8.1 Resonance in A. C. Circuits:

In this chapter we will study the effect of varying the frequency of the sinusoidal source on different type of electric circuits, which refer to *frequency response* of the circuit (voltage or current w.r.t. frequency). According to ohm's law the frequency response represents a relationship between total impedance or admittance w.r.t. frequency.

There are three primary reasons for studying frequency response:

1. The ability to design circuits that are frequency selective circuits, like radio, telephone, and television communication systems.
2. We can predict the response of the circuit to any other *non-sinusoidal inputs*, so the engineer can carry out the design in terms of frequency specification and exert control over the response.
3. We can measure the frequency response in the laboratory and from these data; we can formulate a model for the circuit or device.

A network is in resonance (or resonant) when the voltage and current at the network input terminals are inphase. So the equivalent impedance or admittance consists of *real part* only and the power factor is *unity*. In other words the resonance occurs in electrical circuit at a frequency when the impedance between the input and output of the circuit is minimum (zero) or the transfer impedance is maximum (one). The energy will be oscillating between magnetic field of inductance and electrical field of capacitance. Resonance is a condition in an RLC circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance. Resonant circuits (series or parallel) are useful for constructing filters, as their transfer functions can be highly frequency selective. They are used in many applications such as selecting the desired stations in radio and TV receivers.

Resonant Frequency (ω_o): is the frequency that makes the total impedance seen by the source is purely resistive and the corresponding total admittance purely conductive since $Y_T=1/Z_T$ (imaginary part equal to zero in both).

Bandwidth (β): is the range of frequencies in which the amplitude of the output voltage or current is equal or greater than the maximum value divided by $\sqrt{2}$. So an acceptable output voltage or current has an amplitude is at least $1/\sqrt{2}$=0.707 times the maximum amplitude that can be transmitted by the circuit.

Note: because the amplitude is reduced by $1/\sqrt{2}$, so the average power delivered to the circuit is half its maximum value and this called half power frequency (HPF).

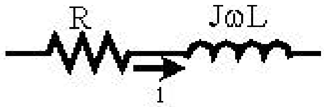
Lower & Upper Side Frequencies (ω_1 & ω_2): these are the two frequencies for which the output voltage or current amplitude is equal to $1/\sqrt{2}$ from its maximum amplitude that can be transmitted by the circuit.

Quality Factor (Q): is the amount of sharpness of the response curve of any resonant (frequency selective) circuit (the circuit response are closely ideal).

$$Q = \text{quality factor} = 2\pi \times \frac{\text{maximum energy stored}}{\text{total energy lost per period (cycle)}}$$

The Quality Factor for Some Circuits:

For series RL circuit:



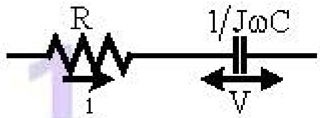
The Sinusoidal Cycle Period = T = 1/f

Maximum Energy (Power) Stored = $\frac{1}{2}LI_{max}^2$

Average Energy (Power) Stored Per Cycle = $\frac{1}{2}RI_{max}^2 \times 1/f$

$$\therefore Q = 2\pi \times \frac{\frac{1}{2}LI_{max}^2}{\frac{1}{2}RI_{max}^2 \times \frac{1}{f_o}} = \frac{2\pi f_o L}{R} = \frac{\omega_o L}{R}$$

For series RC circuit:



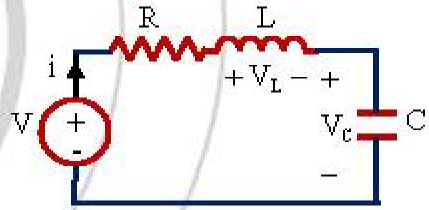
The Sinusoidal Cycle Period = T = 1/f

Maximum Energy (Power) Stored = $\frac{1}{2}CV_{max}^2 = \frac{1}{2} \times \frac{1}{\omega^2 C} I_{max}^2$

Average Energy (Power) = $\frac{1}{2}RI_{max}^2 \times 1/f$ stored per cycle

$$\therefore Q = 2\pi \times \frac{\frac{1}{2} \times \frac{1}{C\omega_o^2} I_{max}^2}{\frac{1}{2}RI_{max}^2 \times \frac{1}{f_o}} = \frac{1}{2\pi f_o CR} = \frac{1}{\omega_o CR}$$

For series RLC circuit:

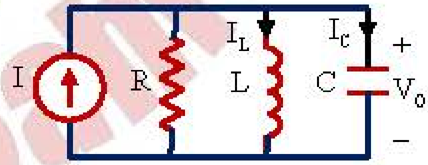


The stored energy in RLC series circuit at resonance are constant, because when the capacitive voltage was maximum then the inductive current equal to zero and vise-versa.

$$\boxed{\frac{1}{2}LI_{max}^2 = \frac{1}{2}CV_{max}^2}$$

$$Q = 2\pi \times \frac{1/2 \times LV_{max}^2 / R^2}{1/2 \times RV_{max}^2 / R^2 \times 1/f} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

For parallel RLC circuit:

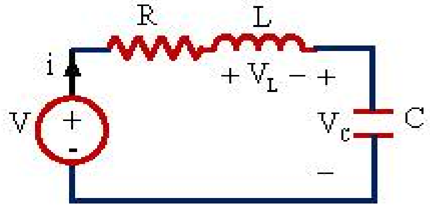


Like series circuit at resonance when the inductive current was maximum then the capacitive voltage equal to zero and vise-versa.

$$\boxed{\frac{1}{2}LI_{max}^2 = \frac{1}{2}CV_{max}^2}$$

$$Q = 2\pi \times \frac{1/2 \times CV_{max}^2}{1/2 \times V_{max}^2 / R \times 1/f} = \frac{R}{\omega_o L} = \omega_o CR$$

8.2 Resonance in Series RLC Circuit:



First we need to change the voltage source frequency:
R=Constant (will not be effected), while

$X_L = \omega L = 2\pi f L$ or $X_L \alpha f$ &

$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ or $X_c \alpha \frac{1}{f}$

$Z = R + J\left(\omega L - \frac{1}{\omega C}\right)$ & at resonance imaginary part=0

$\therefore \omega_o L = \frac{1}{\omega_o C} \Rightarrow \omega_o = \frac{1}{\sqrt{LC}} rad/s$ or $f_o = \frac{1}{2\pi\sqrt{LC}} Hz$

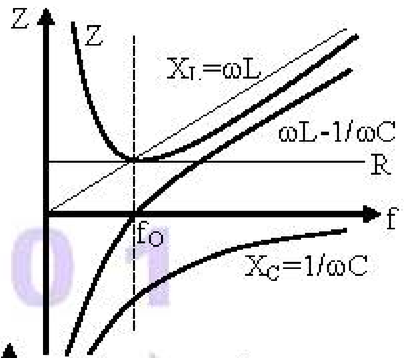
Resonant frequency $f_o = \sqrt{f_1 f_2}$ or $\omega_o = \sqrt{\omega_1 \omega_2}$

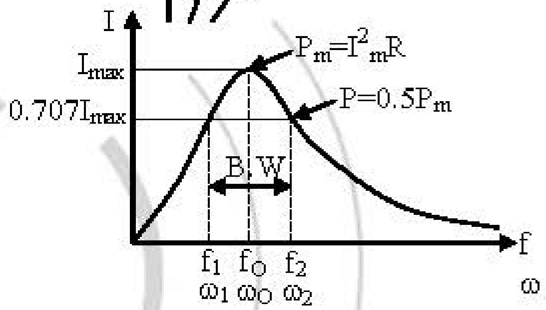
B.W=Band Width $=f_2-f_1$ or $\beta = \omega_2 - \omega_1$

Where: $\beta = 2\pi \times B.W$

$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o C R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$ at f_1 & f_2





$I = \frac{I_m}{\sqrt{2}} = \frac{V_m}{\sqrt{2}R} \quad \Rightarrow \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R$

Solve the equation for two positive values yields:

$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$ and $\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$ upper and lower side frequencies

$\therefore \beta = \omega_2 - \omega_1 = \frac{R}{L} \Rightarrow Q = \frac{\omega_o}{\beta} = \frac{f_o}{B.W}$ Quality factor

$Z = R + J\omega L\left(1 - \frac{1}{\omega^2 LC}\right) = R + J\omega L\left(1 - \left(\frac{\omega_o}{\omega}\right)^2\right)$

$Z = R\left[1 + J\frac{\omega L}{R}\left(1 - \left(\frac{\omega_o}{\omega}\right)^2\right)\right] = R\left[1 + JQ\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)\right]$ Total circuit normalized impedance

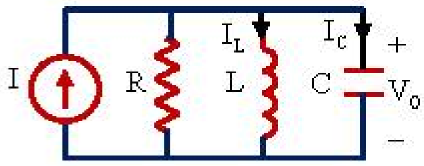
Where: ω/ω_o =normalized frequency variable.

At f_1 or f_2: $I = \frac{I_m}{\sqrt{2}}$, since $P_m = I_m^2 R \quad \therefore I^2 R = \frac{I_m^2}{2} R \quad \Rightarrow$ $P = \frac{P_m}{2}$ half power frequency

As a result $\omega_1 = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$ and $\omega_2 = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$

, or using Quality factor: $\omega_1 = \omega_o\left[-\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$ and $\omega_2 = \omega_o\left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$

8.3 Resonance in Parallel RLC Circuit:



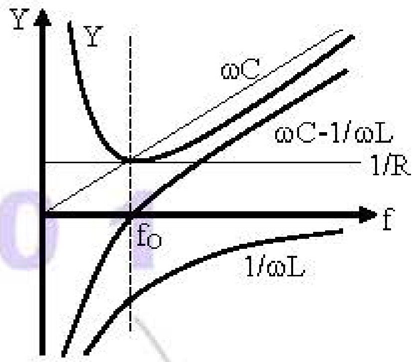
First we need to change the voltage source frequency:
R=Constant (will not be effected), while

$$X_L = \omega L = 2\pi f L \quad \text{or} \quad X_L \alpha f$$ &

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad \text{or} \quad X_c \alpha \frac{1}{f}$$

$Y = \frac{1}{R} + J\left(\omega C - \frac{1}{\omega L}\right)$ & at resonance imaginary part=0

$$\therefore \omega_0 C = \frac{1}{\omega_0 L} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

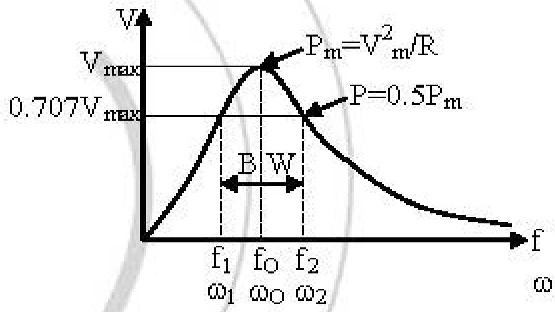


Resonant frequency $f_0 = \sqrt{f_1 f_2}$ or $\omega_0 = \sqrt{\omega_1 \omega_2}$

B.W=Band Width $=f_2-f_1$ or $\beta = \omega_2 - \omega_1$

Where: $\beta = 2\pi \times B.W$

$$Q = \omega_0 CR = \frac{R}{\omega_0 L} = R\sqrt{\frac{C}{L}}$$



$V = \frac{I}{Y} = \frac{I}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}}$ at f_1 & f_2

$$V = \frac{V_m}{\sqrt{2}} = \frac{I_m R}{\sqrt{2}} \Rightarrow \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2} = \frac{\sqrt{2}}{R}$$

Solve the equation for two positive values yields:

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$ and

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$ upper & lower side frequencies

$\therefore \beta = \omega_2 - \omega_1 = \frac{1}{RC} \Rightarrow Q = \frac{\omega_0}{\beta} = \frac{f_0}{B.W}$ Quality factor

$$Y = \frac{1}{R} + J\omega C\left(1 - \frac{1}{\omega^2 LC}\right) = \frac{1}{R} + J\omega C\left(1 - \left(\frac{\omega_0}{\omega}\right)^2\right)$$

$Y = \frac{1}{R}\left[1 + J\omega CR\left(1 - \left(\frac{\omega_0}{\omega}\right)^2\right)\right] = \frac{1}{R}\left[1 + JQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$ Total circuit normalized admittance

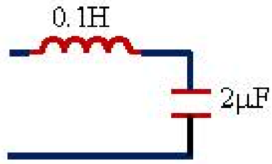
Where: ω/ω_0 =normalized frequency variable.

At f_1 or f_2: $V = \frac{V_m}{\sqrt{2}}$, since $P_m = \frac{V_m^2}{R}$ $\therefore \frac{V^2}{R} = \frac{V_m^2}{2R} \Rightarrow P = \frac{P_m}{2}$ half power frequency

As a result $\omega_1 = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2}$ and $\omega_2 = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2}$

, or using Quality factor: $\omega_1 = \omega_0\left[-\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$ and $\omega_2 = \omega_0\left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]$

Example: For the circuit shown, find the resonant frequency.



Solution:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1\times 2\times 10^{-6}}} = 356\text{Hz}$$

Example: A series circuit is desired that will resonate at 2MHz. if (10μH) inductor is available, what should be the value of capacitor used to form the circuit?

Solution:

$$C = \frac{1}{(2\pi f_0)^2 \times L} = \frac{1}{(2\pi\times 2\times 10^6)^2\times 10\times 10^{-6}} = 633\text{PF}$$

Example: AM band radio station has a center frequency of 1010KHz and B.W of 15KHz. What is the value of Q for the radio transmitter circuit?

Solution:

$$Q = \frac{f_0}{B.W} = \frac{1010\times 10^3}{15\times 10^3} = 67.3$$

Example: For a series circuit with L/C=400 and f_0=2MHz:

1. Find the values of L & C.
2. What is the value of "R" needed so that Q=20.
3. Calculate the upper and lower cutoff frequencies.

Solution:

1. $f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow 2\times 10^6 = \frac{1}{2\pi\sqrt{400C\times C}} \Rightarrow C=3.979\text{nF}$ and $L=400\times 3.979\times 10^{-9}=1.59\mu H$

2. $R = \frac{1}{Q}\sqrt{\frac{L}{C}} = \frac{1}{20}\sqrt{\frac{1.59\times 10^{-6}}{3.979\times 10^{-9}}} = 0.9994\Omega$

3. $B.W = \frac{f_0}{Q} = \frac{2\times 10^6}{20} = 10^5\text{Hz}$

$$\left.\begin{aligned} f_1 &= f_0 - \frac{B.W}{2} = 2\times 10^6 - \frac{10^5}{2} = 1.95\text{MHz} \\ f_2 &= f_0 + \frac{B.W}{2} = 2\times 10^6 + \frac{10^5}{2} = 2.05\text{MHz} \end{aligned}\right\} \text{If } Q \geq 10$$

Example: A series resonant circuit has B.W=100Hz and f_0=2000Hz:

1. Find the value of Q for the circuit.
2. What is the value of "L" & "C" if R=10Ω.
3. Calculate the upper and lower cutoff frequencies.

Solution:

1. $Q = \frac{f_0}{B.W} = \frac{2000}{100} = 20$

2. $L = \frac{QR}{\omega_0} = \frac{20\times 10}{2\pi\times 2000} = 15.9\text{mH}$ and $C = \frac{1}{Q\omega_0 R} = \frac{1}{20\times 2\pi\times 2000\times 10} = 0.398\mu F$

3. $$\left.\begin{aligned} f_1 &= f_0 - \frac{B.W}{2} = 2000 - \frac{100}{2} = 1.95\text{KHz} \\ f_2 &= f_0 + \frac{B.W}{2} = 2000 + \frac{100}{2} = 2.05\text{KHz} \end{aligned}\right\} \text{If } Q \geq 10$$

Example: Design a series resonant circuit with input voltage 2V, B.W=400Hz, and I_m=250mA at resonance, with f_O=10KHz find:

1. The value of Q for the circuit.
2. The value of "L" & "C".
3. The upper and lower cutoff frequencies.

Solution:

1. $Q=\frac{f_o}{B.W}=\frac{10000}{400}=25$ 2. at resonance $R=\frac{V}{I_m}=\frac{2}{250\times10^{-3}}=8\Omega$

$$L=\frac{QR}{\omega_o}=\frac{25\times8}{2\pi\times10000}=3.183\text{mH} \quad \text{and} \quad C=\frac{1}{Q\omega_o R}=\frac{1}{25\times2\pi\times10000\times8}=0.0796\mu\text{F}$$

3. $$\left.\begin{aligned} f_1&=f_o-\frac{B.W}{2}=10000-\frac{400}{2}=9.8\text{KHz}\\ f_2&=f_o+\frac{B.W}{2}=10000+\frac{400}{2}=10.2\text{KHz}\end{aligned}\right\} \text{If } Q\geq10$$

Example: Series RLC circuit with B.W=250Hz, and center frequency 750Hz. Use 100nF capacitor, compute Q, L, R, ω_1, & ω_2.

Solution:

$$Q=\frac{f_o}{B.W}=\frac{750}{250}=3 \qquad L=\frac{1}{(2\pi f_o)^2\times C}=\frac{1}{(2\pi\times750)^2\times100\times10^{-9}}=450\text{mH}$$

$$R=\frac{\omega_o L}{Q}=\frac{2\pi\times750\times450\times10^{-3}}{3}=707\Omega \qquad \text{or} \qquad B.W=\frac{R}{2\pi L}$$

$$\omega_1=2\pi\left[-\frac{B.W}{2}+\sqrt{\left(\frac{B.W}{2}\right)^2+f_o^2}\right]=3992\text{rad/s} \quad \text{or} \quad \omega_1=\omega_o\left[-\frac{1}{2Q}+\sqrt{\left(\frac{1}{2Q}\right)^2+1}\right]$$

$$\omega_2=2\pi\left[\frac{B.W}{2}+\sqrt{\left(\frac{B.W}{2}\right)^2+f_o^2}\right]=5562.8\text{rad/s} \quad \text{or} \quad \omega_2=\omega_o\left[\frac{1}{2Q}+\sqrt{\left(\frac{1}{2Q}\right)^2+1}\right]$$

Example: Design the component values for series RLC circuit with center frequency 4KHz and Q=5, use 500nF capacitor.

Solution:

$$R=\frac{1}{Q\omega_o C}=\frac{1}{5\times2\pi\times4\times10^3\times500\times10^{-9}}=15.92\Omega \qquad L=\frac{QR}{\omega_o}=\frac{5\times15.92}{2\pi\times4\times10^3}=3.17\text{mH}$$

$$B.W=\frac{f_o}{Q}=\frac{4\times10^3}{5}=800\text{Hz} \qquad \text{or} \qquad B.W=\frac{\omega_2-\omega_1}{2\pi}$$

$$\omega_1=\omega_o\left[-\frac{1}{2Q}+\sqrt{\left(\frac{1}{2Q}\right)^2+1}\right]=2\pi\times4\times10^3\left[-\frac{1}{2\times5}+\sqrt{\left(\frac{1}{2\times5}\right)^2+1}\right]=22744.82\text{rad/s}$$

$$\omega_2=\omega_o\left[\frac{1}{2Q}+\sqrt{\left(\frac{1}{2Q}\right)^2+1}\right]=2\pi\times4\times10^3\left[\frac{1}{2\times5}+\sqrt{\left(\frac{1}{2\times5}\right)^2+1}\right]=27771.37\text{rad/s}$$

Example: Design a component values for series RLC circuit with center frequency 20KHz and Q=5, use 100Ω resistor.

Solution:

$$L=\frac{QR}{\omega_0}=\frac{5\times100}{2\pi\times20\times10^3}=3.98\text{mH} \qquad C=\frac{1}{Q\omega_0 R}=\frac{1}{5\times2\pi\times20\times10^3\times100}=15.92\text{nF}$$

$$B.W=\frac{f_0}{Q}=\frac{20\times10^3}{5}=4\text{KHz} \qquad \text{or} \qquad B.W=f_2-f_1=\frac{R}{2\pi L}$$

$$f_1=f_0\left[-\frac{1}{2Q}+\sqrt{\left(\frac{1}{2Q}\right)^2+1}\right]=20\times10^3\left[-\frac{1}{2\times5}+\sqrt{\left(\frac{1}{2\times5}\right)^2+1}\right]=18.1\text{KHz}$$

$$f_2=f_0\left[\frac{1}{2Q}+\sqrt{\left(\frac{1}{2Q}\right)^2+1}\right]=20\times10^3\left[\frac{1}{2\times5}+\sqrt{\left(\frac{1}{2\times5}\right)^2+1}\right]=22.1\text{KHz}$$

Example: Series-resonant circuit has resonant frequency 5MHz and lower half-power frequency 4.5MHz. Use 0.01μF capacitor to find: B.W, Q, L, and R.

Solution:

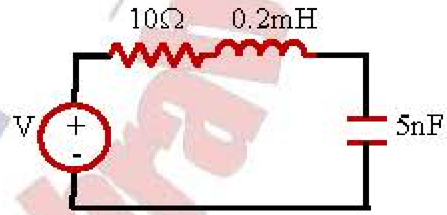
$$f_2=\frac{f_0^2}{f_1}=\frac{(5\times10^6)^2}{4.5\times10^6}=5.56\text{MHz} \qquad B.W=f_2-f_1=(5.56-4.5)\times10^6=1.06\text{MHz}$$

$$Q=\frac{f_0}{B.W}=\frac{5\times10^6}{1.06\times10^6}=4.74$$

$$L=\frac{1}{\omega_0^2 C}=\frac{1}{(2\pi\times5\times10^6)^2\times0.01\times10^{-6}}=0.101\mu\text{H}$$

$$R=\frac{\omega_0 L}{Q}=\frac{2\pi\times5\times10^6\times0.101\times10^{-6}}{4.74}=0.67\Omega$$

Example: For the series RLC circuit shown, calculate: resonant frequency, Q, B.W, f_1, and f_2 (half-power frequencies).



Solution:

$$f_0=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{0.2\times10^{-3}\times5\times10^{-9}}}=0.16\text{MHz}$$

$$Q=\frac{\omega_0 L}{R}=\frac{2\pi\times0.16\times10^6\times0.2\times10^{-3}}{10}=20 \qquad \text{or} \qquad Q=\frac{1}{\omega_0 CR}$$

$$B.W=\frac{f_0}{Q}=\frac{0.16\times10^6}{20}=7.96\text{KHz}\cong8\text{KHz}$$

$$\omega_1=-\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}=0.975\text{Mrad/s} \Rightarrow f_1=\frac{\omega_1}{2\pi}=0.155\text{MHz}$$

$$\omega_2=\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}=1.025\text{Mrad/s} \Rightarrow f_2=\frac{\omega_2}{2\pi}=0.163\text{MHz}$$

Example: If $\left.\frac{V_C}{V_S}\right|_{f_o} = 20$, f_O=20KHz and C=0.001μF find: 1. "L" & "R". 2. V_C, if V_S=1V, 2KHz voltage source. 3. V_L for the condition in "2".

Solution:

1. $L = \frac{1}{\omega_O^2 \times C} = \frac{1}{(2\pi f_O)^2 \times C} = \frac{1}{(2\pi \times 20\times 10^3)^2 \times 0.001\times 10^{-6}} = 63.4\text{mH}$

$\left.V_C\right|_{f_o} = I_O X_C = \frac{\left.V_S\right|_{f_o}}{R_T \omega_O C}$

$\therefore \left.\frac{V_C}{V_S}\right|_{f_o} = 20 = \frac{1}{R_T \times 2\pi \times 20\times 10^3 \times 0.001\times 10^{-6}} \Rightarrow R_T = 398\Omega$

2. $X_c = \frac{1}{2\pi \times 2\times 10^3 \times 0.001\times 10^{-6}} = 79.62\text{K}\Omega$

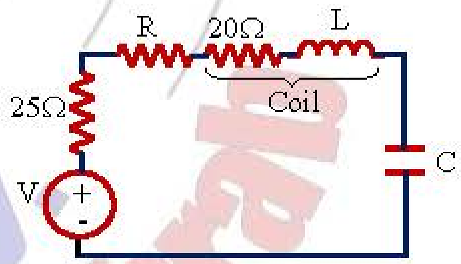
$X_L = 2\pi \times 2\times 10^3 \times 63.4\times 10^{-3} = 796.3\Omega$

$V_c = 1\times \left|\frac{-JX_c}{R_T + J(X_L - X_c)}\right| = \left|\frac{-J79.62\times 10^3}{398 + J(796.3 - 79.62\times 10^3)}\right| = 1.01\text{V}$

3. $V_L = 1\times \left|\frac{JX_L}{R_T + J(X_L - X_c)}\right| = \left|\frac{J796.3}{398 + J(796.3 - 79.62\times 10^3)}\right| = 0.0101\text{V}$

Example: A circuit is operate at 100KHz with B.W of 8KHz. If the magnitude of voltage source is 1V, with internal resistance (25Ω), and the coil available has a resistance (20Ω) and (Q=50). Find the values of R, L, and C, which make up the circuit. Also find current magnitude and voltage across capacitor, inductor, and resistor.

Solution:



$Q_S = \frac{f_o}{B.W} = \frac{100\times 10^3}{8\times 10^3} = 12.5$

For the coil:

$\omega_o L = Q_S \times R_c = 50\times 20 = 1000\Omega$

$Q_S = \frac{\omega_o L}{R_S + R + R_c}$ or $12.5 = \frac{1000}{25 + R + 20}$

$\therefore R = 35\Omega$

$L = \frac{\omega_o L}{2\pi f_o} = \frac{1000}{2\pi \times 100\times 10^3} = 1.59\text{mH}$ at resonance $X_L = X_C$

$C = \frac{1}{2\pi f_o \omega_o L} = \frac{1}{2\pi \times 100\times 10^3 \times 1000} = 1.59\text{nF}$

$I = \frac{V}{R_T} = \frac{1}{25 + 35 + 20} = 12.5\text{mA}$

$V_R = IR = 12.5\times 10^{-3} \times 35 = 0.4375\text{V}$

$V_c = IX_c = 12.5\times 10^{-3} \times 1000 = 12.5\text{V} = V_L$

$V_{R_c} = IR_c = 12.5\times 10^{-3} \times 20 = 0.25\text{V}$

$V_{Coil} = \sqrt{V_L^2 + V_{R_c}^2} = \sqrt{(12.5)^2 + (0.25)^2} = 12.502\text{V}$

Example: Parallel-resonant circuit with resonant frequency 10KHz, B.W=400Hz, input current 20mA, and V_m=50V at resonance, find: Q, R, L, C, f_1 and f_2.

Solution:

$$Q = \frac{f_o}{B.W} = \frac{10\times10^3}{400} = 25 \qquad R = \frac{V_m}{I_m} = \frac{50}{20\times10^{-3}} = 2.5K\Omega$$

$$L = \frac{R}{Q\omega_o} = \frac{2.5\times10^3}{25\times2\pi\times10\times10^3} = 1.59mH \qquad C = \frac{Q}{\omega_o R} = \frac{25}{2\pi\times10\times10^3\times2.5\times10^3} = 0.159\mu F$$

$$\left.\begin{aligned} f_1 &= f_o - \frac{B.W}{2} = 10\times10^3 - \frac{400}{2} = 9.8KHz \\ f_2 &= f_o + \frac{B.W}{2} = 10\times10^3 + \frac{400}{2} = 10.2KHz \end{aligned}\right\} \text{If } Q \ge 10$$

Example: If the resonant frequency of RLC parallel circuit is 100KHz, and B.W=1KHz, use 60nF capacitor, calculate Q, R, L, ω_1, & ω_2.

Solution:

$$Q = \frac{f_o}{B.W} = \frac{100\times10^3}{1\times10^3} = 100 \qquad R = \frac{Q}{\omega_o C} = \frac{100}{2\pi\times100\times10^3\times60\times10^{-9}} = 2652.6\Omega$$

$$L = \frac{R}{Q\omega_o} = \frac{2652.6}{100\times2\pi\times100\times10^3} = 42.22\mu H$$

$$\omega_1 = \omega_o\left[-\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right] = 2\pi\times100\times10^3\left[-\frac{1}{200} + \sqrt{\left(\frac{1}{200}\right)^2 + 1}\right] = 625.2Krad/s = \omega_o - \frac{\beta}{2}$$

$$\omega_2 = \omega_o\left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right] = 2\pi\times100\times10^3\left[\frac{1}{200} + \sqrt{\left(\frac{1}{200}\right)^2 + 1}\right] = 631.5Krad/s = \omega_o + \frac{\beta}{2}$$

Note: $\omega_O = 2\pi f_O$ and $\beta = 2\pi B.W$

Example: Parallel RLC circuit with f_O=10MHz, B.W=100KHz, and R=100KΩ, calculate Q, L, C, ω_1, & ω_2.

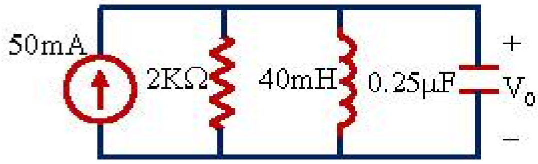
Solution:

$$Q = \frac{f_o}{B.W} = \frac{10\times10^6}{100\times10^3} = 100 \qquad L = \frac{R}{Q\omega_o} = \frac{100\times10^3}{100\times2\pi\times10\times10^6} = 15.9\mu H$$

$$C = \frac{Q}{\omega_o R} = \frac{100}{2\pi\times10\times10^6\times100\times10^3} = 15.9PF$$

$$\omega_1 = \omega_o\left[-\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right] = 2\pi\times10\times10^6\left[-\frac{1}{200} + \sqrt{\left(\frac{1}{200}\right)^2 + 1}\right] = 62.52Mrad/s = \omega_o - \frac{\beta}{2}$$

$$\omega_2 = \omega_o\left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right] = 2\pi\times10\times10^6\left[\frac{1}{200} + \sqrt{\left(\frac{1}{200}\right)^2 + 1}\right] = 63.15Mrad/s = \omega_o + \frac{\beta}{2}$$



Example: For the parallel RLC circuit shown, find:

1. ω_O, Q, ω_1, ω_2, and $|V_o|$ at ω_O, ω_1, and ω_2.
2. Value of "R" for B.W of 80Hz.
3. Value of "Q" in "2".

Solution:

1. $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{40\times10^{-3}\times0.25\times10^{-6}}} = 10^4\,\text{rad/s} \quad \Rightarrow \quad f_0 = \dfrac{\omega_0}{2\pi} = 1591.55\text{Hz}$

$$Q = \omega_0 CR = 10^4\times0.25\times10^{-6}\times2\times10^3 = 5$$

$$B.W = \frac{f_0}{Q} = \frac{1591.55}{5} = 318.31\text{Hz} \quad \text{or} \quad B.W = \frac{1}{2\pi RC}$$

$$\omega_1 = \omega_0\left[-\frac{1}{2Q}+\sqrt{\left(\frac{1}{2Q}\right)^2+1}\right] = 9049.88\text{rad/s} = 2\pi\times\left[-\frac{B.W}{2}+\sqrt{\left(\frac{B.W}{2}\right)^2+f_0^2}\right]$$

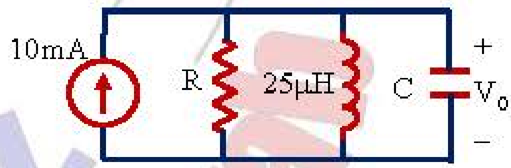
$$\omega_2 = \omega_0\left[\frac{1}{2Q}+\sqrt{\left(\frac{1}{2Q}\right)^2+1}\right] = 11049.88\text{rad/s} = 2\pi\times\left[\frac{B.W}{2}+\sqrt{\left(\frac{B.W}{2}\right)^2+f_0^2}\right]$$

$$V_0(\omega_0) = I_m R = 50\times10^{-3}\times2000 = 100\text{V}$$

$$V_0(\omega_1) = V_0(\omega_2) = 0.707V_0(\omega_0) = 70.7\text{V}$$

2. $R = \dfrac{1}{2\pi B.W C} = \dfrac{1}{2\pi\times80\times0.25\times10^{-6}} = 7957.7\Omega$

3. $Q = \dfrac{f_0}{B.W} = \dfrac{1591.55}{80} = 20$



Example: For the parallel RLC circuit shown, if f_O=2MHz and Q=10, calculate: C, f_1, f_2, B.W, V_O, I_C and I_L at resonance.

Solution:

$$R = Q\omega_0 L = 10\times2\pi\times2\times10^6\times25\times10^{-6} = 3140\Omega$$

$$C = \frac{Q}{\omega_0 R} = \frac{10}{2\pi\times2\times10^6\times3140} = 253\text{PF}$$

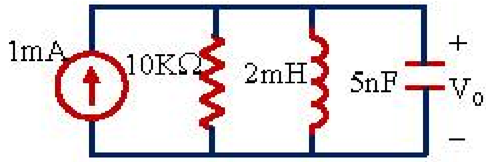
$$B.W = \frac{f_0}{Q} = \frac{2\times10^6}{10} = 0.2\text{MHz}$$

$$\left.\begin{aligned} f_1 &= f_0 - \frac{B.W}{2} = 2\times10^6 - \frac{0.2\times10^6}{2} = 1.9\text{MHz} \\ f_2 &= f_0 + \frac{B.W}{2} = 2\times10^6 + \frac{0.2\times10^6}{2} = 2.1\text{MHz} \end{aligned}\right\} \text{If } Q \ge 10$$

$V_0(\omega_0) = I_m R = 10\times10^{-3}\times3140 = 31.4\text{V}$ output voltage at resonance

$I_C = V_0(\omega_0)Y_C = 31.4\times2\pi\times2\times10^6\times253\times10^{-12} = 0.1\text{A} = I_L$ (phase difference 180^O)

Example: For the circuit shown, find at resonance: f_0, Q, B.W, output voltage, and the current in each element.



Solution:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times10^{-3}\times5\times10^{-9}}} = 3.16\times10^5\,\text{rad/s} \quad\Rightarrow\quad f_0 = \frac{\omega_0}{2\pi} = 50.3\,\text{KHz}$$

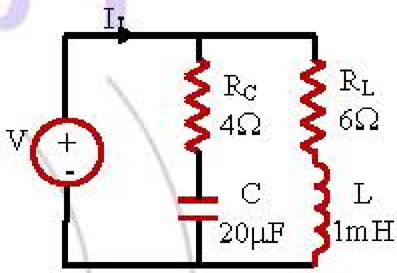
$$Q = \omega_0 CR = 3.16\times10^5\times5\times10^{-9}\times10\times10^3 = 15.8$$

$$B.W = \frac{f_0}{Q} = \frac{50.3\times10^3}{15.8} = 3.19\,\text{KHz} \quad \text{or} \quad B.W = \frac{1}{2\pi RC}$$

$$V_0(\omega_0) = I_m R = 1\times10^{-3}\times10^4 = 10\,V \quad \text{output voltage at resonance}$$

$$I_c = V_0(\omega_0)Y_c = 10\times2\pi\times50.3\times10^3\times5\times10^{-9} = 15.8\,mA = I_L$$ (phase difference 180^O)

Example: For the circuit shown: 1. Estimate ω_O w.r.t. circuit elements. 2. Find that resonance frequency.



Solution:

$$Y = \frac{1}{R_c - JX_c} + \frac{1}{R_L + JX_L}$$

$$Y = \left(\frac{R_c}{R_c^2 + X_c^2} + \frac{R_L}{R_L^2 + X_L^2}\right) + J\left(\frac{X_c}{R_c^2 + X_c^2} - \frac{X_L}{R_L^2 + X_L^2}\right)$$

Since at resonance imaginary part of total admittance=0

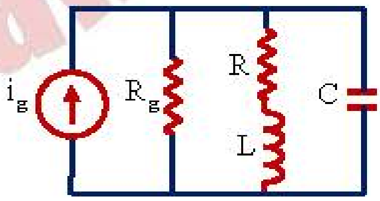
$$\therefore \frac{X_c}{R_c^2 + X_c^2} = \frac{X_L}{R_L^2 + X_L^2} \Rightarrow X_c(R_L^2 + X_L^2) = X_L(R_c^2 + X_c^2)$$

$$\frac{1}{\omega_0 C}(R_L^2 + \omega_0^2 L^2) = \omega_0 L\left(R_c^2 + \frac{1}{\omega_0^2 C^2}\right) \Rightarrow R_L^2 + \omega_0^2 L^2 - \omega_0^2 LCR_c^2 - \frac{L}{C} = 0$$

$$\boxed{\therefore \omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_c^2 - L/C}}}$$ where: $\omega_0 = 2\pi f_0$ for real frequency $\boxed{R_L^2 > L/C \,\&\, R_c^2 > L/C}$

$$\therefore \omega_0 = \frac{1}{\sqrt{10^{-3}\times20\times10^{-6}}}\sqrt{\frac{6^2 - 10^{-3}/20\times10^{-6}}{4^2 - 10^{-3}/20\times10^{-6}}} = 4537.42\,\text{rad/sec or } f_O = \omega_0/2\pi = 722\,\text{Hz}$$

Example: For the circuit shown, estimate ω_O w.r.t. circuit elements.



Solution:

$$Y = \frac{1}{R_g} + \frac{1}{R + J\omega L} + J\omega C$$

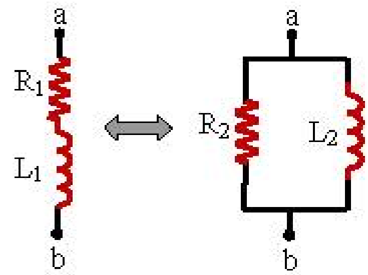
$$Y = \left(\frac{1}{R_g} + \frac{R}{R^2 + (\omega L)^2}\right) + J\left(\omega C - \frac{\omega L}{R^2 + (\omega L)^2}\right) \quad \text{at resonance imaginary part of (Y)=0}$$

$$\therefore \omega_0 C = \frac{\omega_0 L}{R^2 + (\omega_0 L)^2} \Rightarrow \omega_0^2 = \frac{1}{LC} - \left(\frac{R}{L}\right)^2 \quad \text{or} \quad \boxed{\omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}}$$

8.4 More on Serial and Parallel Resonant Circuits:

a. RL Serial to Parallel:

$$\left.\begin{aligned} R_2 &= R_1\left(1+Q_S^2\right) \\ L_2 &= L_1\left(1+\frac{1}{Q_S^2}\right) \end{aligned}\right\} Q_S = \frac{\omega_0 L_1}{R_1}$$

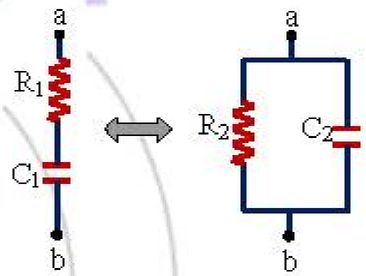


b. RL Parallel to Serial:

$$\left.\begin{aligned} R_1 &= R_2\left(\frac{1}{1+Q_P^2}\right) \\ L_1 &= L_2\left(\frac{Q_P^2}{1+Q_P^2}\right) \end{aligned}\right\} Q_P = \frac{R_2}{\omega_0 L_2}$$

c. RC Serial to Parallel:

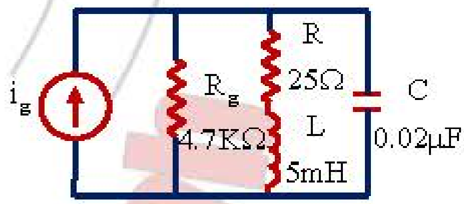
$$\left.\begin{aligned} R_2 &= R_1\left(1+Q_S^2\right) \\ C_2 &= C_1\left(\frac{Q_S^2}{1+Q_S^2}\right) \end{aligned}\right\} Q_S = \frac{1}{\omega_0 R_1 C_1}$$



d. RC Parallel to Serial:

$$\left.\begin{aligned} R_1 &= R_2\left(\frac{1}{1+Q_P^2}\right) \\ C_1 &= C_2\left(1+\frac{1}{Q_P^2}\right) \end{aligned}\right\} Q_P = \omega_0 R_2 C_2$$

Example: For the circuit shown, find the quality factor and Bandwidth.



Solution:

$$\omega_0 = \sqrt{\frac{1}{LC}-\left(\frac{R}{L}\right)^2} = \sqrt{\frac{1}{5\times10^{-3}\times0.02\times10^{-6}}-\left(\frac{25}{5\times10^{-3}}\right)^2}$$

$$=100\text{Krad/s} \quad \text{or} \quad f_0 = \frac{\omega_0}{2\pi} = 15.9\text{KHz}$$

$$Q_S = \frac{\omega_0 L}{R} = \frac{100\times10^3\times5\times10^{-3}}{25} = 20$$

$$R_P = R\left(1+Q_S^2\right) = 25\left(1+(20)^2\right) = 10025\Omega$$

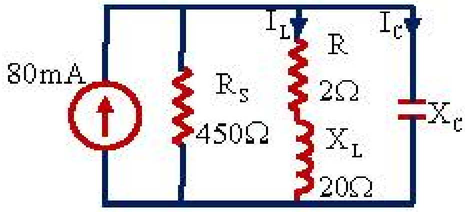
$$R_T = \frac{R_g\times R_P}{R_g+R_P} = \frac{4.7\times10^3\times10025}{4.7\times10^3+10025} = 3.2\text{K}\Omega$$

$$L_P = L\left(1+\frac{1}{Q_S^2}\right) = 5\times10^{-3}\left(1+\frac{1}{20^2}\right) = 5.0125\text{mH}$$

$$Q_P = \frac{R_T}{\omega_0 L_P} = \frac{3.2\times10^3}{100\times10^3\times5.0125\times10^{-3}} = 6.38 = \omega_0 R_T C$$

$$B.W = \frac{f_0}{Q_P} = \frac{15.9\times10^3}{6.38} = 2.49\text{KHz}$$

Example: For the circuit shown, find:



1. The value of X_C at resonance.
2. The total circuit impedance at resonance.
3. The currents I_L & I_C at resonance.
4. The value of "L" & "C" at resonance frequency 20KHz.
5. Parallel quality factor and Bandwidth.

Solution:

1. $$Y = \frac{1}{R_S} + \frac{1}{R + J\omega L} + J\omega C = \left(\frac{1}{R_S} + \frac{R}{R^2 + (\omega L)^2}\right) + J\left(\omega C - \frac{\omega L}{R^2 + (\omega L)^2}\right)$$

At resonance the imaginary part of (Y)=0

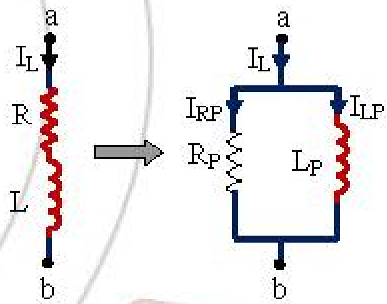
$$\therefore \omega_0 C = \frac{\omega_0 L}{R^2 + (\omega_0 L)^2} = \frac{X_L}{R^2 + X_L^2} = \frac{20}{2^2 + 20^2} = 0.0495℧$$

$$\boxed{X_C = \frac{1}{\omega_0 C} = 20.2\Omega}$$

2. $$Y = \frac{1}{450} + \frac{2}{2^2 + 20^2} = 0.00717℧$$ Just real part of (Y) or real value at resonance

$$\boxed{Z = \frac{1}{Y} = 139.42\Omega}$$

3. $$Q_S = \frac{\omega_0 L}{R} = \frac{X_L}{R} = \frac{20}{2} = 10$$



$$R_P = R\left(1 + Q_S^2\right) = 2(1 + (10)^2) = 202\Omega$$

$$R_T = \frac{R_S \times R_P}{R_S + R_P} = \frac{450 \times 202}{450 + 202} = 139.4\Omega$$

$$V_0(\omega_0) = I_m R_T = 80 \times 10^{-3} \times 139.4 = 11.15V$$

$$\boxed{I_C = V_0(\omega_0) Y_C} = 11.15 \times 0.0495 = \boxed{0.552A = I_{LP}}$$ (phase difference 180°)

$$I_{RP} = \frac{V_0}{R_P} = \frac{11.15}{202} = 0.0552A$$

$$\boxed{I_L = \sqrt{(I_{RP})^2 + (I_{LP})^2}} = \sqrt{(0.0552)^2 + (0.552)^2} = \boxed{0.555A}$$

4. $$\boxed{L = \frac{Q_S R}{\omega_0}} = \frac{10 \times 2}{2\pi \times 20 \times 10^3} = \boxed{0.159mH}$$

$$L_P = L\left(1 + \frac{1}{Q_S^2}\right) = 0.159 \times 10^{-3}\left(1 + \frac{1}{10^2}\right) = 0.16mH$$

$$\boxed{C = \frac{1}{\omega_0^2 L_P}} = \frac{1}{(2\pi \times 20 \times 10^3)^2 \times 0.16 \times 10^{-3}} = \boxed{0.396\mu F}$$

5. $$\boxed{Q_P = \frac{R_T}{\omega_0 L_P}} = \frac{139.4}{2\pi \times 20 \times 10^3 \times 0.16 \times 10^{-3}} = \boxed{6.94 = \omega_0 R_T C}$$

$$\boxed{B.W = \frac{f_0}{Q_P}} = \frac{20 \times 10^3}{6.94} = \boxed{2.9KHz}$$