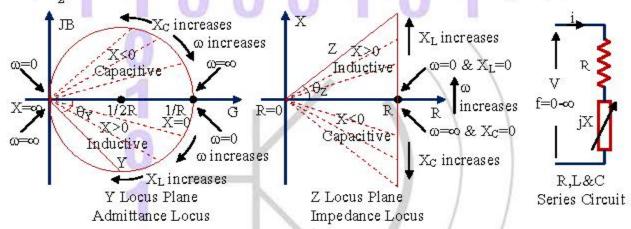
## 7.1 Admittance and Current Locus:

Electric circuit analysis could be implemented by using engineering locus shapes for circle area, where: I=VY ohm's low and constant voltage "V", so the admittance locus "Y" represent the change in current "I" value with respect to change in element value. So Locus curves are vector diagrams where only the tip of the vector is shown dependent on some parameter ( $\omega$ , R, L & C).

Admittance "Y": It's reciprocal of impedance "Z" or vice versa (Z = 1/Y or Y =1/Z,), that is a complex number consists of real and imaginary parts, its unit is mho (υ).

For series circuit:

Z=R+j(X<sub>1</sub>-X<sub>C</sub>) consists of Cartesian coordinates: R=real part and X=imaginary part.  $Y=G+jB=\frac{1}{2}$  consists of Cartesian coordinates G=real part and B=imaginary part.



Hint: 
$$X_L = 2\pi f L \implies X_L \propto f$$
 and  $X_C = \frac{1}{2\pi f C} \implies X_C \propto \frac{1}{f}$ 

$$\therefore f = 0 \implies X_L = 0 \& X_C = \infty \text{ and } \therefore f = \infty \implies X_L = \infty \& X_C = 0$$

Where the admittance locus is a circle area as shown above depending on the following equations:

$$Z = R + jX = \frac{1}{Y} = \frac{1}{G + jB}$$
 ...(1)

And from real part (R is constant) of equation (1) and by using denominator conjugate to eliminate it's complex vector we find:

$$R = \frac{G}{G^2 + B^2}$$
 or  $G^2 - \frac{G}{R} + B^2 = 0$  ...(2)  
By added 1/4R<sup>2</sup> to both side of equation (2) to simplify it to:

$$\left(G - \frac{1}{2R}\right)^2 + B^2 = \left(\frac{1}{2R}\right)^2 \qquad ...(3)$$

By comparison between equation (3) and the general circle equation  $(x-h)^2+(y-k)^2=r^2$  we find that the admittance locus is a circle with (1/2R,0) center and 1/2R radius as shown above. Each point in the impedance locus is represented by another point in the admittance locus. By using Ohm's law with constant source voltage we can compute the complex value of total current supplied from the source to the total load.

Note: Current loci is the same as admittance loci multiplied by the applied voltage "V", so there are three cases: (V=1) current & admittance locus are the same, (V>1) current>admittance locus, & (V<1) current<admittance locus.

# 7.2 Admittance Locus for R&L Series Circuit with Variable "L":

Fixing the resistance and change the reactance will give straight line parallel to the imaginary axis in the Z-plane (Impedance Locus) while it will be half lower circle in the Y-plane (Admittance Locus). Also it will be half lower circle in the current Locus.

$$Z = R + jX_L = \frac{1}{Y} = \frac{1}{G + jB} \times \frac{G - jB}{G - jB} = \frac{G - jB}{G^2 + B^2}$$

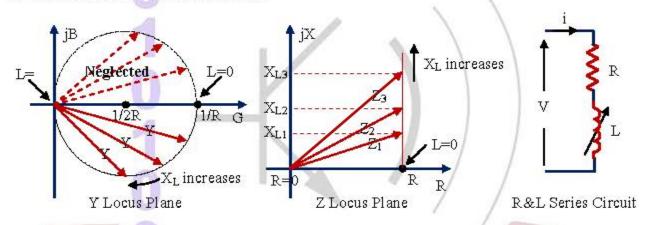
From the real part of the equation (constant element value):

$$R = \frac{G}{G^2 + B^2} \quad \Rightarrow \quad \left\{ G^2 - \frac{G}{R} + B^2 = 0 \right\} + \left( \frac{1}{2R} \right)^2$$

 $R = \frac{G}{G^2 + B^2} \implies \left\{ G^2 - \frac{G}{R} + B^2 = 0 \right\} + \left( \frac{1}{2R} \right)^2$   $\therefore \left( G - \frac{1}{2R} \right)^2 + B^2 = \left( \frac{1}{2R} \right)^2 \text{ Circle equation with (1/2R,0) center and (1/2R)}$ radius.

From imaginary part of the equation (variable element):  $X_L = \frac{-B}{C^2 + B^2}$ 

∴ B must be always negative to make (because) X<sub>L</sub> always positive, so the admittance loci is half lower circle.



Example: 20V applied to series R&L circuit with variable "L" and R=5 $\Omega$ . Draw the admittance and current locus. Y Locus Plane

Solution:

Due to the above derived equations for the admittance loci its half lower circle with:

Centered at 
$$\left(\frac{1}{2R}, 0\right) = (0.1,0)$$

Radius= $\frac{1}{2n}$  = 0.10

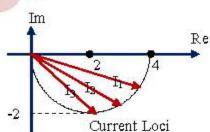
The current locus (I=VY Ohm's law) is half lower circle too with:

Center=
$$\left(\frac{V}{2R}, 0\right) = (2,0)$$
  
Radius= $\frac{V}{2R} = 2A$ 

Radius=
$$\frac{V}{2R} = 2A$$

Note: Y<sub>1</sub>, Y<sub>2</sub>, Y<sub>3</sub>,...,etc represents complex value of total admittance for the circuit. While

 $l_1, l_2, l_3, \dots$ , etc represents complex value of total current in the circuit.



X<sub>L</sub> increases

-0.1

# 7.3 Admittance Locus for R&L Series Circuit with Variable "R":

Fixing the reactance and change the resistance will give straight line parallel to the real axis in the Z-plane (Impedance Locus) while it will be half right circle in the Y-plane (Admittance Locus). Also it will be half right circle in the current Locus.

$$Z = R + jX_L = \frac{1}{Y} = \frac{1}{G + jB} \times \frac{G - jB}{G - jB} = \frac{G - jB}{G^2 + B^2}$$

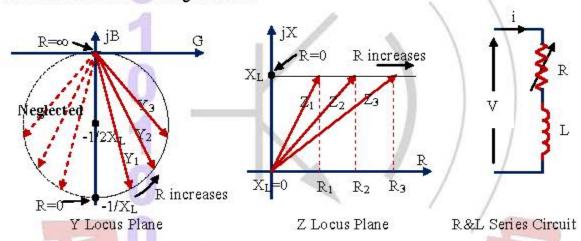
From the imaginary part of the equation (constant element value):

$$X_L = \frac{-B}{G^2 + B^2} \implies \left\{ G^2 + \frac{B}{X_L} + B^2 = 0 \right\} + \left( \frac{1}{2X_L} \right)^2$$

 $\therefore G^2 + \left(B + \frac{1}{2X_L}\right)^2 = \left(\frac{1}{2X_L}\right)^2 \text{ Circle equation with } (0,-1/2X_L) \text{ center and } (1/2X_L)$  radius.

From real part of the equation (variable element):  $R = \frac{G}{G^2 + B^2}$ 

.. G must be always positive to give (because) R always positive, so the admittance loci is half right circle.



Example: 100V applied to series R&L circuit with variable "R" and  $X_L=10\Omega$ . Draw the admittance and current locus.

Solution:

Due to the above derived equations for the admittance loci its half right circle with:

Centered at 
$$\left(0, \frac{-1}{2X_L}\right) = (0, -0.05)$$

Radius=
$$\frac{1}{2X_L}$$
 = 0.05 $\upsilon$ 

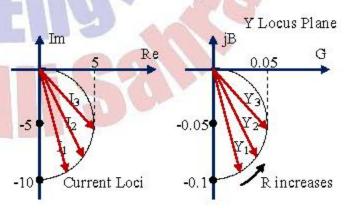
The current locus (I=VY Ohm's law) is half right circle too with:

Center=
$$\left(0, \frac{-V}{2X_L}\right) = (0, -5)$$

$$\mathsf{Radius} = \frac{v}{2X_L} = 5\mathsf{A}$$

Note:  $Y_1, Y_2, Y_3, ...$ , etc represents complex value of total admittance for the circuit. while  $I_1, I_2, I_3, ...$ , etc represents complex value of total current in the circuit.

-60-



# 7.4 Admittance Locus for R&C Series Circuit with Variable "C":

Fixing the resistance and change the capacitance will give straight line parallel to the imaginary axis in the Z-plane (Impedance Locus) while it will be half upper circle in the Y-plane (Admittance Locus). Also it will be half upper circle in the current Locus.

$$Z = R - jX_G = \frac{1}{V} = \frac{1}{G + iR} \times \frac{G - jR}{G - iR} = \frac{G - jR}{G^2 + R^2}$$

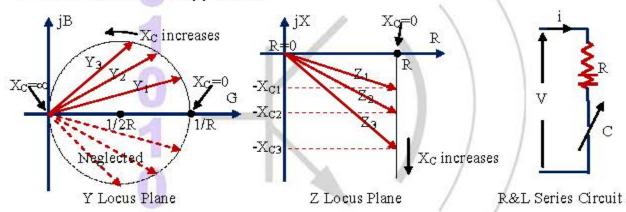
 $Z=R-jX_C=rac{1}{Y}=rac{1}{G+jB} imesrac{G-jB}{G-jB}=rac{G-jB}{G^2+B^2}$  From the real part of the equation (constant element value):

$$R = \frac{G}{G^2 + B^2} \quad \Rightarrow \quad \left\{ G^2 - \frac{G}{R} + B^2 = 0 \right\} + \left( \frac{1}{2R} \right)^2$$

 $R = \frac{G}{G^2 + B^2} \implies \left\{ G^2 - \frac{G}{R} + B^2 = 0 \right\} + \left( \frac{1}{2R} \right)^2$   $\therefore \left( G - \frac{1}{2R} \right)^2 + B^2 = \left( \frac{1}{2R} \right)^2 \text{ Circle equation with (1/2R,0) center and (1/2R)}$ radius.

From imaginary part of the equation (variable element):  $X_C = \frac{B}{C^2 + B^2}$ 

∴ B must be always positive to make (because) X<sub>C</sub> always positive, so the admittance loci is half upper circle.



Example: 50V applied to series R&C circuit with variable "C" and R=20 $\Omega$ .

Draw the admittance and current locus.

Solution:

Due to above equations the admittance loci is half upper circle with:

Centered at 
$$\left(\frac{1}{2R}, 0\right) = (0.025, 0)$$

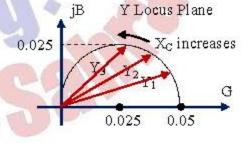
Radius=
$$\frac{1}{2R} = 0.0250$$

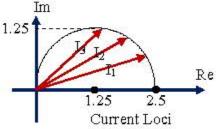
The current locus (I=VY Ohm's law) is half upper circle too with:

Center=
$$\left(\frac{v}{2R}, 0\right) = (1.25, 0)$$

Radius=
$$\frac{\sqrt{2R}}{2R}$$
 = 1.25A

Note:  $Y_1, Y_2, Y_3, \dots$ , etc represents complex value of total admittance for the circuit. while





 $I_1,I_2,I_3,...$ , etc represents complex value of total current in the circuit.

# 7.5 Admittance Locus for R&C Series Circuit with Variable "R":

Fixing the capacitance and change the resistance will give straight line parallel to the real axis in the Z-plane (Impedance Locus) while it will be half right circle in the Y-plane (Admittance Locus). Also it will be half right circle in the current Locus.

$$Z = R - jX_C = \frac{1}{Y} = \frac{1}{G + iB} \times \frac{G - jB}{G - iB} = \frac{G - jB}{G^2 + B^2}$$

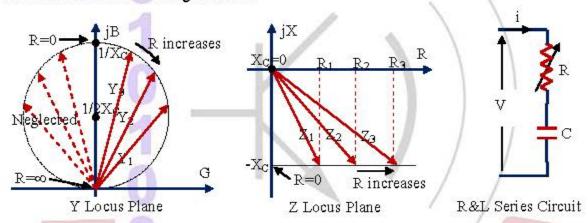
 $Z=R-jX_C=rac{1}{Y}=rac{1}{G+jB} imesrac{G-jB}{G-jB}=rac{G-jB}{G^2+B^2}$  From the imaginary part of the equation (constant element value):

$$X_C = \frac{B}{G^2 + B^2} \implies \left\{ G^2 - \frac{B}{X_C} + B^2 = 0 \right\} + \left( \frac{1}{2X_C} \right)^2$$

 $\therefore G^2 + \left(B - \frac{1}{2X_C}\right)^2 = \left(\frac{1}{2X_C}\right)^2 \text{ Circle equation with } (0,1/2X_C) \text{ center and } (1/2X_C)$ radius.

From real part of the equation (variable):  $R = \frac{G}{G^2 + R^2}$ 

∴ G must be always positive to give (because) R always positive, so the admittance loci is half right circle.



Example: 30V applied to series R&C circuit with variable "R" and  $X_c = 8\Omega$ . Draw the admittance and current locus.

Solution: Due to above equations the admittance loci is half right circle with:

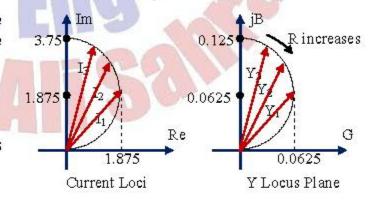
Centered at 
$$\left(0, \frac{1}{2X_C}\right) = (0, 0.0625)$$

Radius=
$$\frac{1}{2X_C} = 0.0625v$$

The current locus (I=VY Ohm's law) is half right circle too with:

Center=
$$\left(0, \frac{v}{2x_c}\right) = (0, 1.875)$$
  
Radius= $\frac{v}{2x_c} = 1.875$ A

$$\mathsf{Radius} = \frac{v}{2X_C} = 1.875 \,\mathsf{A}$$



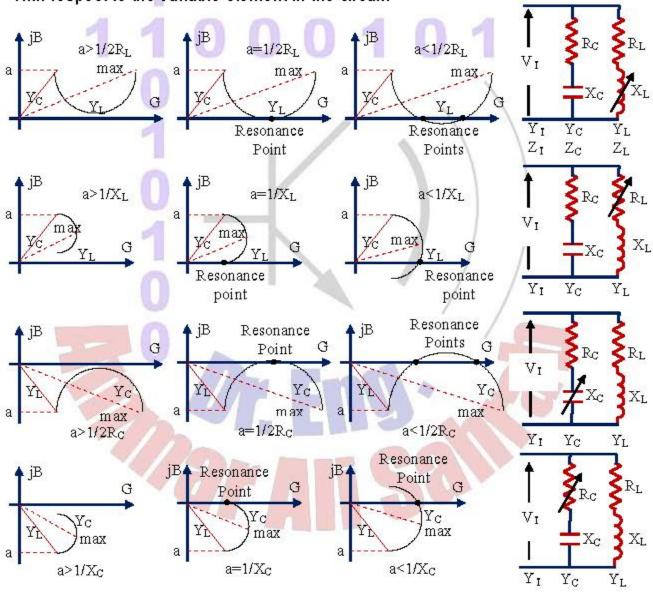
Note: Y<sub>1</sub>,Y<sub>2</sub>,Y<sub>3</sub>,...,etc represents complex value of total admittance for the circuit. While  $l_1, l_2, l_3, ...,$ etc represents complex value of total current in the circuit.

# 7.6 <u>Admittance Locus for Two Parallel Branch Electric Circuits with</u> One Variable Element:

The total admittance for the two parallel branch electric circuits equal to the sum of the individual admittances for each branch as follows:

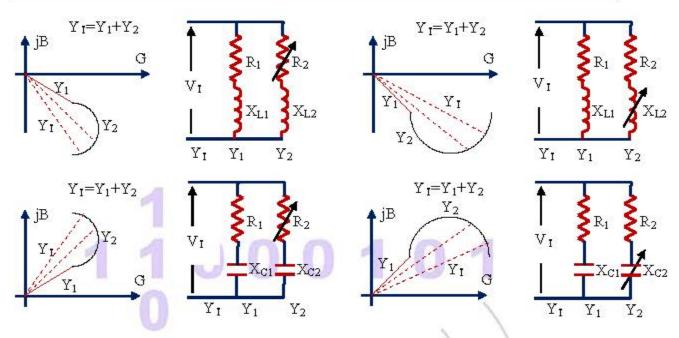
$$Y_T = Y_1 + Y_2$$
 or  $\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2}$ 

So the total admittance is any line from the origin to that point on the half circle. The resonance points for the locus could be found by intersecting the locus with the real axis (G) (i.e. real part of  $Y_T$ ), while, the maximum and minimum points for the locus could be found by deriving the equation of  $Y_T$  with respect to the variable element in the circuit.



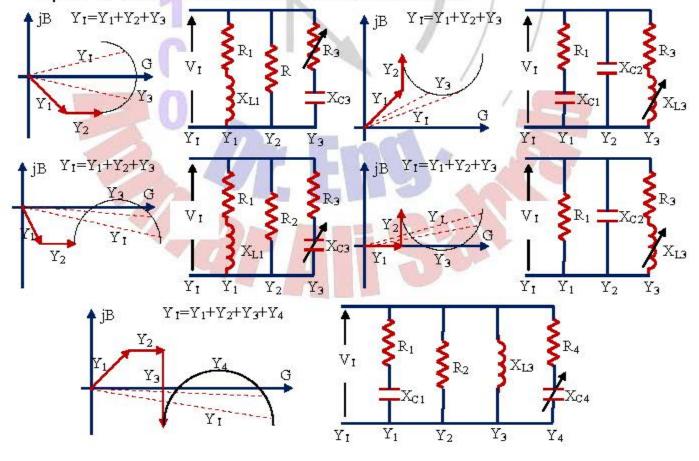
<u>Note:</u> In the previous circuits the resonance could be happened when the admittance locus intersect the real axis (G), then the total current will be inphase with the total applied voltage. In the next circuits there are no resonance case (the admittance locus does not intersect the real axis).

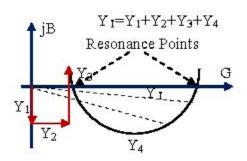


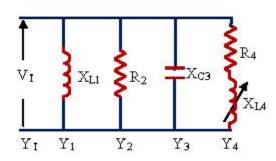


# 7.7 <u>Admittance Locus for Circuits with More Than Two Parallel Branches:</u>

The total admittance for these parallel electric circuits is equal to the sum of the admittances of each branch. Also, the resonance may be happened or not depend on the intersection of the locus with the real axis.







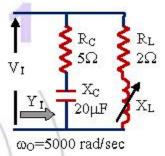
0.08

Note: The total impedance is the reciprocal of the total admittance for the circuit, and the total current, power & power factor for the circuit could be implemented.

Example: For the circuit shown:

- 1. Derive an equation for suitable value of R<sub>1</sub> that give resonance at two points.
- 2. Find the value of "L" that makes the circuit in resonance?
- 3. Find the value of total impedance at resonance for the values found in point "2"?

Note: Explain the resonance using admittance locus. Solution:



Points

1. For two points at resonance the radius of the circle must be greater than the imaginary part of admittance locus (Y<sub>C</sub>). Resonance

$$\therefore \frac{1}{2R_L} > \frac{X_C}{R_C^2 + X_C^2} \quad \Rightarrow \quad R_L < \frac{1}{2} \left( \frac{R_C^2}{X_C} + X_C \right)$$

2. Draw the admittance locus with the following points:

Yo is a straight line with 0.04 real part and 0.08 imaginary part.

Y<sub>L</sub> is half down circle with 0.25 radius and (0.25,0) center. The Y<sub>L</sub> locus is

drawn after the Y<sub>C</sub> locus end point as seen in the locus shown. 
$$X_C = \frac{1}{\omega C} = \frac{1}{500 \times 20 \times 10^{-6}} = 10\Omega$$

$$Y_T = Y_C + Y_L = \left(\frac{5}{125} + \frac{2}{4 + X_L^2}\right) + j\left(\frac{10}{125} - \frac{X_L}{4 + X_L^2}\right)$$

At resonance the imaginary part equal to zero.

$$\therefore \frac{10}{125} = \frac{X_L}{4 + X_L^2} \implies X_L^2 - 12.5X_L + 4 = 0$$

Or 
$$(X_L-0.33)(X_L-12.12)=0$$

At 
$$X_L = 0.33\Omega$$
  $\Rightarrow$   $L_{O1} = 0.33/5000 = 0.066 mH$ 

At 
$$X_1 = 12.12\Omega \implies L_{02} = 12.12/5000 = 2.43 \text{ mHz}$$

At 
$$X_L = 12.12\Omega$$
  $\Rightarrow$   $L_{O2} = 12.12/5000 = 2.43 \text{mH}$   
3. At  $X_L = 0.33\Omega$   $Y_{O1} = \frac{5}{125} + \frac{2}{4 + (0.33)^2} = 0.52 \text{v}$   $\Rightarrow$   $Z_{O1} = \frac{1}{Y_{O1}} = 1.92\Omega$   
At  $X_L = 12.12\Omega$   $Y_{O2} = \frac{5}{125} + \frac{2}{4 + (12.12)^2} = 0.053 \text{v}$   $\Rightarrow$   $Z_{O2} = \frac{1}{Y_{O2}} = 18.8\Omega$ 

Note: As shown there are two resonance points from the locus and by using the equation of total admittance for the circuit seen by the source.

### Example: For the circuit shown:

- 1. Find the value of " $R_L$ " that makes the circuit in resonance at  $R_C$ =10 $\Omega$ .
- 2. Find the value of " $R_L$ " that makes the circuit in resonance at  $R_C$ =4 $\Omega$ . If not what changes should be made in the circuit elements value?

Note: Prove the resonance using admittance locus. Solution:

$$\mathbf{1}.\,Y_T = Y_C + Y_L = \left(\frac{10}{125} + \frac{R_L}{R_L^2 + 100}\right) + j\left(\frac{5}{125} - \frac{10}{R_L^2 + 100}\right)$$

At resonance the imaginary part equal to zero.

$$\frac{5}{125} = \frac{10}{R_L^2 + 100} \implies R_L^2 + 100 = 250$$

$$\therefore$$
  $R_L = 12.25\Omega \Rightarrow Y_{T_O} = 0.1290$  and  $Z_{T_O} = 7.75\Omega$ 

2. 
$$Y_T = Y_C + Y_L = \left(\frac{4}{41} + \frac{R_L}{R_L^2 + 100}\right) + j\left(\frac{5}{41} - \frac{10}{R_L^2 + 100}\right)$$

At resonance the imaginary part equal to zero.

$$\frac{5}{41} = \frac{10}{R_L^2 + 100}$$
  $\Rightarrow$   $R_L^2 + 100 = 82$  or  $R_L^2 = -18$ 

.. The resonance cannot be happened, to solve this problem the value of X<sub>L</sub> must be changed to make the circuit work at resonance as follows:

$$\begin{array}{ll} 1/X_L {=}\, 0.12 & \Rightarrow & X_L {=}\, 8.3\Omega \\ \text{or} & X_1 {\leq} 8.3\Omega \end{array}$$

# Example: The circuit shown work at resonance:

- 1. Find the values of "c".
- 2. Draw the admittance locus.
- 3. Find the total current.

### Solution:

1. 
$$Y_T = Y_C + Y_L = \left(\frac{4}{16 + X_C^2} + \frac{5}{34}\right) + j\left(\frac{X_C}{16 + X_C^2} - \frac{3}{34}\right)$$

At resonance the imaginary part equal to zero.

$$\frac{X_C}{16+X_C^2} = \frac{3}{34} \implies X_C^2 - 11.3X_C + 16 = 0$$

 $X_C = 9.68\Omega$ 

and c=1/500×9.68=20.6μF

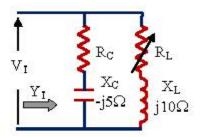
∴X<sub>C</sub>=1.65Ω

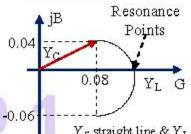
and c=1/500×1.65=121 $\mu$ F

3. at X<sub>C</sub>=9.68Ω

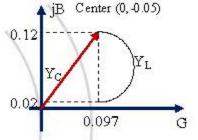
$$Y_{O1} = \frac{4}{16 + (9.68)^2} + \frac{5}{34} = 0.18v \implies I_{O1} = VY_{O1} = 1.8A$$

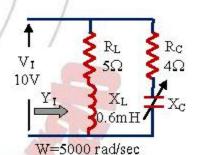
$$Y_{02} = \frac{4}{16 + (1.65)^2} + \frac{5}{34} = 0.36v \implies I_{02} = VY_{02} = 3.6A$$





Y<sub>c</sub> straight line & Y<sub>L</sub> is Circle Radius=1/2X<sub>L</sub>=0.05





D.09

Resonance
Points

O.14

Yc

O.125

2. Y<sub>L</sub> straight line & Y<sub>C</sub> is Circle Radius=1/2R<sub>C</sub>=0.125 Center (0.125,0)

<u>Example:</u> For the total admittance locus shown that represent an electrical circuit with two parallel branches, find the value of each element in the circuit. Note: Use the angular frequency W=500 rad/sec.

Solution: From the locus we find that:

$$Y_C = 0.05 + j0.05 = 0.0707 \angle 45^O \upsilon$$

$$Z_{\rm C} = \frac{1}{Y_{\rm C}} = 14.14 \angle -45^{\rm O} \Omega = (10 - j10)\Omega$$

.. From real and imaginary parts we found that:

 $R_C = 10\Omega$  and  $X_C = 10\Omega$ 

$$c = \frac{1}{\omega X_C} = \frac{1}{500 \times 10} = 200 \mu F$$

Again from the locus we find that:

Diameter=
$$1/X_L = 0.05v \Rightarrow X_L = 20\Omega$$

$$\therefore L = \frac{X_L}{\omega} = \frac{20}{500} = 40 \text{mH}$$

<u>Example:</u> For the total admittance locus shown that represent an electrical circuit with two parallel branches. The maximum value of real part of total admittance is equal to (0.5), what is the value and type of each element in the circuit.



From the locus we find that:

$$\frac{1}{X_C} \times \frac{1}{\tan 30^{\circ}} + \frac{1}{2X_C} = 0.5$$

Solving this equation for  $X_C$  we get:  $X_C=4.464\Omega$ Again from the locus we find that:

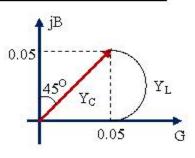
$$Y_{L} = \frac{1}{X_{C}} - j \left( \frac{1}{X_{C}} \times \frac{1}{\tan 30^{O}} \right)$$

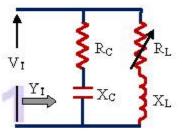
Were  $\frac{1}{X_C}$  representing the real part of  $Y_L$  and  $\left(\frac{1}{X_C} \times \frac{1}{\tan 30^O}\right)$  representing

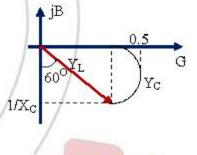
the imaginary part of Y<sub>L</sub>.

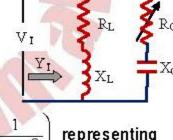
Substituting the value of X<sub>C</sub> to get: Y<sub>L</sub>=(0.224-j0.388)v

$$\therefore Z_{L} = \frac{1}{Y_{L}} = (1.938 + j1.119)\Omega = R_{L} + jX_{L}$$











Example: For the total current locus shown that represent an electrical circuit with two parallel branches, find the value of each element in the circuit.

Note: Use the angular frequency W=2000 rad/sec.

#### Solution:

$$Z_L = \frac{V}{I_L} = \frac{250 \angle 30^0}{25 \angle -15^0} = 10 \angle 45^0 \Omega = (7.07 + j7.07) \Omega$$

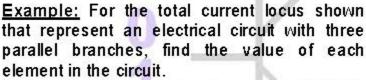
 $\therefore$  From real and imaginary parts we found that:  $R_L$ =7.07 $\Omega$  and  $X_L$ =7.07 $\Omega$ 

$$\therefore L = \frac{X_L}{\omega} = \frac{7.07}{2000} = 3.53 \text{mH}$$

From the admittance locus we find that:

$$Y_L = \frac{1}{Z_L} = \frac{1}{10\angle 45^O} = 0.1\angle -45^O \upsilon = (0.0707 + j0.0707)\upsilon$$

Radius= $1/2R_C=0.0707v \Rightarrow R_C=7.07\Omega$ 



Note: Use the angular frequency W=5000 rad/sec.

#### Solution:

$$Z_1 = \frac{V}{I_1} = \frac{150\angle - 25^{\circ}}{18\angle - 40^{\circ}} = 8.33\angle 15^{\circ}\Omega = (8.05 + j2.16)\Omega$$

∴ From real and imaginary parts we found that:  $R_{L1}=8.05\Omega$  and  $X_{L1}=2.16\Omega$ 

$$\therefore L_1 = \frac{X_{L1}}{\omega} = \frac{2.16}{5000} = 0.433 \text{mH}$$

$$Z_2 = \frac{V}{I_2} = \frac{150\angle - 25^{\circ}}{18\angle 35^{\circ}} = 8.3\angle - 60^{\circ} \Omega = (4.16 - j7.22)\Omega$$

∴ From real and imaginary parts we found that:  $R_C$  =4.16Ω and  $X_C$  =7.22Ω

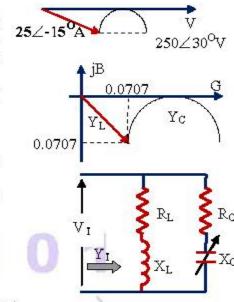
$$\therefore c = \frac{1}{\omega X_C} = \frac{1}{5000 \times 7.22} = 27.7 \mu F$$

From the admittance locus we find that:

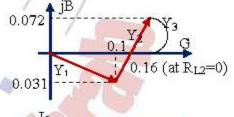
Diameter=
$$1/X_{L2}$$
=Im( $Y_2$ )-Im( $Y_1$ )=0.072 $\upsilon$   $\Rightarrow$   $X_{L2}$ =13.7 $\Omega$ 

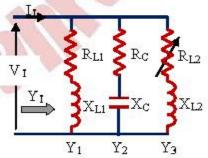
Using:  $Y_1 = 1/Z_1$  and  $Y_2 = 1/Z_2$ 

$$\therefore L_2 = \frac{X_{L2}}{\omega} = \frac{13.7}{5000} = 2.74 \text{mH}$$



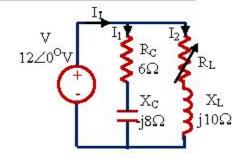








Example: Find the value of "R<sub>L</sub>" which result in parallel for the circuit shown besides using the I-Locus diagram. Also find the value of  $I_{max}$ , p. $f_{max}$ ,  $I_{min}$ and  $P_{max}$  (maximum power dissipated). Solution:



1. 
$$I_1 = \frac{12\angle 0^0}{6-j8} = 1.2\angle 53.1^0 = 0.72 + j0.96 = 0D + jAD$$

From the current locus we find that:

Radius=
$$\frac{v}{2X_L} = \frac{12\angle 0^0}{2\times 10} = 0.6A = CB$$

And by using Pythagoras rule for triangle (CDB):

$$DB = \sqrt{(CB)^2 - (CD)^2} = \sqrt{(0.6)^2 - (0.36)^2} = 0.48A$$

Were: CD=AD-AC=0.96-0.6=0.36A

Again using Pythagoras rule for triangle (ADB):

$$|I_{20}| = \sqrt{(AD)^2 + (DB)^2} = \sqrt{(0.96)^2 + (0.48)^2} = 1.1A$$
  
 $\alpha = tan^{-1} \frac{KB}{KA} = tan^{-1} \frac{0.96}{0.48} = -63.43^{\circ}$ 

$$\therefore$$
  $I_{2O}$  = 1.0733 $\angle$ -63.43 $^{\circ}$ A (is the current in the second branch at resonance)  $Z_2 = R_L + j10 = \frac{v}{I_2} = \frac{12 \angle 0^{\circ}}{1.1 \angle -63.43^{\circ}} = 11.18 \angle 63.43^{\circ} = (5 + j10)\Omega$ 

2. 
$$I_T = V \times \left[ \left( \frac{6}{100} + \frac{R_L}{R_L^2 + 100} \right) + j \left( \frac{8}{100} - \frac{10}{R_L^2 + 100} \right) \right]$$

To get the maximum value condition the above equation must be derived with respect to the variable element in the equation, and equating it with

$$\frac{dI_T}{dR} = V \times \left[ \frac{-R_L^2 + 100}{(R_L^2 + 100)^2} + j \frac{20R_L}{(R_L^2 + 100)^2} \right] = 0 + j0$$

From real part  $R_L=10\Omega$  (maximum) and from imaginary part  $R_L=0\Omega$ (minimum)

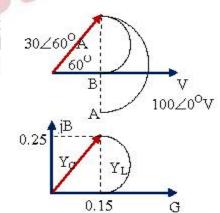
$$\begin{split} I_{Tmax} &= 12 \angle 0^{O} \times \left[ \left( \frac{6}{100} + \frac{10}{200} \right) + j \left( \frac{8}{100} - \frac{10}{200} \right) \right] = 1.368 \angle 15.255^{O} A \\ I_{Tmin} &= 12 \angle 0^{O} \times \left[ \left( \frac{6}{100} \right) + j \left( \frac{8}{100} - \frac{10}{100} \right) \right] = 0.76 \angle -18.4^{O} A \end{split}$$

 $P_{\text{max}} = V \times I_{\text{Tmax}} \times \cos \theta = 12 \times 1.368 \times \cos(0.15.255) = 15.84W$ 

Or 
$$P_{\text{max}} = (I_{\text{Tmax}})^2 \times \text{Re}(Z_{\text{Tmax}}) = (1.368)^2 \times 8.463 = 15.84 \text{W}$$

$$p.f_{max} = cos\theta_{max} = cos0 = 1$$

Example: For the total current locus shown that represent an electrical circuit with two parallel branches. What is the change required in the element value to make the locus ends at point B. Solution:



$$Y_C = \frac{I_C}{V} = \frac{30\angle 60^O}{100\angle 0^O} = 0.3\angle 60^O \upsilon = (0.15 + j0.25)\upsilon$$

From the admittance locus we find that:

Diameter=1/X₁ =0.250  $\Rightarrow$  $X_1 = 4\Omega$  at point "B"