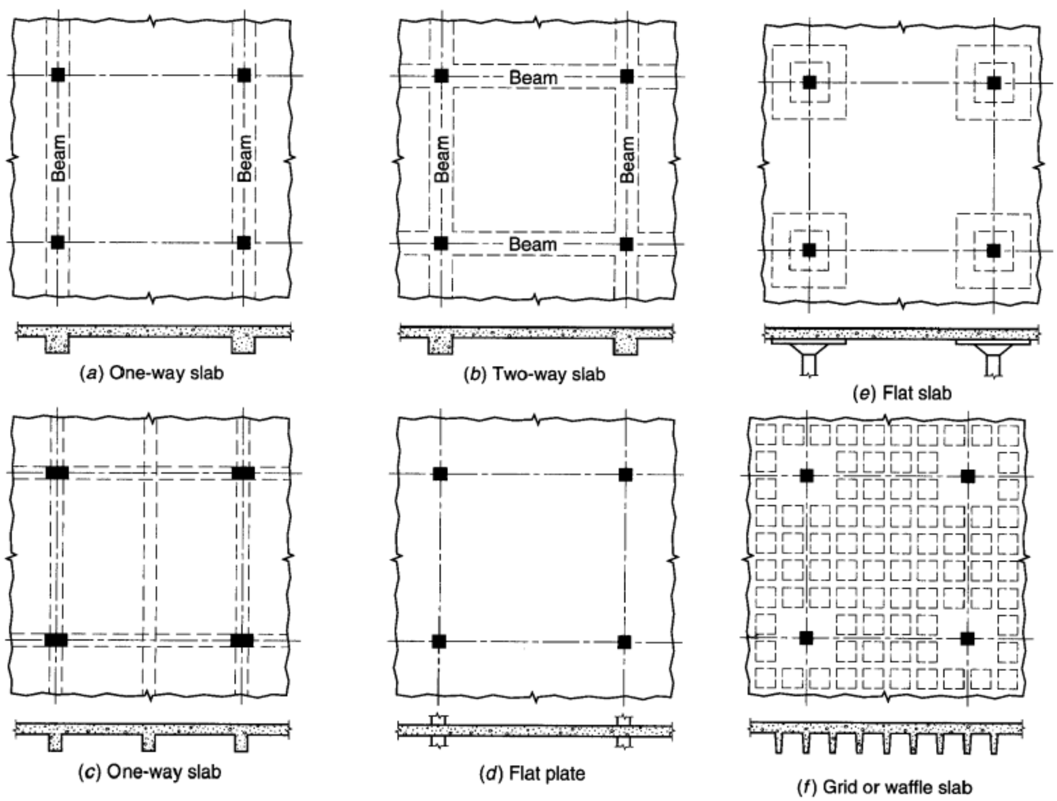
Two-Way Slabs: When a rectangular slab is supported on all the sides (by Beams, Columns and Walls) and the length-to-breadth ratio is less than two ($L/S \leq 2$), it is considered to be a two-way slab. Rectangular two-way slabs can be divided into the following types:-

1-Slab with Beams

2-Flat Plate (With or Without Edge Beam)

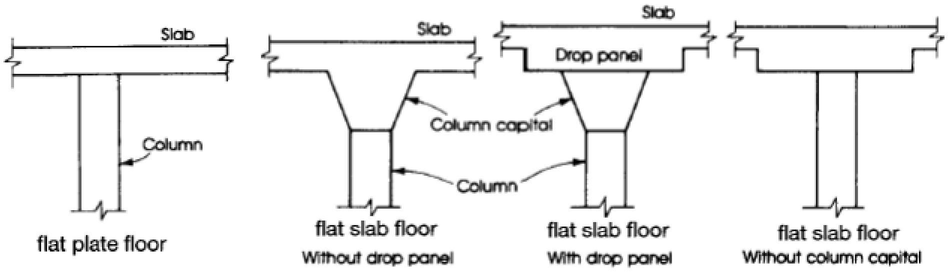
3- Flat Slab (With Drop Panel or Column Capital or Both).

4- Waffle Slab.



Types of structural slabs.

Commons types of One-way and Two-way Slabs



Two-way slabs without beams

2-Methods of Analysis of Two-way Slabs

1- ACI-318 Coefficient Method (Method-I, Method-II and Method-III) →3rd Stage

2- **Direct Design Method (DDM)**

3-Equivalent Frame Method (EFM)

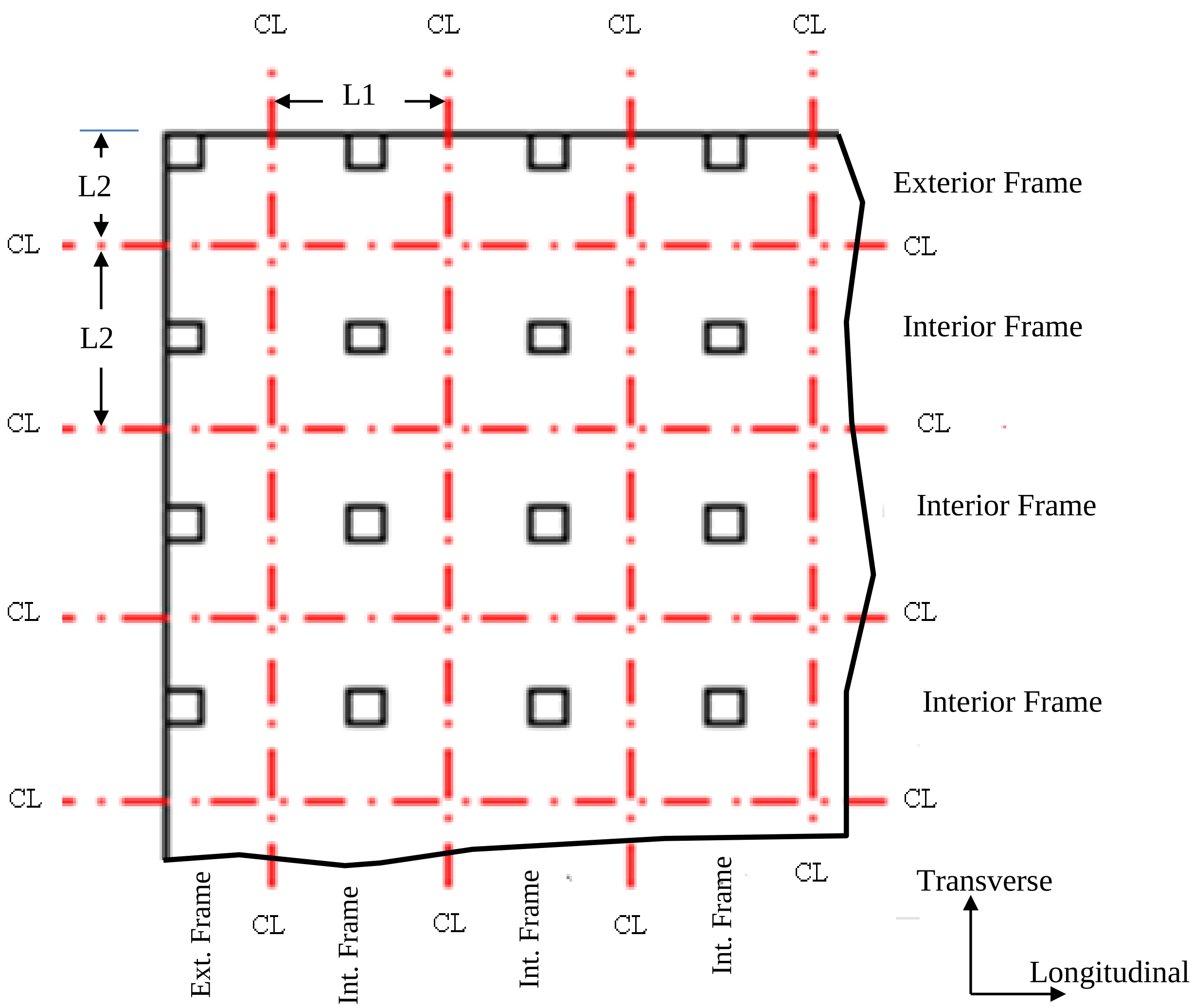
4-**Yield Line Method (YL)**

5-Finite Element Method (FEM) (most powerful)

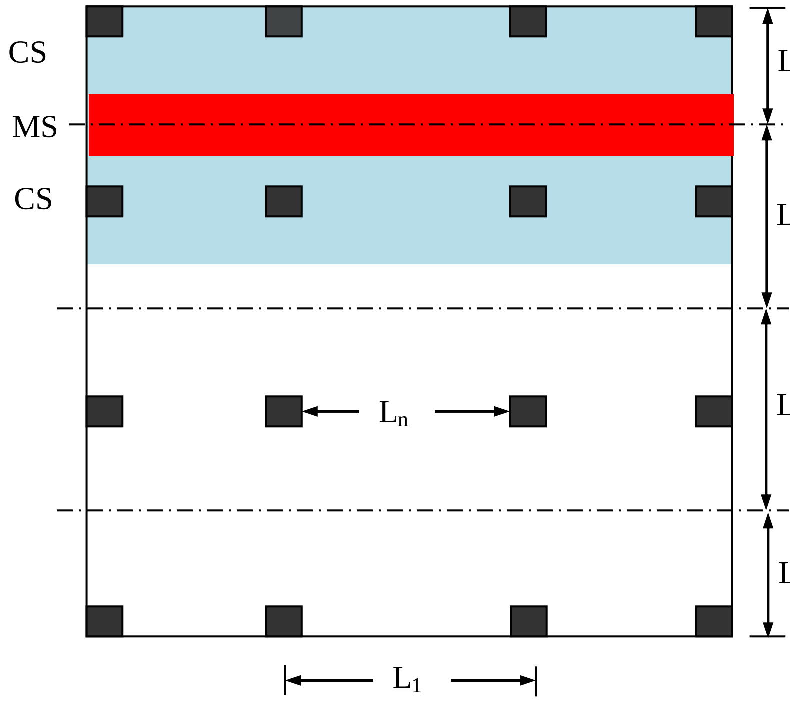
3-General Design Concept of ACI-318 Code

The procedure consists of the following steps:-

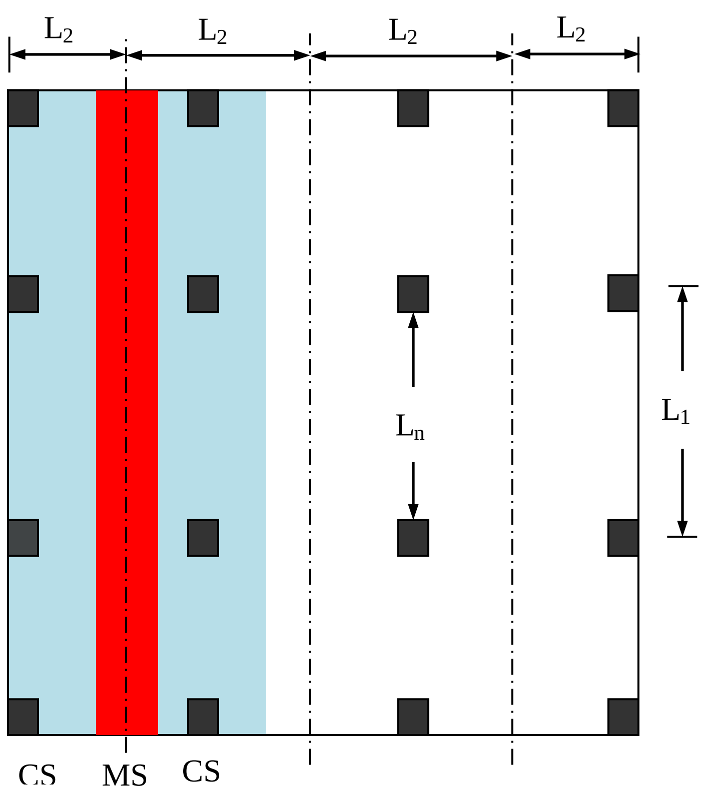
1-Divide the floor (slab) into lines mid-way between columns (supports). This creates a series of frames in two orthogonal directions.



Creation of Interior and Exterior Frames



Subdivision in Frames in Longitudinal Direction (X-direction)



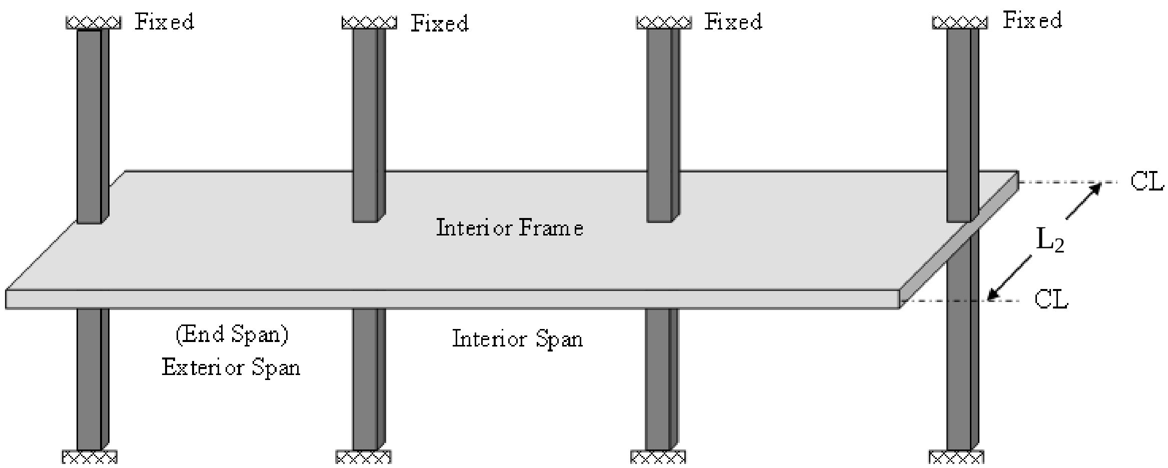
Subdivision in Frames in Transverse Direction (Y-direction)

Where:

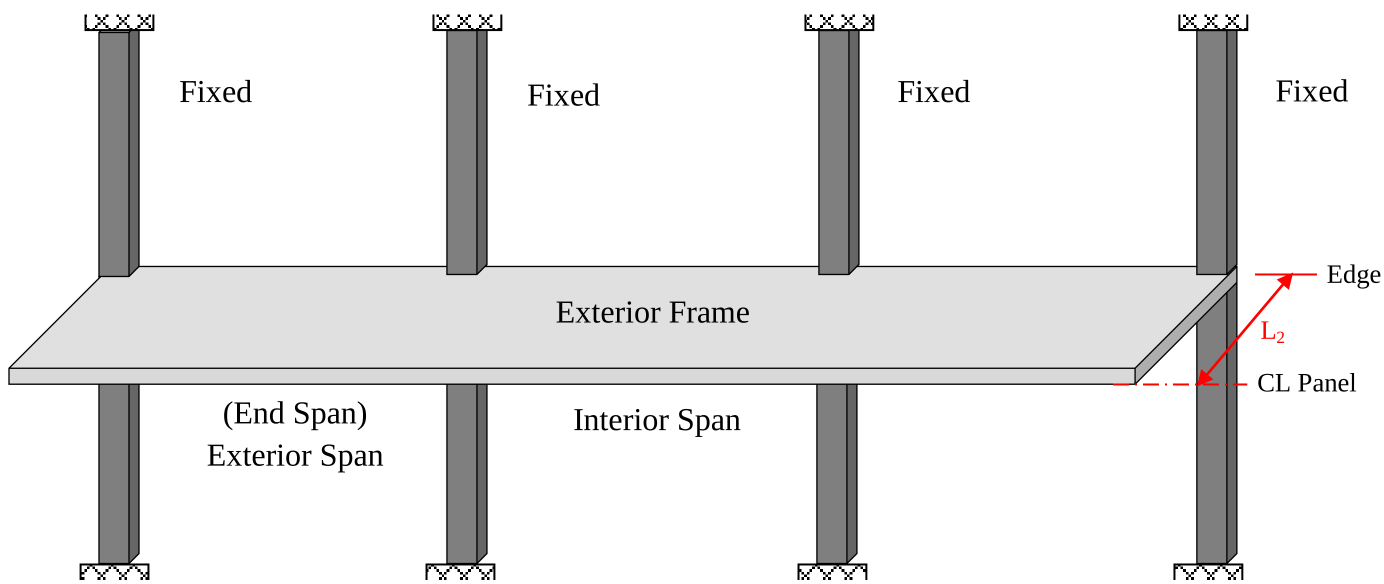
L_1= Length in Direction of Frame (Span of Frame).

L_2 = Transverse of the Frame (Width of Frame)

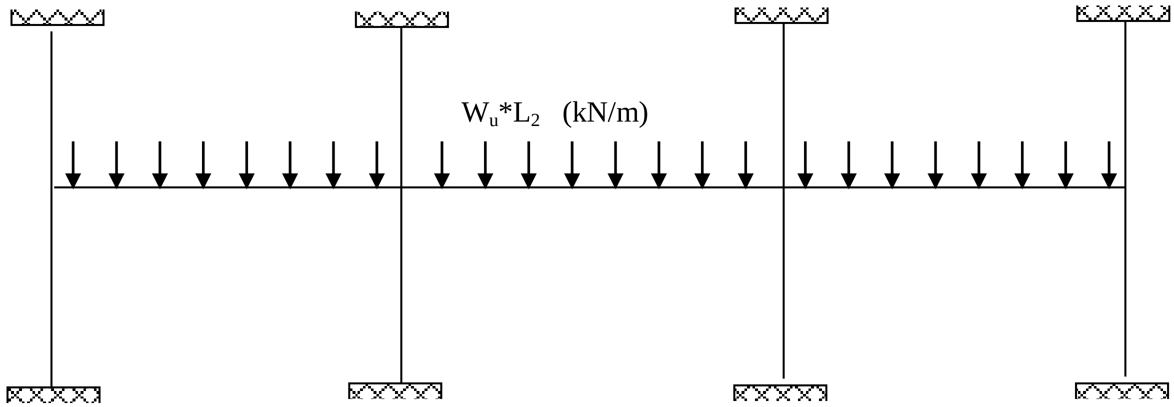
L_n= Clear Length (Clear Span) in Direction of Frame.



Typical Interior Frame



Typical Exterior Frame



Typical 2D Frame

2- Divide the panel into **column strips** and **middle strips**.

Width of column strip= min. (L_1/4 , L_2/4)

3- Design each frame separately using either:-
i- Direct Design Method (DDM)
ii-Equivalent Frame Method (EFM)

4- In either method, the design includes:-
i- Determining the longitudinal positive and negative moments in the direction of the frame ($\mathbf{M}^-$ at Supports and $\mathbf{M}^+$ at Mid Span).
ii- Transverse distribution of longitudinal moments (Distribute $\mathbf{M}^-$ and $\mathbf{M}^+$ to column strips and middle strips , then to slab and beam).
iii- Performing design procedure to find section dimensions and reinforcement.

5- The procedure of analysis described in previse steps is actually based on assumed dimensions for beams and slabs; these assumed dimensions have to compared with the calculated dimensions, otherwise repeat design procedure using the new found dimensions.

4- Direct Design Method (DDM)

The direct design method (DDM) is an approximate procedure for the analysis and design of two-way slabs. It is limited to slab systems subjected to uniformly distributed loads and supported on equally or nearly equally spaced columns. The method uses a set of coefficients to determine the design moments at the critical sections. The Two-way slab systems that do not meet the limitations of the direct design method must be analyzed by more accurate method (procedures) such as Equivalent Frame Method (EFM).

4-1-Limitations of Direct Design Method

1- There must be three or more continuous spans exist in each direction.

2- Panels should be rectangular and the long span is no more than twice the short span (measured center-to-center of supports).

3- Successive span lengths center-to-center of supports in each direction shall not differ by more than 1/3 of the longer span.

4- Columns must be near the corners of each panel with an offset from the general column line of no more 10% of the span in each direction.

5- The live load should not exceed (2) time the dead load in each direction. All loads shall be due gravity only and uniformly distributed over an entire panel.

6- If there are beams, there must be beams in both directions, and the relative stiffness of the beam in the two directions must be related as follows:-

$$\boxed{0.2 \le \frac{\alpha_1 l_2^{\,2}}{\alpha_2 l_1^{\,2}} \le 5.0}$$

(Which means, that beams in the two directions should not differ too much in their stiffness)

$$\boxed{\alpha = \frac{E_{cb} I_b}{E_{cs} I_s}}$$

α= Ratio of flexural stiffness of beam sections to flexural stiffness of a width of slab bounded laterally by center lines of adjacent panels (if any) on each side of the beam.

4-2-Minimum Thickness of Two-Way Slabs to Control Deflection (ACI 318-14/ CL 8.3.1)

i- Slabs without Interior Beams (Table 8.3.1.1)

Table (8.3.1.1) Minimum Thickness of Two-Way Slabs to Control Deflection

f_y (MPa)	Without Drop Panels ≥125mm			With Drop Panels ≥100mm		
	Exterior Panels		Internal Panels	Exterior Panels		Internal Panels
	No Edge Beams	With Edge Beams*		No Edge Beams	With Edge Beams*	
280	Ln / 33	Ln / 36	Ln / 36	Ln / 36	Ln / 40	Ln / 40
420	Ln / 30	Ln / 33	Ln / 33	Ln / 33	Ln / 36	Ln / 36
520	Ln / 28	Ln / 31	Ln / 31	Ln / 31	Ln / 34	Ln / 34

* Slab with Edge Beams along exterior edges, α ≥ 0.8 for the Edge Beam.

L_n= Clear distance between columns.

$$\boxed{\alpha = \frac{E_{cb} I_b}{E_{cs} I_s}}$$

α= Ratio of flexural stiffness of beam sections to flexural stiffness of a width of slab bounded laterally by center lines of adjacent panels (if any) on each side of the beam.

E_{cb}= Modulus of elasticity of the beam concrete.

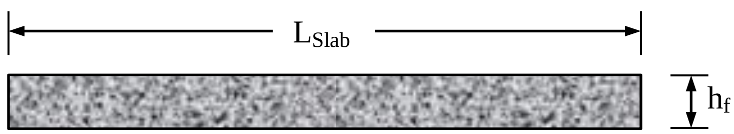
E_{cs}= Modulus of elasticity of the slab concrete.

Note

α=0 for flat plate slab; why?

Moment of Inertia of Slabs and Beams

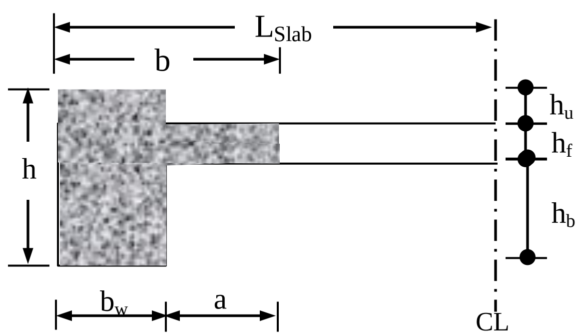
Moment of Slab (I_s)

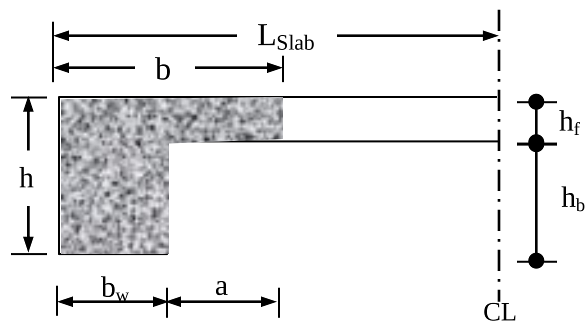


$$\boxed{\mathbf{I_s} = \frac{(\mathbf{L}_{\boldsymbol{Slab}}).(\mathbf{h_f})^3}{\mathbf{12}}}$$

Moment of Inertia of Beams (I_b)

i- Edge Beams (Exterior Beams)





Exact (I_b)

$b=b_w+a$

$a=\min.(4h_f, \max. (h_b , h_u))$

To find (I_b); (1) Find $y'=\sum A_y/\sum A$

(2) Find $I_b=I_o+Ad^2$

Exact (I_b)

$b=b_w+a$

$a=\min.(4h_f, h_b)$

To find (I_b); (1) Find $y'=\sum A_y/\sum A$

(2) Find $I_b=I_o+Ad^2$

Note

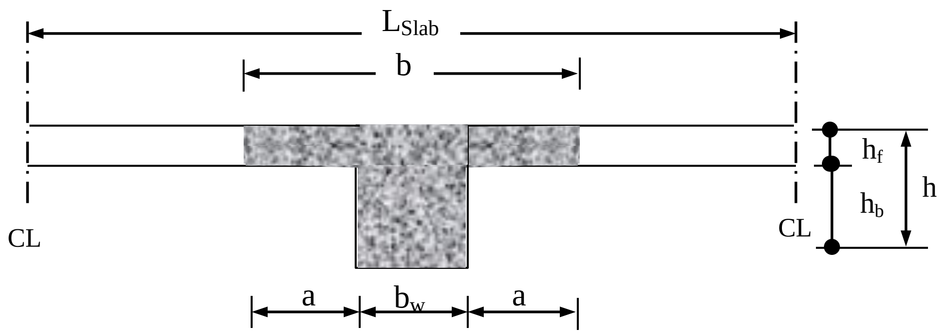
(I_b) can be estimated for edge beams as follows:-

$$\boxed{I_b = \frac{(b_w).(h)^3}{12} * 1.5} \leftrightarrow \text{Approximate } (I_b)$$

*For exterior frames, (I_s) can be calculated as follows:-

$$\boxed{\mathbf{I_s = \frac{(L_{\mathit{Slab}}).(h_f)^3}{12}}}$$

ii- Interior Beams



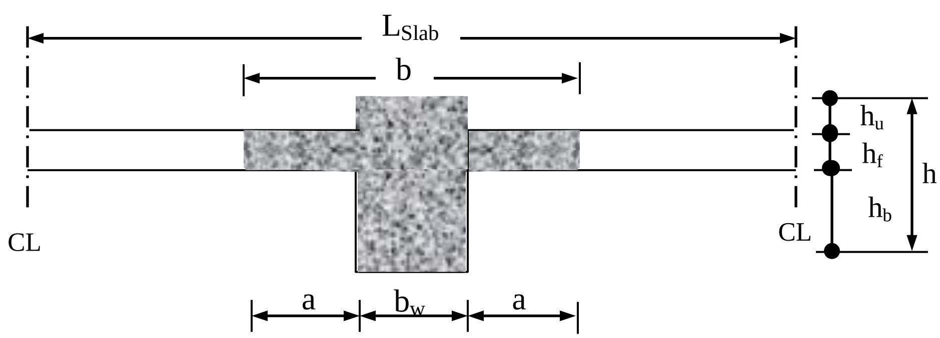
Exact (I_b)

$b=b_w+2a$

$a=\min.(4h_f, h_b)$

To find (I_b); (1) Find $y'=\sum A_y/\sum A$

(2) Find $I_b=I_o+Ad^2$



Exact (I_b)

$b=b_w+2a$

$a=\min.(4h_f, \max.(h_b , h_u))$

To find (I_b); (1) Find $y'=\sum A_y/\sum A$

(2) Find $I_b=I_o+Ad^2$

Note

(I_b) can be estimated for interior beams as follows:-

$$\boxed{I_b = \frac{(b_w).(h)^3}{12} * 2} \leftrightarrow \text{Approximate } (I_b)$$

*For interior frames, (I_s) can be calculated as follows:-

$$\boxed{\mathbf{I_s = \frac{(L_{Slab}).(h_f)^3}{12}}}$$

ii- Slabs with Interior Beams

Find L_n, β, α_m where:

Ln= the clear span in the long. direction, measured face to face, of :-

(a) Columns for slabs without beams.

(b) Beams for slab with beams.

β = the ratio of the long to the short clear span of slab.

α_m = the average value of the ratios of beam-to-slab stiffness on all sides of a panel

1- For $\alpha_m \leq 0.2$ $\rightarrow$ Use Table **Table (8.3.1.1)**

2- For $0.2 < \alpha_m \leq 0.2$

$$h = \frac{\ell_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta(\alpha_m - 0.2)} \geq 125 \text{ mm} \qquad \text{(Table 8.3.1.2-b)}$$

3- For $\alpha_m > 0.2$

$$h = \frac{\ell_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} \geq 90 \text{ mm} \qquad \text{(Table 8.3.1.2-d)}$$

Note

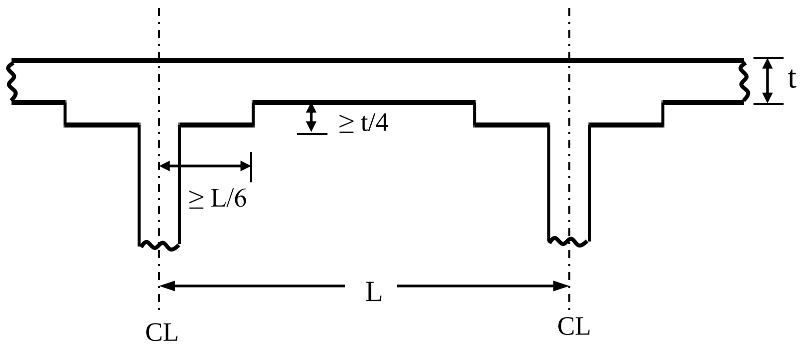
If the discontinuous edge of a panel is not supported on an edge beam whose stiffness ratio (α) is at least (0.8), the slab thickness required by equation (Table 8.3.1.2-b) and (Table 8.3.1.2-d) must be increased by (10%) in the panel with a discontinuous edge.

4-5 Drop Panel Dimensions

According to (ACI318-14 / CL 8.2.4), the drop panel shall satisfy (a) and (b):-

(a) The drop panel shall project below the slab at least **one-fourth of the adjacent slab thickness**.

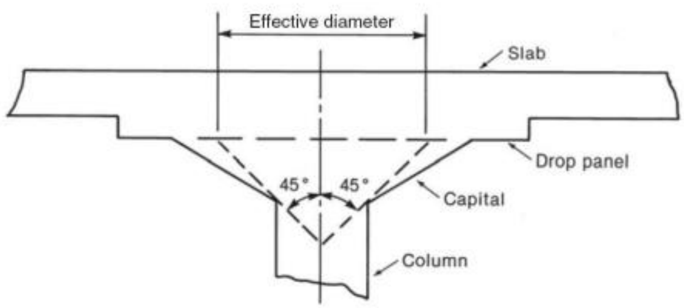
(b) The drop panel shall extend in each direction from the centerline of support a distance not less than **one-sixth the span** length measured from center-to-center of supports in that direction.



Minimum Drop Panel Dimensions

4-6 Column Capital Dimensions

The column capital is normally 20 to 25% of the average span length.



4-7-Total Static Moment (M_o) (ACI 318-14/CL 8.10.3.2)

According to (ACI 318-14/CL 8.10.3.2), the total factored static moment for a span can be defined as follows:

$$\boxed{M_o = \frac{w_u * l_2 * l_n^2}{8}}$$

Where

w_u= Factored load per unit area.

l_2= Transverse width of the strip.

l_n= Clear span between columns.

4-8 Longitudinal Distribution of Total Moment (M_O) in an Interior and End Spans

a- Interior spans:

(Mo) is apportioned (distributed) between the critical positive and negative bending sections according to the following ratios:-

Neg. Mu = 0.65 Mo

Pos. Mu = 0.35 Mo

The critical section for a negative bending is taken at the face of rectangular supports, or at the face of an equivalent square support having the same sectional area.

b- End span:

In end spans, the apportionment (distribution) of the total static moment (Mo) among the three critical moment sections (interior negative, positive, and exterior negative) depends upon the flexural restraint provided for the slab by the exterior column or the exterior wall and upon the presence or absence of beams on the column lines. End span, Mo shall be distributed in accordance with Table (8.10.4.2) (ACI 318-14/CL 8.10.4.2).

***Table (8.10.4.2) Moment Distribution Coefficients for End Spans**

Moment Location	1	2	3		4
	Exterior Edge Unrestrained	Slab with Beams Between all Supports	Slab without Beams between Interior Supports		Exterior Edge Fully Restrained
			Without Edge Beam	With Edge Beam	
Interior Negative Factored Moment	**0.75**	**0.70**	**0.70**	**0.70**	**0.65**
Positive Factored Moment	**0.63**	**0.57**	**0.52**	**0.50**	**0.35**
Exterior Negative Factored Moment	**0**	**0.16**	**0.26**	**0.30**	**0.65**

Description of Exterior Edge

1- Exterior Edge Unrestrained→ **The Exterior edge is Hinge or Brick Wall support.**

2- Slab with Beams Between all Supports→ **The Exterior edge is RC Beam.**

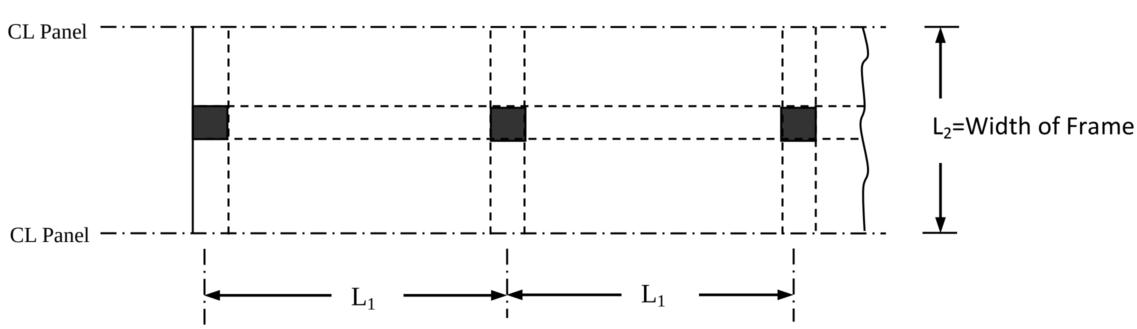
3- Slab without Beams between Interior Supports**:-**

3-1- Without Edge Beam→ **The Exterior edge is RC Column.**

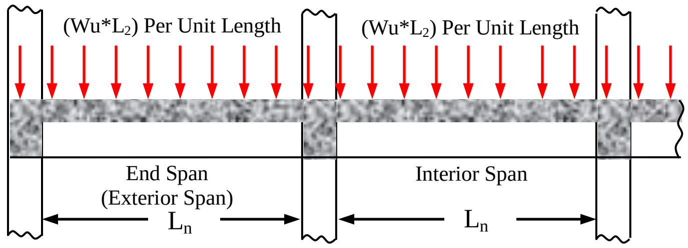
3-2- With Edge Beam→ **The Exterior edge is Edge Beam.**

4- Exterior Edge Fully Restrained→ **The Exterior edge is RC Wall.**

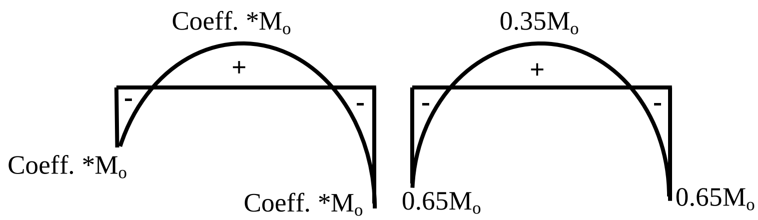
Note: At interior supports, negative moment may differ for spans framing into the common support. In such a case **the slab should be designed to resist the larger of the two moments**.



(Plan)



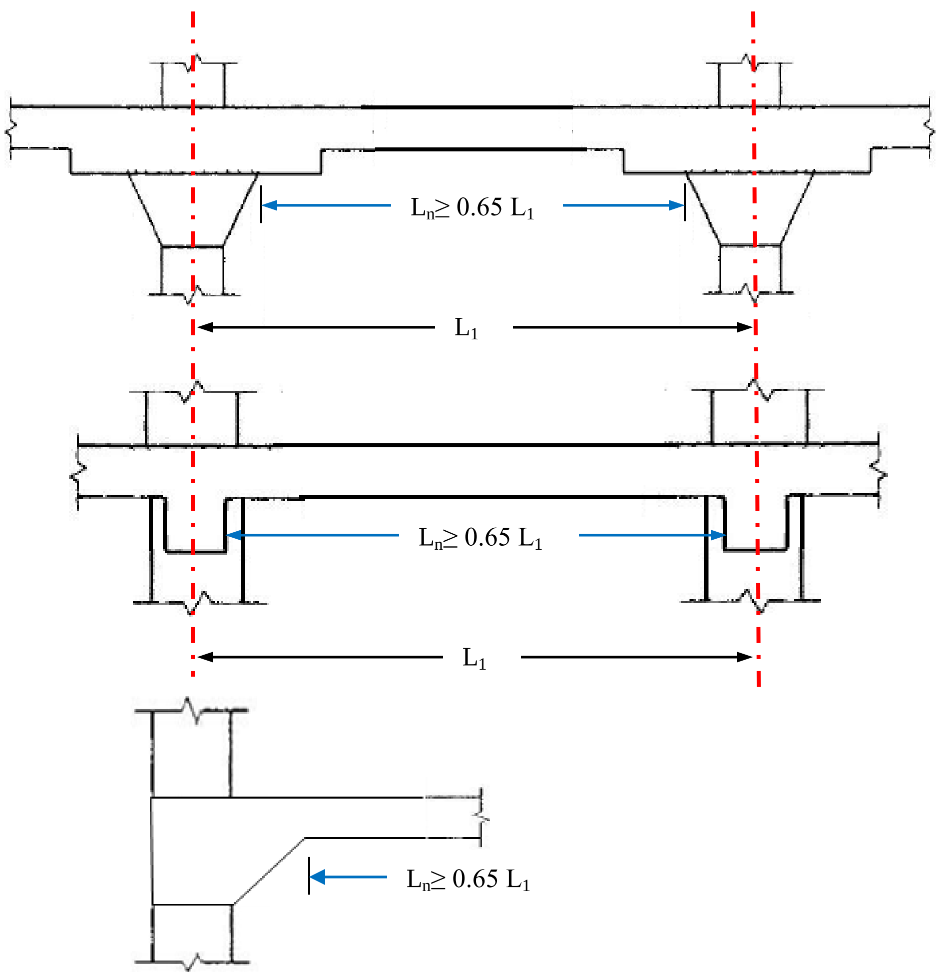
Vertical Section of the Frame



Values of Longitudinal Moment according to DDM

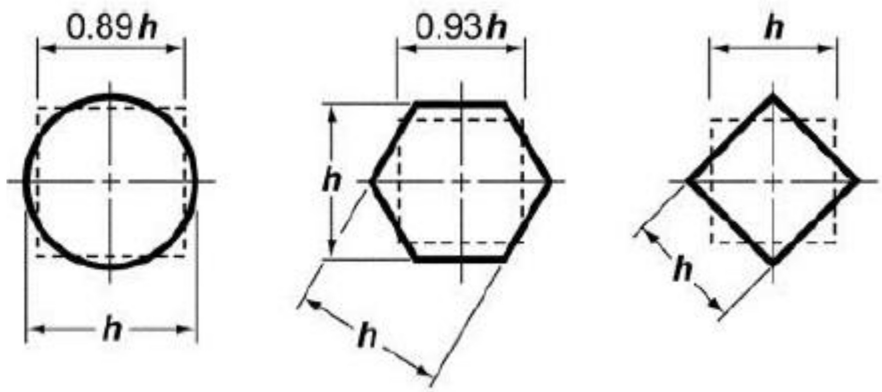
<u>4.9 Clear Span (L_n) (ACI 318-14/CL 8.10.3.2)</u>

The clear span (L_n) shall extend from face to face of the columns, capitals, brackets, or walls but is not to be less than (65%) of the span (L_1) measured c/c of supports.



Note

Circular or regular polygon-shaped supports shall be treated as square supports with the same area.



4-10 Transverse Distribution of Positive and Negative Longitudinal Moments

i- Distribution to CS (rest to MS) according to Tables given by (ACI 318-14/CL 8.10.5)

1-Find aspect ratio of l_2/l_1,

2-Find the relative stiffness of beam and the slab **(ACI 318-14/CL 8.10.2.7b)**,

$$\alpha = \frac{E_{cb} I_b}{E_{cs} I_s}$$

3-Find $\alpha_1\ l_2/l_1$,

4-Find the degree of torsional restraint provided by the edge beam **(ACI 318-14/CL 8.10.5.2a)**,

$$\beta_t = \frac{E_{cb} C}{2 E_{cs} I_s}$$

Where

l_2/l_1= Panel dimensions (measured c/c of span)

β_t= Ratio of torsional stiffness of edge beam section to flexural stiffness of a width of slab equal to the span length of the Torsional stiffness of edge beam c/c of supports.

α_1 =value of (α) in direction of l_1= $\dfrac{E_{cb} I_b}{E_{cs} I_s}$

C = Cross–section constant for torsional member **(ACI 318-14/CL 8.10.5.2b)**;

$$C = \sum (1 - 0.63\frac{x}{y})\frac{x^3 y}{3}$$

x=Shorter dimension of rectangular part.

y= Larger dimension of rectangular part.

The **Column Strip** shall resist the portion of **Interior Negative M_u** in accordance with Table (8.10.5.1).

Table (8.10.5.1) Portion of interior negative Mu in Column Strip

$\alpha_{f1}\, l_2/l_1$	l_2/l_1		
	0.5	**1.0**	**2.0**
0	**75**	**75**	**75**
≥1.0	**90**	**75**	**45**

Note: Linear interpolations shall be made between values shown

The **Column Strip** shall resist the portion of **Exterior Negative M_u** in accordance with Table (8.10.5.2).

Table (8.10.5.2) Portion of exterior negative Mu in Column Strip

$\alpha_{f1}\, l_2/l_1$ and β_t		l_2/l_1		
		0.5	**1.0**	**2.0**
$\alpha_{f1}\, l_2/l_1 = 0$	$\beta_t=0$	**100**	**100**	**100**
	$\beta_t \geq 2.5$	**75**	**75**	**75**
$\alpha_{f1}\, l_2/l_1 \geq 1.0$	$\beta_t=0$	**100**	**100**	**100**
	$\beta_t \geq 2.5$	**90**	**75**	**45**

Note: Linear interpolations shall be made between values shown

The **Column Strip** shall resist the portion of **Positive M_u** in accordance with Table (8.10.5.3).

Table (8.10.5.3) Portion of positive Mu in Column Strip

$\alpha_{f1}\, l_2/l_1$	l_2/l_1		
	0.5	**1.0**	**2.0**
0	**60**	**60**	**60**
≥1.0	**90**	**75**	**45**

Note: Linear interpolations shall be made between values shown

Note

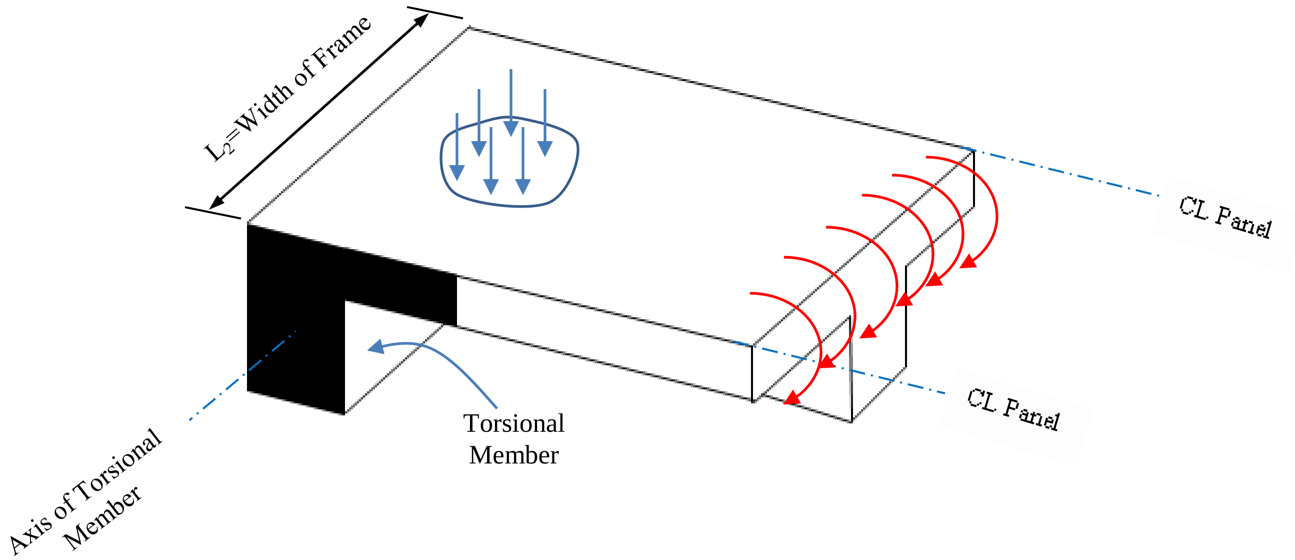
1-When the exterior support is a wall perpendicular to the direction in which moment are being determined:-

$\beta_t = 0$ for brick wall or flat plate without edge beam.

$\beta_t \geq 2.5$ for RC wall.

2- Torsional member

When the RC slab subjected to the gravity load (Dead and Live load), the slab bent due to flexural moment stresses, but, this flexural moment reflected on an edge beam as torsion stresses, therefore, the edge beam called "torsional member".

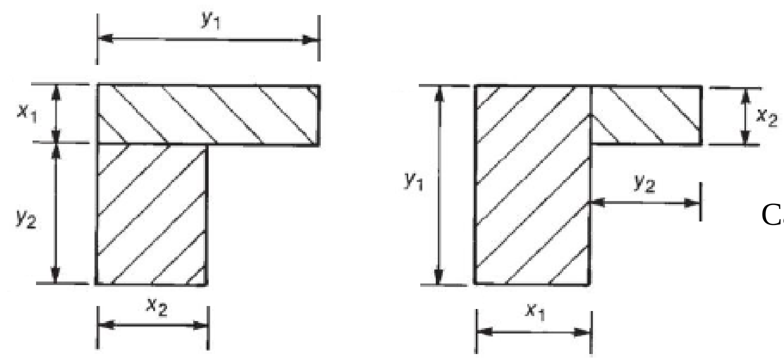


Torsional Member

Cross–section constant for torsional member (C) is calculated by dividing the section into its component rectangles, each having smaller dimension (x) and larger dimension (y) and summing the contributions of all the parts by means of the equation **(ACI 318-14/CL 8.10.5.2b)**:

$$C = \sum \left(1 - 0.63\,\frac{x}{y}\right)\frac{x^3\,y}{3}$$

Case-I



Case-II

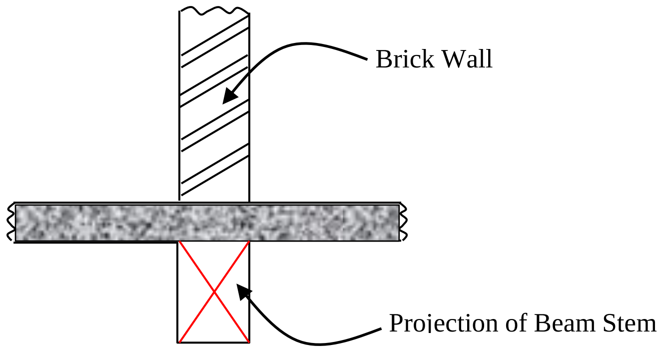
Use the Larger Value of (C) either from (Case-I) or (Case-II)

ii- Distribution of CS moment (M_{cs}) to beam (rest to slab) according to (ACI 318-14/CL 8.10.5.7.1)
Beams resist 85% of the column strip moment if $\alpha_1\ (l_2/l_1) \geq 1.0$; For values between 1.0 and 0, the moment allotted to the beam is determined by linear interpolation.

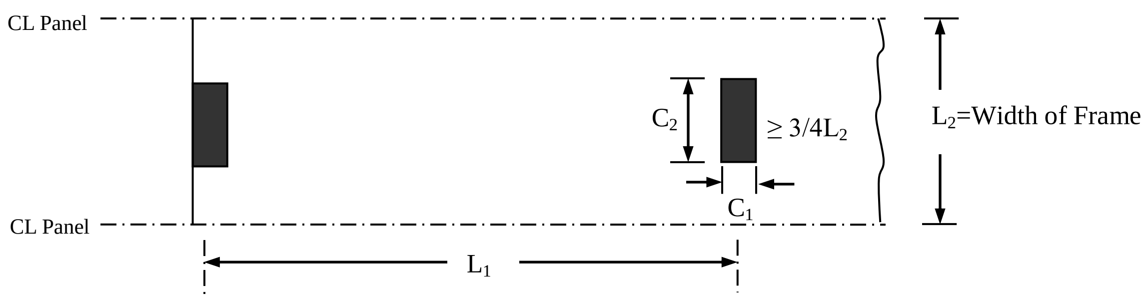
$$M_{beam}=0.85(\alpha_1 * l_2/l_1) * M_{cs} \leq 0.85\ M_{cs}$$

Note

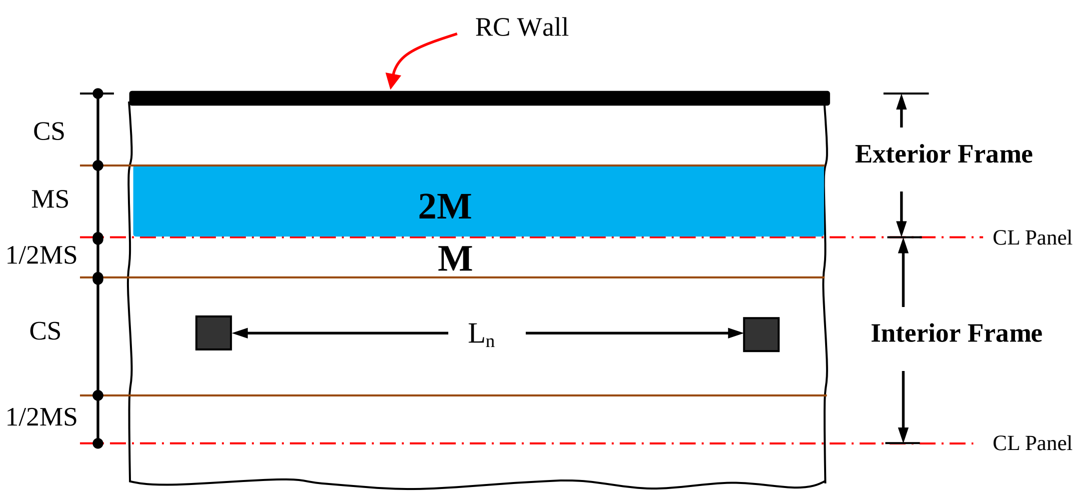
i-In addition to moment calculated according to **(ACI 318-14/CL 8.10.5.7.1)**, beams shall resist moments caused by factored loads applied directly to the beams, including the weight of the beam stem, above and below the slab.



ii-When the size of column (support) , [$C_2 \geq 3/4*L_2$], the negative moment (M^-) must be uniformly distribution across (L_2), **(ACI 318-14/CL 8.10.5.4)**.



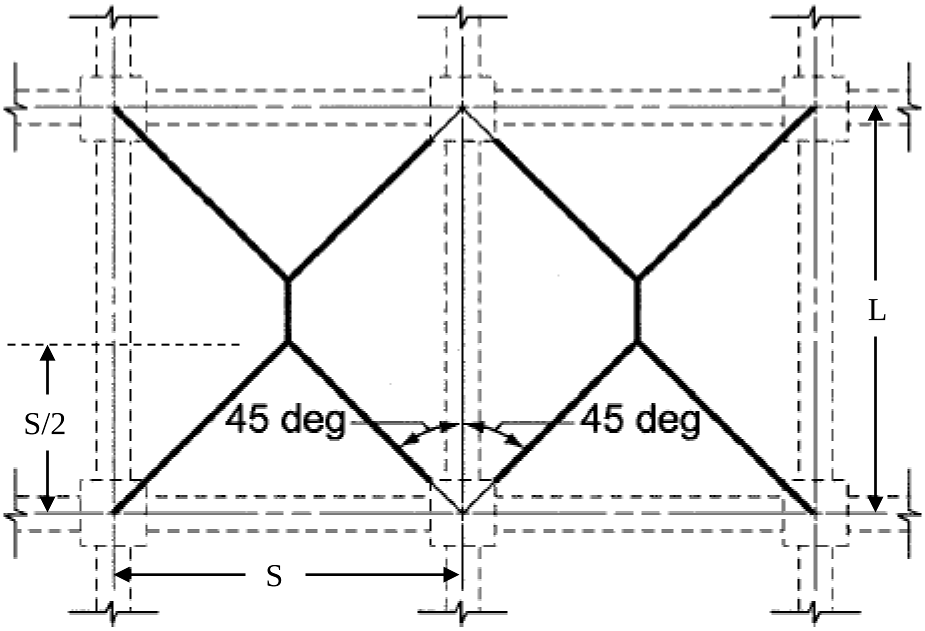
iii- A middle strip adjacent to and parallel to a **wall-supported edge**, shall be proportioned to resist **twice the moment** assigned to the haft middle strip corresponding to the first row of interior supports**(ACI 318-14/CL 8.10.6.3)**.



4-11 Shear in Slab System (Load Transformed from the RC Slab to Supports)

i- Slab-beam system (ACI 318-14/CL 8.10.8.1).

Beams with ($\alpha_1 * l_2/l_1 \geq 1.0$) are assumed to carry load on the tributary floor areas bounded by (45^o) lines drawn from corners to centerline of the panel parallel to long side.



To convert the **Triangular Load** to equivalent UDL→**$UDL=W_u*S/3$**

To convert the **Trapezoidal Load** to equivalent UDL→**$UDL=[W_u*S/3*(3-m^2)/2]$**

Where

S=Short Side of Slab.

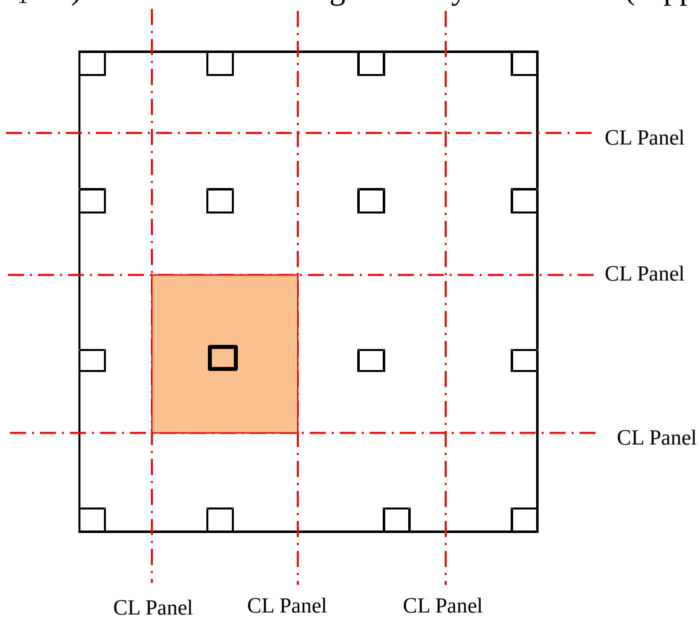
L=Long Side of Slab.

Wu= Factored load.

m=S/L

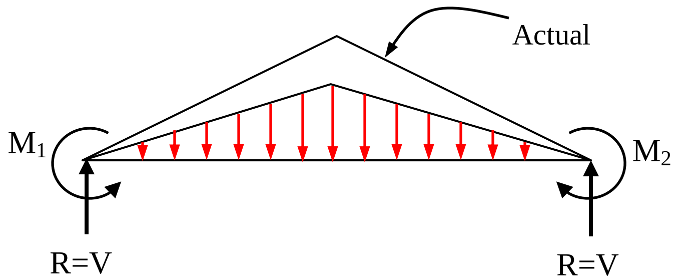
ii- Slab System without Interior Beams

For slabs with no beams ($\alpha_1 * l_2/l_1 = 0$) the load from slab go directly to columns (supports).



ii- Slab System with Interior Beams having ($0< \alpha_1 * l_2/l_1 < 1.0$)

For slabs with beams having ($0< \alpha_1 * l_2/l_1 < 1.0$) the percentage of floor load going to the beams should be obtained by linear interpolation (multiply $W_u * l * (\alpha_1 * l_2/l_1)$).

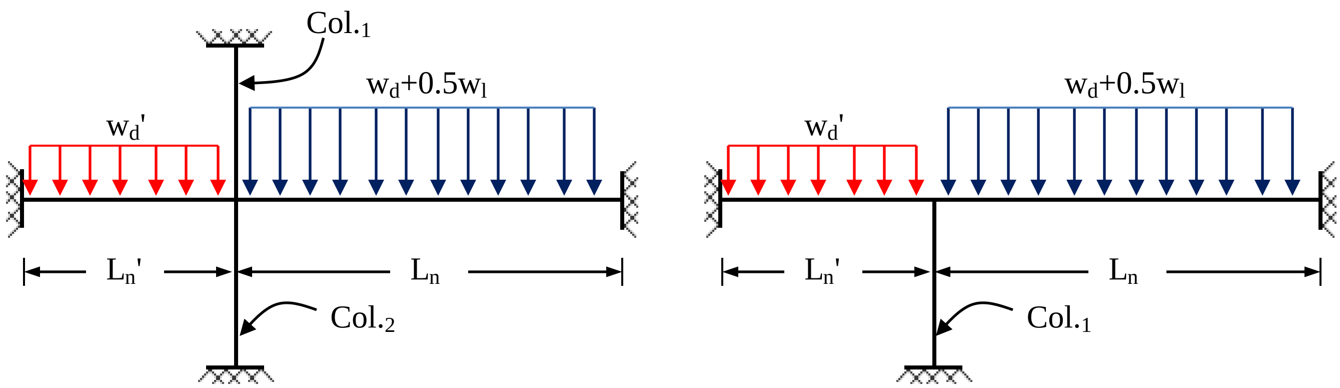


4-12 Factored Moments in Columns and Walls (ACI 318-14/CL 8.10.7.2).

i- Interior Columns

At an interior support, columns or walls above and below the slab shall resist the unbalanced factored moment calculated by the following equation (for upper and lower columns) in direct proportion to their stiffness.

$$\boxed{M_{eq.} = 0.07[(w_d + 0.5w_l)l_2 * l_n^2 - w_d' * l_2' * l_n'^2]}$$



Where

w_d= Factored dead load per unit area.

w_l= Factored live load per unit area.

L_n'= Clear span in short direction.

L_n= Clear span in long direction.

The prime (') quantities refer to the shorter of the two adjacent spans; the shorter span should carry (w_d) only, while, the longer span should carry ($w_d+0.5\ w_l$).

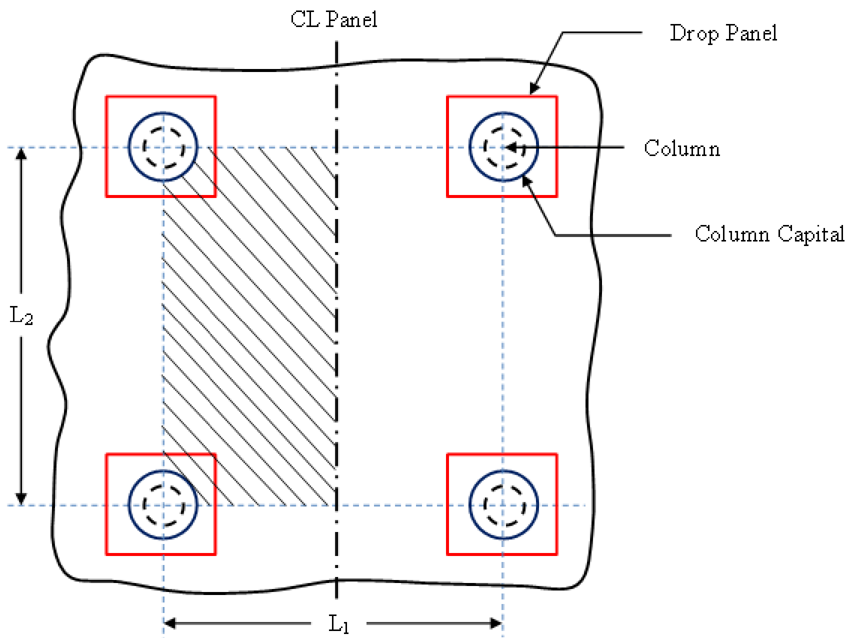
$$M_{Col.1} = \frac{K_{Col.1}}{K_{Col.1}+K_{Col.2}} * M_{eq.} \quad \& \quad M_{Col.2} = \frac{K_{Col.2}}{K_{Col.1}+K_{Col.2}} * M_{eq.}$$

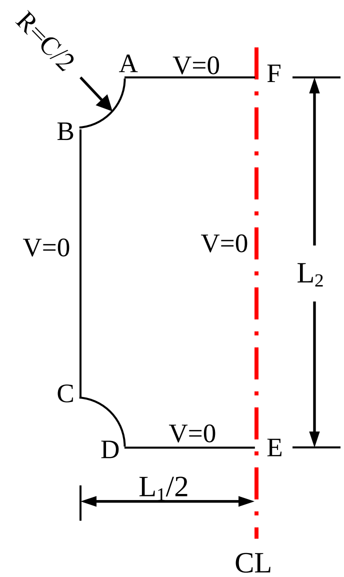
Where $\quad K_{Col.} = \frac{4EI_{Col.}}{L_{Col.}}$

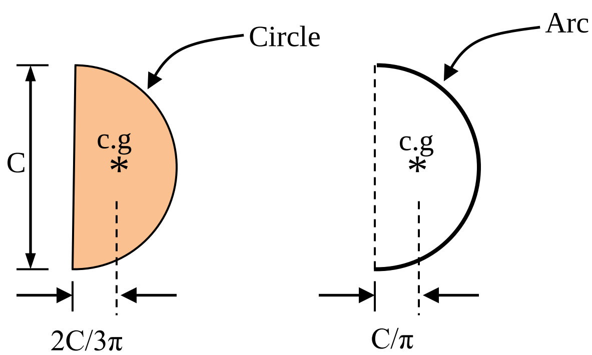
<u>i- Exterior Columns (Edge Columns) (ACI 318-14/CL 8.10.7.3).</u>

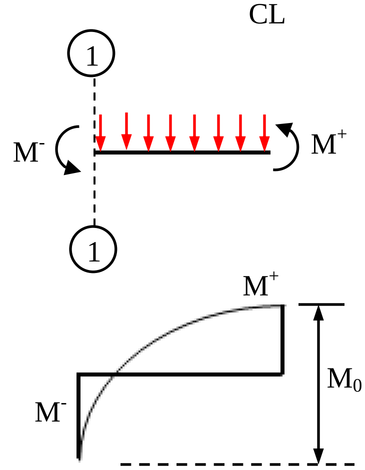
The gravity load moment to be transferred between slab and edge column in accordance with **(ACI 318-14/CL 8.4.2.3)** shall not be less than ($0.3M_o$).

<u>4-13 Total factored static moment (Mo) in flat slab with column capital</u>









Assume C=Diameter of Column Capital

$\sum$ Reactions on arcs AB & CD=W_u (L_2*L_1/2-π C^2/8)

$\sum M_{1\text{-}1}=0$ +

$$M^+ + M^- - \left(W_u\frac{L_2L_1}{2}\right)\left(\frac{L_1}{2}\right) + \left(W_u\frac{\pi}{8}C^2\right)\left(\frac{2C}{3\pi}\right) + W_u\left(\frac{L_2L_1}{2} - \frac{\pi}{8}C^2\right) * \frac{C}{\pi} = 0$$

$M_0=M^+ + M^-$

$$M_0 - \left(\frac{W_uL_2L_1^2}{8}\right) + \left(\frac{W_uC^3}{12}\right) + \left(\frac{W_uL_2L_1C}{2\pi}\right) - \left(\frac{W_uC^3}{8}\right) = 0$$

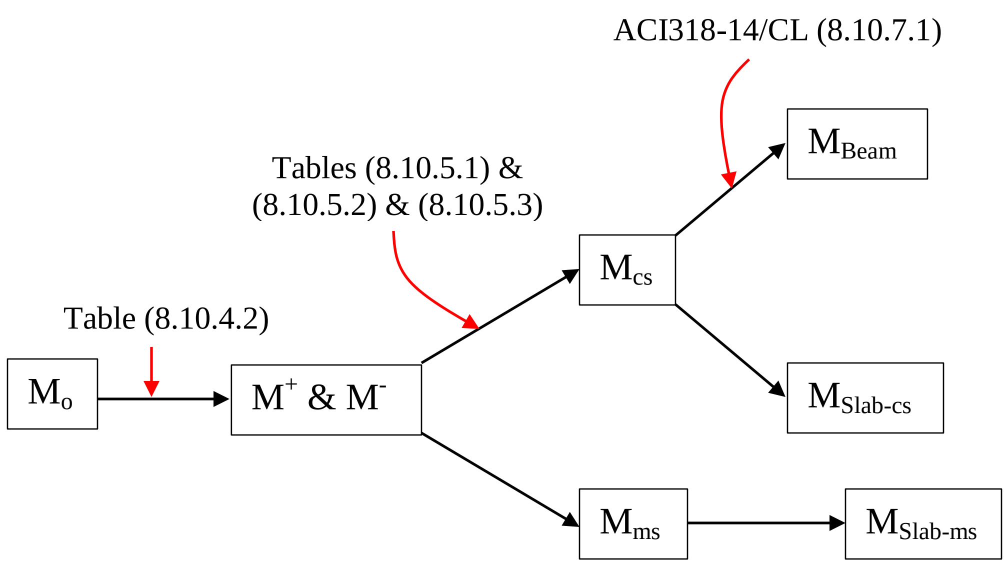
$$M_0 = \left(\frac{W_u L_2 L_1^2}{8}\right) + \left(\frac{W_u C^3}{24}\right) - \left(\frac{W_u L_2 L_1 C}{2\pi}\right)$$

$$M_0 = \left(\frac{W_u L_2 L_1^2}{8}\right)\left(1 + \frac{C^3}{3 L_2 L_1^2} - \frac{4C}{\pi L_1}\right)$$

$$\boxed{M_0 \simeq \left(\frac{W_u L_2 L_1^2}{8}\right)\left(1 - \frac{2C}{3L_1}\right)^2}$$

The above equation is more suitable for the flat slab with circular column capital.

<u>4-14 Summary of DDM</u>



References

1- Design of Concrete Structure, Arthur Nilson; David Darwin; Charles Dolan Nilson 15th Edition.

2-Reinforced Concrete Design, Wang C.K. and Salmon C.G.,4th Edition 1985

3- Building Code Requirements for Reinforced Concrete ACI 318M-14

4- Reinforced Concrete Fundamentals, Ferguson Phil M., 4th Editions 1981.

5- Design of Reinforced Concrete, McCormac, J. and Nelson, K., 7th Edition, (2006).

6- Reinforced Concrete Design, Leet, K. and Bernal, D., 3rd Edition, (1997).

7- Reinforced Concrete, Nawy, E., 6th Edition, (2009).

8- تصميم المنشاءات الخرسانية المسلحة وفقا لمتطلبات الكود ACI 318-02 جمال عبد الواحد

9-محاضرات تصاميم الخرسانة المسلحة- (ا.د.هشام الحسني)-المرحلة الرابعة-1993-قسم الهندسة المدنية-جامعة بغداد.