Theory of Structures

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Introduction:

The goal of this subject is to provide the skills for the analysis of statically determinate trusses, beams, frames, and cables. The internal forces in various members of the structure and displacements at some controlling points are discussed in the theory of structures.

Types of structural loads:

1. Live load it is consists of:

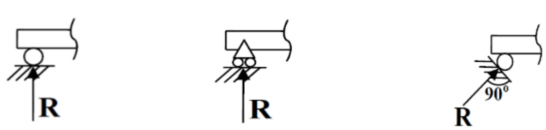
   a. Vertical live load
      - ✓ Movable load.
      - ✓ Moving load.
      - ✓ Snow load.

   b. Horizontal live load
      - ✓ Wind load.
      - ✓ Earthquake load
      - ✓ Soil pressure.
      - ✓ Hydrostatic load.
      - ✓ Thermal force.

2. Dead Load it is consist of:
   - ✓ Immovable loads that are constant in magnitude and permanently attached to the structure (for ceiling, columns …. ext).
   - ✓ Own weight of the structure.

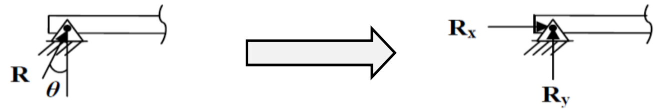
Types of supports:

1. Roller: (One unknown element)



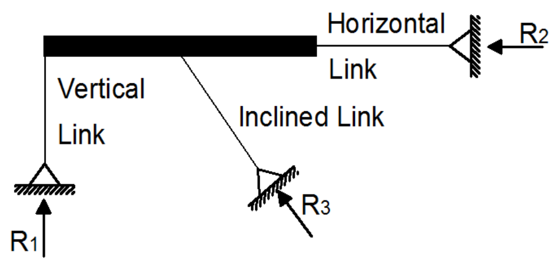
One reaction perpendicular to the moving surface, R.

2. Hinge: (Two unknown elements)

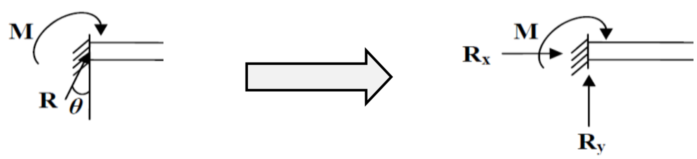


Two reactions perpendicular to the supported surface and parallel to it, Rx and Ry

3. Link: (One unknown element)
   Link is a straight element of a pin end support and no external acting force a long its length.



4. Fixed End: (Three unknown elements)



Three reactions, two force and one moment. Rx, Ry and M.

Stability and Determinacy of Structures:

With few exceptions, structures must be stable under all conditions of load; that is, they must be able to support applied loads (their own weight, anticipated live loads, wind, and so forth) without changing shape, undergoing large displacements, or collapsing.

By determinacy, we mean procedures to establish if the equations of statics alone are sufficient to permit a complete analysis of a structure. If the structure cannot be analyzed by the equations of statics, the structure is termed indeterminate. To analyze an indeterminate structure, we must supply additional equations by considering the geometry of the deflected shape.

By stability, we mean the geometric arrangement of members and supports required to produce a stable structure, that is, a structure that can resist load from any direction without undergoing either a radical change in shape or large rigid-body displacements. we consider the stability and determinacy of structures that can be treated as either a single rigid body or as several interconnected rigid bodies.

A rigid body is in equilibrium if it is either:

- ✓ At rest “velocity = 0”.
- ✓ State of constant motion “acceleration = 0”.

This require that:

- ✓ The resultant force on the body must be zero to prevent linear acceleration ΣF=0.
- ✓ The resultant moment on the body must be zero to prevent angular acceleration ΣM=0.

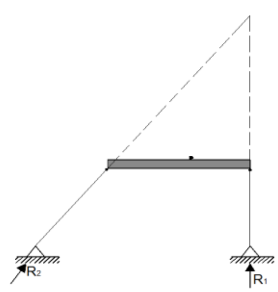
For plane truss or frame (two dimensions), these two conditions are usually designed by the three equilibrium equations:

$$\Sigma Fx = 0 \qquad \Sigma Fy = 0 \qquad \Sigma M = 0$$

The structure is said to be unstable if:

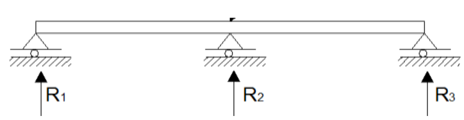
1. All reactions are concurrent (meet at one point).

$\sum M \neq 0$



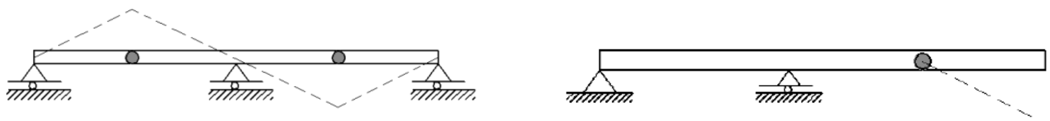
2. All reactions are parallel

$\sum M \neq 0$



3. Numbers of Unknown (reactions) < Total numbers of equilibrium equations

4. When the structure is geometrically unstable.



Stability and Determinacy of Beams:

The equations of equilibrium are:

$$\sum Fx = 0 \qquad \Sigma Fy = 0 \qquad \Sigma M = 0$$

∴ The total number of the equilibrium equations = 3 + C

$$r \begin{matrix} < \\ = \\ > \end{matrix} 3+C$$

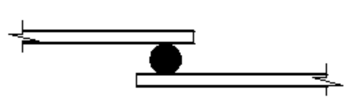
(r) = Number of reactions.

(C) = The total number of equations of conditions.

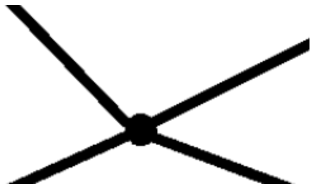
Where:

C = 1 for an internal hinge.

C = 2 for an internal roller and C = 0 for beams without internal connection.

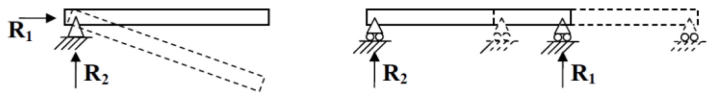


C = number of members connected at joint – 1, for interior hinge that is connecting (n) members.

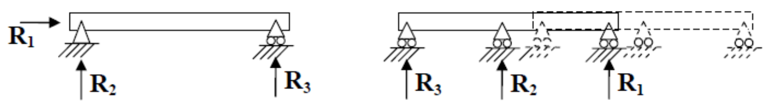


The beam is set to be:

a. Unstable if (r < 3+c)

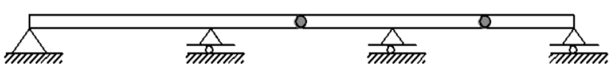


b. Stable and determinate if (r = 3 + c)



c. Stable and indeterminate if (r > 3 + c)

Example 1



Sol.

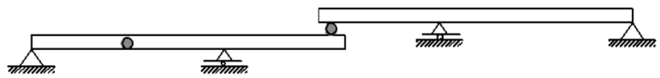
r = 5

C = 2

r = (3 + C) = 5   The beam is determinate and stable

(No parallel reactions, No concurrent reactions)

Example 2



Sol.

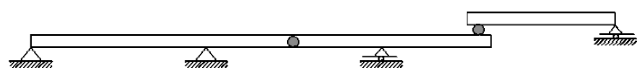
$r = 6$

$C = 3$

$r = (3 + C) = 6$ *The beam is determinate and stable.*

(No parallel reactions, No concurrent reactions)

Example 3



Sol.

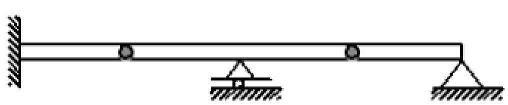
$r = 6$

$C = 3$

$r = (3 + C) = 6$

(The reactions are parallel, $\therefore$ *The beam is unstable)*

Example 4



Sol.

$r = 6$

$C = 2$

$r > (3 + C) = 5$ *The beam is indeterminate and stable*

(No parallel reactions, No concurrent reactions)

Stability and Determinacy of Trusses:

$$r + b \; \begin{matrix} > \\ = \\ < \end{matrix} \; 2J$$

Let: r = Number of reactions
b = Number of bars
J = Number of joints

2J = Number of equilibrium equations (In trusses two equilibrium Eqs. can be written at each joint $\Sigma Fx=0$, $\Sigma Fy=0$)

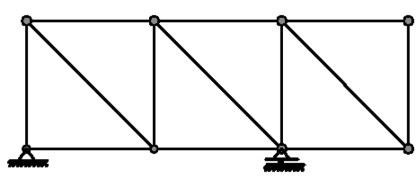
b + r = Number of unknown

If r+b < 2J (The truss is unstable).

r +b= 2J (The truss is determinate, if stable).

r+b > 2J (The truss is indeterminate, if stable).

Example 1



Sol.

r = 3

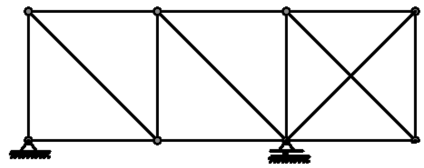
b = 13 ⇨ *2J = 16*

r + b = 16

J = 8

$\therefore$ *r + b = 2J* *The truss is determinate and stable*

Example 2



Sol.

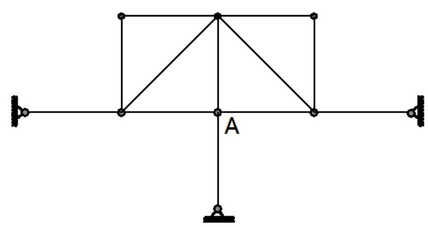
$r = 3$

$b = 14$ ⇨ $2J = 16$

$J = 8$ $r + b = 17$

$\therefore r + b > 2J$ *The truss is determinate and stable*

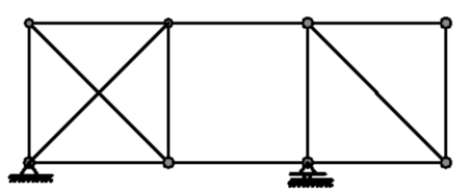
Example 3



Sol.

The truss is unstable because all the reactions intersect at point (A)

Example 4



Sol.

$r = 3$

$b = 13$ ⇨ $2J = 16$

$J = 8$ $r + b = 17$

$r + b = 2J$ But the truss is unstable ((No diagonal bar to carry shear))

Stability and Determinacy of Frames:

$$r + 3b \quad \begin{matrix} > \\ = \\ < \end{matrix} \quad 3J + C$$

Let: r = Number of reactions.
b = Number of members.
J = Number of joints.
C = The total number of equations of conditions.

3b + r = Number of unknowns. (Note: number "three" denotes to the three interior unknown force P, V and M that is generated at each members).

3J + C = Total equations of equilibrium. (Note: number "three" denotes to the equilibrium equations ($\Sigma Fy=0$, $\Sigma Fx=0$, $\Sigma M=0$) at each joints).

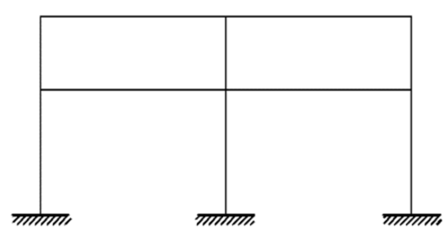
C = Number of members connected at joint – 1

If 3b+r > 3j + C (The frame is indeterminate, if stable).

3b+r = 3j + C (The frame is determinate, if stable).

3b+r < 3j + C (The frame is unstable).

__Example 1__



__Sol.__

$r = 9 \quad b = 10 \quad C = 0 \quad J = 9$

$3b + r = 3 \times 10 + 9 = 39$

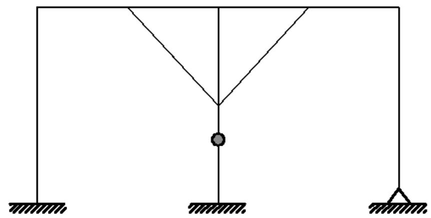
$3j + C = 3 \times 9 + 0 = 27$

$39 > 27$

$3b + r > 3j + C$ *The frame is stable and indeterminate to the 12th degree*

Example 2

Sol.



$r = 8 \quad b = 11 \quad C = 1 \quad J = 10$

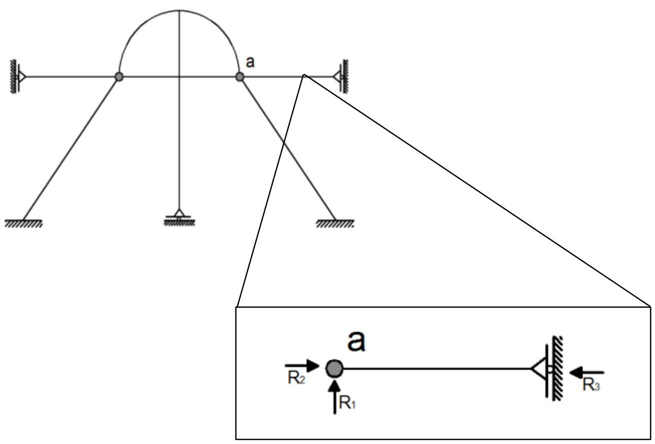
$3b + r = 3 \times 11 + 8 = 41$

$3J + C = 3 \times 10 + 1 = 31$

$3b + r > 3J + C$ *The frame is stable indeterminate to the 10th degree*

Example 3

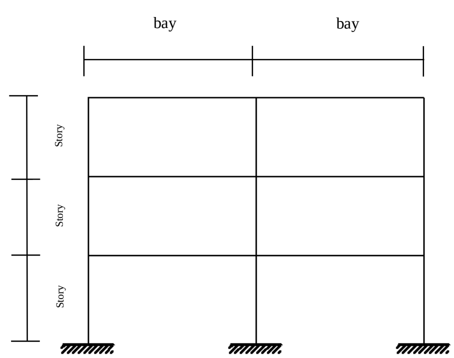
Sol.



Unstable because $\Sigma M \neq 0$

Example 4

Sol.



$3b + r = 3 \times 15 + 9 = 54$

$3j + C = 3 \times 12 + 0 = 36$

$3b + r > 3j + C$

Stable and Indeterminate to the 18th degree

Stability and Determinacy of composite structures:

$$U \;\substack{>\\=\\<}\; E$$

Let: *U = Number of unknowns.*
E = Number of equilibrium equations.

If *U > E (Indeterminate if stable).*

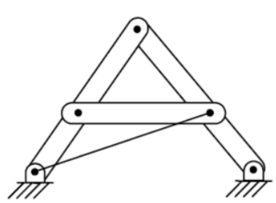
U = E (Determinate if stable).

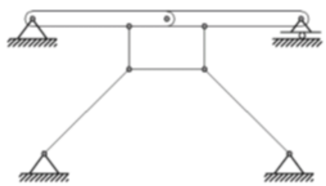
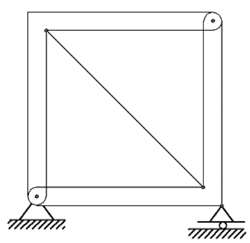
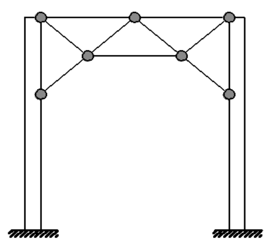
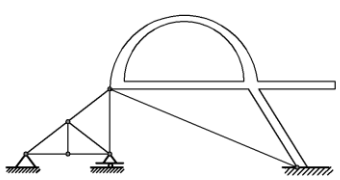
U < E (Unstable).

The degree of indeterminacy (m) can be obtained by: m = U - E

ملاحظة:

- تعتبر كل قطعة (Frame or Beam) فيها ثلاث معادلات للاتزان.
- يوجد في كل (Joint in Truss) معادلتين للاتزان.
- نقاط الاتصال للـ (Truss) مع (Beam or Frame) لاتؤخذ عند حساب معادلات الاتزان (E).

Composite Structure	U	E	$U \;\substack{>\\=\\<}\; E$	Classification
	*11*	*9*	*11 > 9*	*Stable & indeterminate* *2nd degree*

Composite Structure	U	E	U ≷ E	Classification
	*9*	*9*	*9 = 9*	*Stable & determinate*
	*8*	*6*	*8 > 6*	*Stable & Indeterminate* *2nd degree*
	*12*	*9*	*12 > 9*	*Stable & Indeterminate* *3rd degree*
	*12*	*6*	*12 > 6*	*Stable & Indeterminate* *6th degree*