12.1 Electric Circuits Transient (Natural and Time Response):

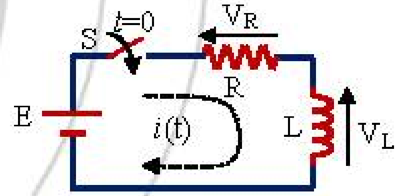
When an electric circuit transferred from steady-state to another steady-state the currents and voltages values depends on time (changed with time) this is called transient state (response), this occurs when a switch is turned on\off or tri-state a rather common event in electrical circuits.

A first-order circuit is characterized by a first-order differential equation. We consider circuits that contain various combinations of two or three of the passive elements (resistors, capacitors, and inductors). First, we shall examine two types of simple circuits: a circuit comprising a resistor and inductor and a circuit comprising resistor and capacitor. These are called RL and RC circuits, respectively. As simple as these circuits are, they find continual applications in electronics, communications, and control systems.

We carry out the analysis of RL and RC circuits by applying Kirchhoff's laws, as we did for resistive circuits. The only difference is that applying Kirchhoff's laws to purely resistive circuit's results in algebraic equations, while applying the laws to RL and RC circuits produces differential equations, which are more difficult to solve than algebraic equations. The differential equations resulting from analyzing RL and RC circuits are of the first order. Hence, the circuits are collectively known as first-order circuits. Finally, there are four typical applications of RC and RL circuits: delay and relay circuits, a photoflash unit, and an automobile ignition circuit.

a. Step Response of Series RL Circuit:

Using KVL method:



Our goal is to determine the circuit response, which we will assume to be the current $i(t)$ through the inductor.

Applying KVL around the loop in figure beside:

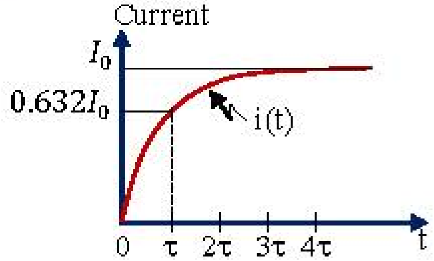
$E = V_R + V_L = Ri + L\frac{di}{dt}$ Dividing both sides by L

$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$ first order differential equation, then rearranging terms and integrating gives: $i(t) = Ke^{-\frac{R}{L}t} + e^{-\frac{R}{L}t}\int e^{\frac{R}{L}t}\cdot\frac{E}{L}dt = Ke^{-\frac{R}{L}t} + \frac{E}{R}$

We now determine the constant K from the initial value of i. Let I_0 be the initial current through the inductor, which may come from a source other than E. If there is no other source the current equal to zero when the switch is open because the coil is open circuit. Since the current through the inductor cannot change instantaneously

$\therefore\ i(0) = I_0 = K + \frac{E}{R} \Rightarrow K = I_0 - \frac{E}{R}$

$\therefore\ i(t) = \frac{E}{R} + \left(I_0 - \frac{E}{R}\right)e^{-\frac{R}{L}t}$ at $t \geq 0$



Current response of RL circuit

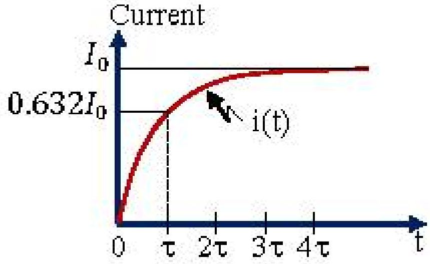
This shows that the natural response of the RL circuit is an exponential increased of the initial current. The current response is shown in figure bedside. It is evident from current equation that the time constant for the RL circuit is $\tau = \frac{L}{R}\ (sec)$

The complete response of the RL circuit is:

$\therefore\ i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}}$ at $t \geq 0$

where i(0) and i(∞) are the initial and final values of i, respectively. Thus, to find the step response of an RL circuit requires three things:

1. The initial inductor current i(0) at t=0.
2. The final inductor current i(∞).
3. The time constant τ.



Current response of RL circuit

When $i(0) = I_0 = 0$ (special case) then

$\therefore\ i(t) = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$ at $t \geq 0$

The steady-state case for long time at $t = \infty$ is $i(\infty) = I_0 = \frac{E}{R}$

Again, if the switching takes place at time $t = t_0$ instead of $t = 0$, then

$\therefore\ i(t) = i(\infty) + [i(t_0) - i(\infty)]e^{-\frac{t-t_0}{\tau}}$ at $t \geq t_0$

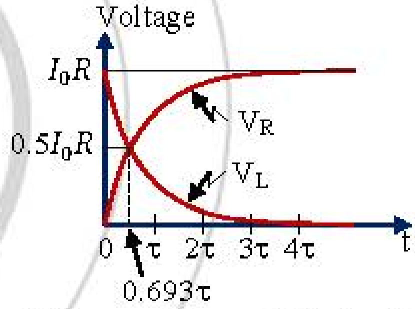
The resistor and inductor voltages are:

$V_R = iR = I_0R(1 - e^{-\frac{t}{\tau}})$ at $t \geq 0$

$V_L = L\frac{di}{dt} = I_0Re^{-\frac{t}{\tau}}$ at $t \geq 0$

The sum of any two points (V_R, V_L) equal "I_0R" at any time (i.e. time constant) and this time make the exponential term equal to one.

The two voltages V_R and V_L will be equaled at a time of 0.693τ and $V_R=V_L=0.5I_0R$.



Voltage response of RL circuit

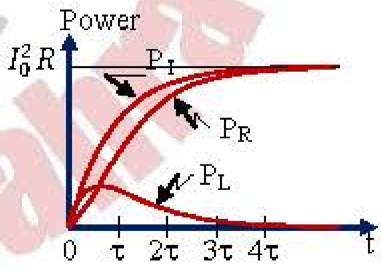
And the instantaneous power dissipated in the resistor is:

P_R= Resistance power $=V_R \times i$

$= I_0R\left(1 - e^{-\frac{R}{L}t}\right) \times I_0\left(1 - e^{-\frac{R}{L}t}\right) = I_0^2R(1 - 2e^{-\frac{R}{L}t} + e^{-2\frac{R}{L}t})$

P_L= Inductance power $=V_L \times i$

$= I_0Re^{-\frac{R}{L}t} \times I_0\left(1 - e^{-\frac{R}{L}t}\right) = I_0^2R(e^{-\frac{R}{L}t} - e^{-2\frac{R}{L}t})$



P_T= Total power = $P_R + P_L$ = $I_0^2R(1 - e^{-\frac{R}{L}t})$ Watt

Power = average consumed energy $=\frac{\text{energy}}{\text{time}}$

The energy absorbed by the resistor is:

W = energy stored in the coil = $\int_0^\infty P_L dt$

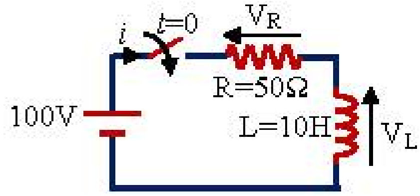
$= \int_0^\infty I_0^2R(e^{-\frac{R}{L}t} - e^{-2\frac{R}{L}t})dt$

$= I_0^2R\left[-\frac{L}{R}e^{-\frac{R}{L}t} + \frac{L}{2R}e^{-2\frac{R}{L}t}\right]_0^\infty = \frac{1}{2}I_0^2L\ Jouls$

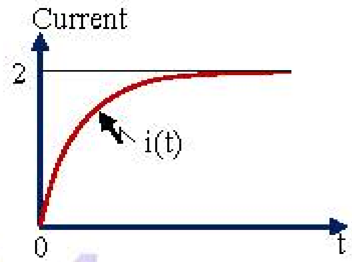
<u>Note:</u> At TC $(1 - e^{-1}) = (1 - 0.368) = 0.632$ and this time give a current of 63.2% of its initial value. At 2TC $(1 - e^{-2}) = (1 - 0.135) = 0.865$ gives a current of 86.5% of its initial value.

Example: For the circuit shown, if the switch closed at t=0 then find:

1. The time equation of i, V_R & V_L.
2. Current at t=0.5 sec.
3. Time when V_L = V_R.



Solution:



1. $i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}} Amp. \quad t \geq 0$

$i(0) = 0\ A$ open circuit

$i(\infty) = 2\ A$ short circuit

$\tau = \frac{L}{R} = \frac{10}{50} = 0.2 sec$

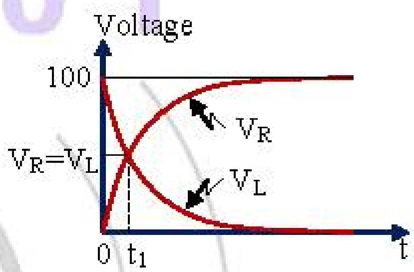
$i(t) = 2 + (0-2)e^{-\frac{t}{0.2}} = 2(1 - e^{-5t})$

$V_R = I_0 R = 100(1 - e^{-5t})\ Volt \quad t \geq 0$

$V_R(0) = 0\ V$, $V_R(\infty) = 100\ V$ (Coil is S/C)

$V_L = L\frac{di}{dt} = I_0 R e^{-\frac{t}{\tau}} = 100e^{-5t}\ Volt \quad t \geq 0$

$V_L(0) = 100\ V$, $V_L(\infty) = 0\ V$



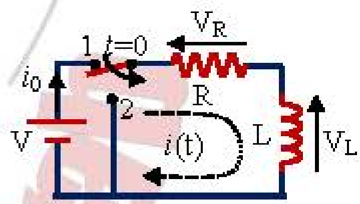
2. $i(0.5) = 2(1 - e^{-5*0.5}) = 1.836\ Amp.$
3. $V_R = V_L \Rightarrow 100(1 - e^{-5t_1}) = 100e^{-5t_1}$

or $e^{-5t_1} = \frac{1}{2} \Rightarrow t_1 = 0.1386\ sec$

Note: When the switch closed at t=0 all the equations of currents and voltages starts at t=0 and ends at t=∞ (i.e. $t \geq 0$). The case t=∞ is the most public case that the circuit is in steady state.

b. Step Response of Discharge Path Series RL Circuit:

When $t < 0$ the inductor acts like a short circuit (the switch has been in position "1" for a long time).

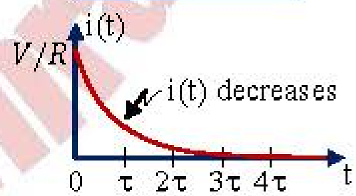


The current through the inductor at $t = \infty = 0^-$ (i.e., just before) $t = 0$:

For the first time the switch in position "1":

Since the inductor current cannot change instantaneously,

$i_0 = i(0^+) = i(0^-) = \frac{V}{R}$ steady-state in position "1"



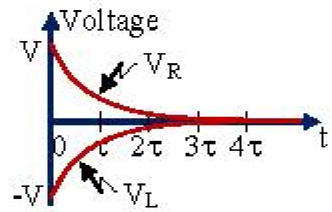
Now the switch in position "2":

$i(\infty) = 0A$ there is no source in the circuit.

$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}} Amp. \quad t \geq 0$

TC = Time constant $= \tau = \frac{L}{R}\ (sec)$

$i(t) = 0 + \left[\frac{V}{R} - 0\right]e^{-\frac{R}{L}t} = \frac{V}{R}e^{-\frac{R}{L}t} Amp. \quad t \geq 0$

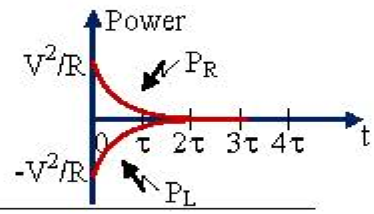


$V_R = Ri = Ve^{-\frac{R}{L}t}\ Volt \quad t \geq 0$

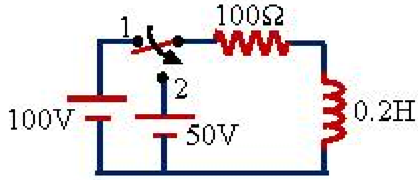
$V_L = L\frac{di}{dt} = -Ve^{-\frac{R}{L}t}\ Volt \quad t \geq 0$

$P_R = \frac{V^2}{R}e^{-2\frac{R}{L}t}$ and $P_L = -\frac{V^2}{R}e^{-2\frac{R}{L}t}$

$W = \int_0^{\infty} P_L dt = \frac{1}{2}\left(\frac{V}{R}\right)^2 L = \frac{1}{2}I^2 L\ Jouls$



Example: In the circuit shown, if the switch closed to position "1" at t=0 and then moved to position "2" at t=500μsec. Find and draw the transient current in each case.



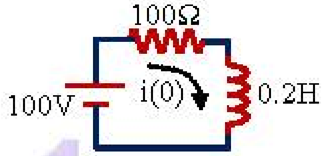
Solution:

The switch in position "1"

$i_0 = i(0^+) = i(0^-) = \frac{V}{R} = \frac{100}{100} = 1A$

but the switch moved at t_0=500μsec

$\tau = \frac{L}{R} = \frac{0.2}{100} = 2msec$ applied to the current equation

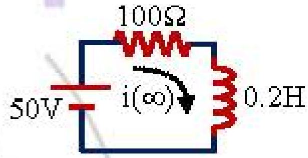


$\therefore\ i_0 = i(500\mu sec) = \frac{100}{100}\left(1 - e^{-500\times500\times10^{-6}}\right) = 0.221A$

The switch in position "2"

$i(\infty) = \frac{V}{R} = \frac{50}{100} = 0.5A$

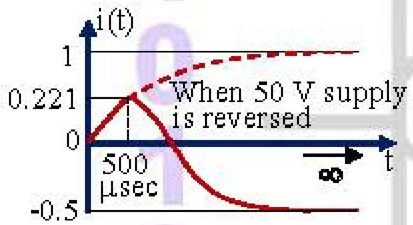
$\tau = \frac{L}{R} = \frac{0.2}{100} = 2msec$

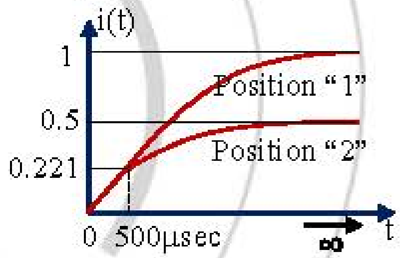


$i(t) = i(\infty) + [i(t_0) - i(\infty)]e^{-\frac{t-t_0}{\tau}}$

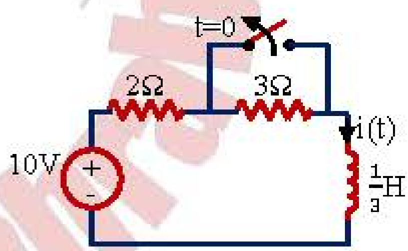
$= 0.5 + [0.221 - 0.5]e^{-500(t-500\times10^{-6})}$

$= 0.5 - 0.279e^{-500(t-500\times10^{-6})}\ Amp.\quad t \geq 500\mu sec$





Example: Find i(t) in the circuit shown for t≥0 Assume that the switch has been closed for a long time.



Solution:

When t<0, the 3Ω resistor is short-circuited, and the inductor acts like a short circuit. The current through the inductor at $t = 0^-$ (i.e., just before $t = 0$) is:

$i(0^-) = \frac{10}{2} = 5A$

Since the inductor current cannot change instantaneously,

$i(0) = i(0^+) = i(0^-) = 5A$

When t>0, the switch is open. The 2Ω and 3Ω resistors are in series, so that The Thevenin resistance across the inductor terminals is:

$R_{Th} = 2 + 3 = 5\Omega$

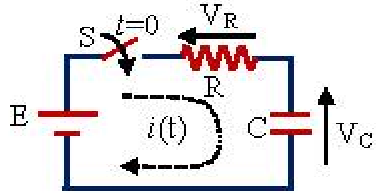
$i(\infty) = \frac{10}{2+3} = 2A$

TC = Time constant $= \tau = \frac{L}{R_{Th}} = \frac{\frac{1}{3}}{5} = \frac{1}{15}\ sec$

$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}}$

$= 2 + [5 - 2]e^{-15t} = 2 + 3e^{-15t}\ Amp.\quad t \geq 0$

c. Step Response of Series RC Circuit:



Using KCL method:

$0 = \frac{E - v_C}{R} + C\frac{dv_C}{dt}$ Rearranging terms gives:

$\frac{dv_C}{dt} + \frac{v_C}{RC} = \frac{E}{RC}$

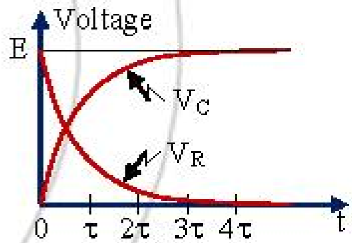
first order differential equation, then rearranging terms and integrating gives: $v_C(t) = Ke^{-\frac{1}{RC}t} + e^{-\frac{1}{RC}t}\int e^{\frac{1}{RC}t}\cdot\frac{E}{RC}dt = Ke^{-\frac{1}{RC}t} + E$

We now determine the constant K from the initial value of v_C. Let V_0 be the initial voltage across the capacitor, which may come from a source other than E. If there is no other source the voltage equal to zero when the switch is open because the coil is open circuit. Since the voltage across the capacitor cannot change instantaneously

$\therefore\ v_C(0) = V_0 = K + E \Rightarrow K = V_0 - E$

$\therefore\ v_C(t) = E + (V_0 - E)e^{-\frac{1}{RC}t}$ at $t \geq 0$

This shows that the natural response of the RC circuit is an exponential increased of the initial voltage. The voltage response is shown in figure bedside. It is evident from voltage equation that the time constant for the RC circuit is $\tau = RC\ (sec)$

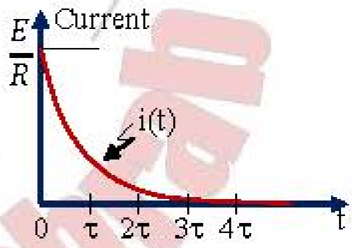


The complete response of the RC circuit is:

$\therefore\ v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]e^{-\frac{t}{\tau}}$ volt at $t \geq 0$

where v_C (0) and v_C (∞) are the initial and final values of v_C, respectively. Thus, to find the step response of an RC circuit requires three things:

1. The initial capacitor voltage v_C (0) at t=0.
2. The final capacitor voltage v_C (∞).
3. The time constant τ.



When $v_C(0) = V_0 = 0$ (special case) then

$\therefore\ v_C(t) = V_0\left(1 - e^{-\frac{t}{\tau}}\right)$ volt at $t \geq 0$

The steady-state case for long time at $t = \infty$ is $v_C(\infty) = V_0 = E$

Again, if the switching takes place at time $t = t_0$ instead of $t = 0$, then

$\therefore\ v_C(t) = v_C(\infty) + [v_C(t_0) - v_C(\infty)]e^{-\frac{t-t_0}{\tau}}$ volt at $t \geq t_0$

where $v_C(t_0)$ is the initial value at $t = t_0^+$ Keep in mind that the previous equation applies only to step responses, that is, when the input excitation is constant.

The instantaneous current equation is:

$i(t) = C\frac{dv_C}{dt} = \frac{v_C(0) - v_C(\infty)}{R}e^{-\frac{1}{RC}t}\ \ Amp.\ \ t \geq 0$

The instantaneous resistance voltage equation is:

$V_R = Ri = [v_C(0) - v_C(\infty)]e^{-\frac{1}{RC}t}\ \ volt\ \ t \geq 0$

and the instantaneous power is:

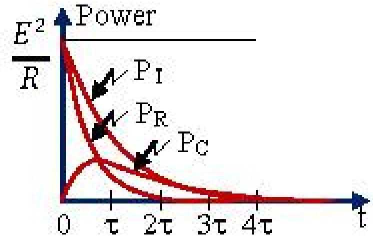
P_R= Resistance power $=V_R \times i$

$$= Ee^{-\frac{1}{RC}t} \times \frac{E}{R} e^{-\frac{1}{RC}t} = \frac{E^2}{R} e^{-2\frac{1}{RC}t}$$

P_C= Capacitance power $=V_C \times i$

$$= E\left(1 - e^{-\frac{1}{RC}t}\right) \times \frac{E}{R} e^{-\frac{1}{RC}t} = \frac{E^2}{R}\left(e^{-\frac{1}{RC}t} - e^{-2\frac{1}{RC}t}\right)$$

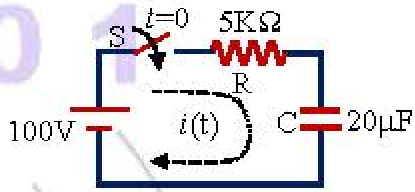
P_T= Total power = $P_R + P_C = \frac{E^2}{R} e^{-\frac{1}{RC}t}$ Watt



W = energy stored in the capacitor = $\int_0^\infty P_C dt$

$$= \int_0^\infty \frac{E^2}{R}\left(e^{-\frac{1}{RC}t} - e^{-2\frac{1}{RC}t}\right) dt = \frac{E^2}{R}\left[-RCe^{-\frac{1}{RC}t} + \frac{RC}{2} e^{-2\frac{1}{RC}t}\right]_0^\infty = \frac{1}{2} CE^2 \; Jouls$$

Example: For the circuit shown, find $i(t)$, $V_R(t)$, $V_C(t)$ and the time when V_R=V_C. Assume zero initial capacitor charge.



Solution:

$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]e^{-\frac{t}{\tau}} volt \quad t \geq 0$

$v_C(0) = 0\ V$ short circuit

$v_C(\infty) = 100\ V$ open circuit

$\tau = RC = 5 \times 10^3 \times 20 \times 10^{-6} = 0.1 sec$

$v_C(t) = 100 + (0 - 100)e^{-\frac{t}{0.1}} = 100(1 - e^{-10t})\ Volt \quad t \geq 0$

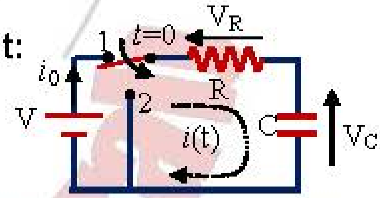
$i(t) = C\frac{dv_C}{dt} = \frac{V}{R} e^{-\frac{1}{RC}t} = 0.02e^{-10t}\ \ Amp. \quad t \geq 0$

$V_R = Ri = Ve^{-\frac{1}{RC}t} = 100e^{-10t}\ \ Volt \quad t \geq 0$

time when V_R=V_C is

$100e^{-10t} = 100(1 - e^{-10t}) \quad \Rightarrow \quad t = 69.3 msec$

d. Step Response of Discharge Path Series RC Circuit:



When $t < 0$ the capacitor acts like a open circuit (the switch has been in position "1" for a long time).

The voltage across the capacitor at $t = \infty = 0^-$ (i.e., just before) $t = 0$:

For the first time the switch in position "1"

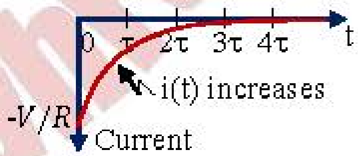
$v_C(0^-) = 100\ V$ open circuit

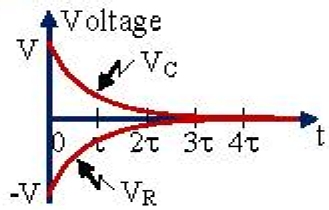
opposite charge and discharge path.

The switch in position "2"

$v_C(\infty) = 0\ V$ there is no voltage source

$\tau = RC\ sec$





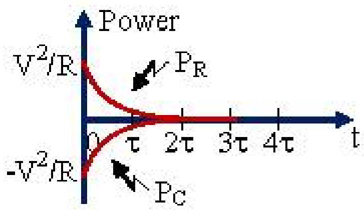
$v_C(t) = 0 + (100 - 0)e^{-\frac{t}{RC}} = 100e^{-\frac{t}{RC}} Volt \quad t \geq 0$

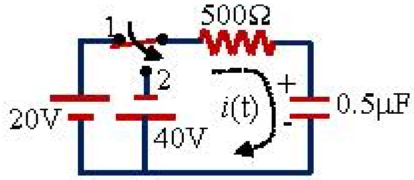
$i(t) = C\frac{dv_C}{dt} = -\frac{V}{R} e^{-\frac{1}{RC}t}\ \ Amp. \quad t \geq 0$

$V_R = Ri = -Ve^{-\frac{1}{RC}t}\ \ Volt \quad t \geq 0$

$P_R = \frac{V^2}{R} e^{-2\frac{1}{RC}t}$ and $P_C = -\frac{V^2}{R} e^{-2\frac{1}{RC}t}$

$W = \int_0^\infty P_C dt = -\frac{1}{2} CV^2\ Jouls$





Example: In the circuit shown, the switch closed to position "1" at t=0 and moved to position "2" at t=TC. Find and draw the transient current in both cases, assume zero initial charge.

Solution:

The switch in position "1":

$v_C(0^-) = 20\ V$ open circuit

but the switch moved at TC=RC=250μsec

applied to the voltage equation

$V_C(t) = 20(1 - e^{-4000t})$ Volt $\Rightarrow V_C(TC) = 12.65$ Volt

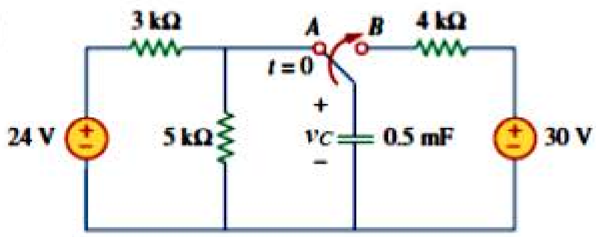
The switch in position "2":

$v_C(\infty) = -40\ V$

$\tau = RC = 250\ \mu sec$

$v_C(t - t_0) = -40 + (12.65 + 40)e^{-4000(t-t_0)} = -40 + 52.65e^{-4000(t-t_0)}\ Volt \quad t \geq t_0$

$i(t - t_0) = C\frac{dv_C}{dt} = -0.1053e^{-4000(t-t_0)}\ Amp. \quad t \geq t_0$



Example: The switch in the circuit shown has been in position A for a long time. At $t = 0$ the switch moves to B. Determine $v_C(t)$ for $t \geq 0$ and calculate its value at $t = 1\ s$ and $4\ s$.

Solution:

For $t < 0$ the switch is at position A.

The capacitor acts like an open circuit to DC, but v_C is the same as the voltage across the 5KΩ resistor. Hence, the voltage across the capacitor just before $t = 0$ is obtained by voltage division as

$v_C(0^-) = V_{Th} = 24 \times \frac{5}{5+3} = 15\ V$

Using the fact that the capacitor voltage cannot change instantaneously,

$v_C(0) = v_C(0^-) = v_C(0^+) = 15\ V$

For $t \geq 0$ the switch is in position B.

The Thevenin resistance connected to the capacitor is $R_{Th} = 4K\Omega$ and the time constant is

$\tau = R_{Th}C = 4 \times 10^3 \times 0.5 \times 10^{-3} = 2\ sec$

Since the capacitor acts like an open circuit to dc at steady state,

$v_C(\infty) = 30\ V$ Thus,

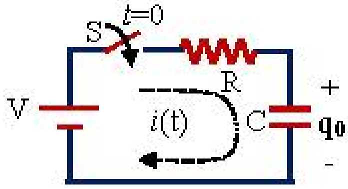
$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]e^{-\frac{t}{\tau}} volt \quad t \geq 0$

$\quad = 30 + [15 - 30]e^{-\frac{t}{2}} = 30 - 15e^{-0.5t}\ volt \quad t \geq 0$

At $t = 1\ s$, $v_C(1) = 30 - 15e^{-0.5} = 20.9\ volt$

At $t = 4\ s$, $v_C(4) = 30 - 15e^{-2} = 27.97\ volt$

12.2 Transient in Series RC with Charge:



Sometimes the capacitor is initially charged by a previous or external charging circuit. This charge will produces a voltage across the capacitor terminals equal to the initial charge divided by the capacitor value. The direction of initial charge "q_0" assumed to be in the same direction of the current "i", and the initial voltage across the capacitor is:

$v_C = \frac{q_0}{C}\ Volt.$

Hence, the voltage across the capacitor just before $t = 0$ is:

$v_C(0) = v_C(0^-) = v_C(0^+) = \frac{q_0}{C}\ Volt$

The voltage across the capacitor at $t \geq 0$ is:

$v_C(\infty) = V$ Thus,

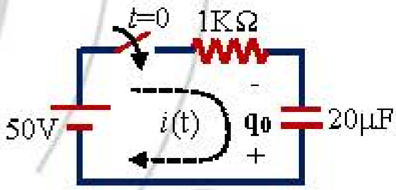
$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]e^{-\frac{t}{\tau}} volt \quad t \geq 0$$

$\tau = RC\ sec$

$$v_C(t) = V + \left[\frac{q_0}{C} - V\right]e^{-\frac{t}{RC}}\ volt \quad t \geq 0$$

$$\therefore\ q(t) = Cv_C = CV + \left[\frac{q_0}{C} - CV\right]e^{-\frac{t}{RC}} \quad Coulomb \quad t \geq 0$$

$$\therefore\ i(t) = \frac{dq(t)}{dt} = C\frac{dv_C}{dt} = \frac{1}{R}\left(V - \frac{q_0}{C}\right)e^{-\frac{1}{RC}t} \quad Amp. \quad t \geq 0$$

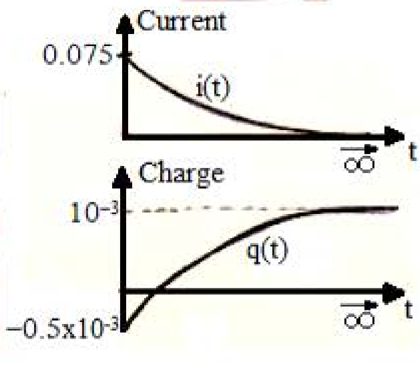


Example: Find the transient current when the initial charge on the capacitor terminals are q_0=500µ Coulomb for circuit shown.

Solution: at $t \geq 0$

Hence, the voltage across the capacitor just before $t = 0$ is:

$$v_C(0) = -\frac{q_0}{C} = -\frac{500*10^{-6}}{20*10^{-6}} = -25\ Volt$$



The voltage across the capacitor at $t \geq 0$ is:

$v_C(\infty) = 50\ Volt$ Thus,

$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]e^{-\frac{t}{\tau}} volt \quad t \geq 0$$

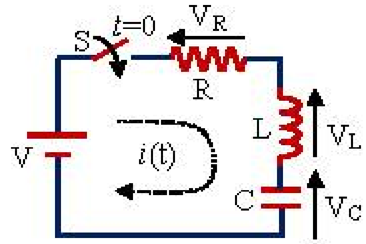
$\tau = RC = 1 \times 10^3 \times 20 \times 10^{-6} = 20\ msec$

$$v_C(t) = 50 + [-25 - 50]e^{-\frac{t}{RC}}$$
$$= 50 - 75e^{-50t}\ volt \quad t \geq 0$$

$$q(t) = Cv_C = CV + \left[\frac{q_0}{C} - CV\right]e^{-\frac{t}{RC}}$$
$$= 1 - 1.5e^{-50t} \quad mCoulomb \quad t \geq 0$$

$$i(t) = \frac{dq(t)}{dt} = C\frac{dv_C}{dt} = \frac{1}{R}\left(V - \frac{q_0}{C}\right)e^{-\frac{1}{RC}t}$$
$$= \frac{1}{1*10^3}(50 + 25)e^{-50t} = 0.075e^{-50t} \quad Amp. \quad t \geq 0$$

12.3 Series RLC Circuit:



Series RLC circuits containing two storage elements. These are known as second-order circuits because their responses are described by differential equations that contain second derivatives. A second-order circuit is characterized by a second-order differential equation. It consists of resistors and the equivalent of two energy storage elements. An understanding of the natural response of the series RLC circuit is a necessary background for future studies in filter design and communications networks. Consider the series RLC circuit shown in figure beside. The circuit is being excited by the energy initially stored in the capacitor and inductor. The energy is represented by the initial capacitor voltage V_0 and initial inductor current I_0. Thus, at $t = 0$,

$v_C(0) = \frac{1}{C}\int_{-\infty}^{0} i\,dt = V_0$ and $i(0) = I_0$

Apply KVL around the loop:

$V = V_R + V_L + V_C = \mathrm{R}i + \mathrm{L}\frac{di}{dt} + \frac{1}{C}\int i\,dt$

Differentiating both sides with respect to t, and rearrange terms. We get

$0 = \mathrm{R}\frac{di}{dt} + \mathrm{L}\frac{d^2i}{d^2t} + \frac{1}{C}i \quad \Rightarrow \quad \frac{d^2i}{d^2t} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$

This quadratic equation is known as the characteristic equation of the differential equation, since the roots of the equation dictate the character of i. The two roots are:

$\therefore\ s_{12} = \frac{-R}{2L} \mp \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}}$

A more compact way of expressing the roots is

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \quad and \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

where $= \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$ and $\beta = \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}} = \sqrt{\alpha^2 - \omega_0^2}$

Depending on the value of "β" there are three cases:

1- <u>Over-Damping:</u> $\alpha > \omega_0 \ \Rightarrow$ both roots s_1 and s_2 are negative, real and unequal roots.

$s_1 = -\alpha + \beta$ and $s_2 = -\alpha - \beta$

$\therefore\ i(t) = K_1e^{(-\alpha+\beta)t} + K_2e^{(-\alpha-\beta)t} = e^{-\alpha t}(K_1e^{\beta t} + K_2e^{-\beta t})$

which decays and approaches zero as t increases. Figure (a) illustrates a typical over-damped response.

2- <u>Critical-Damping:</u> $\alpha = \omega_0 \ \Rightarrow$ both roots s_1 and s_2 are negative and equal roots.

$\beta = 0$, $s_1 = s_2 = -\alpha$

$\therefore\ i(t) = e^{-\alpha t}(K_1 + K_2t)$

A typical critically damped response is shown in Figure (b). In fact, Figure (b) is a sketch of $i(t) = te^{-\alpha t}$ which reaches a maximum value of e^{-1}/α at $t = 1/\alpha$, one time constant, and then decays all the way to zero.

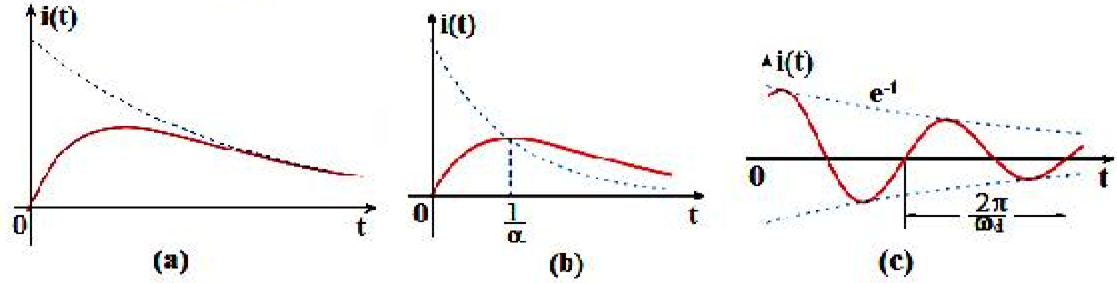
3- <u>Oscillatory (Under)-Damping:</u> $\alpha < \omega_0$ $\Rightarrow$ both roots s_1 and s_2 are unequal and complex conjugate roots.

$s_1 = -\alpha + j\beta = -\alpha + \omega_d$

$s_2 = -\alpha - j\beta = -\alpha - \omega_d$

$\therefore\ i(t) = e^{-\alpha t}(K_1 cos\beta t + K_2 sin\beta t)$

With the presence of sine and cosine functions, it is clear that the natural response for this case is exponentially damped and oscillatory in nature. The response has a time constant of $1/\alpha$ and a period of $T = 2\pi/\omega_d$ Figure (c) depicts a typical under-damped response.



where:

α = damping factor (coefficient), neper frequency.

ω_0 = resonant radian frequency (rad/sec)

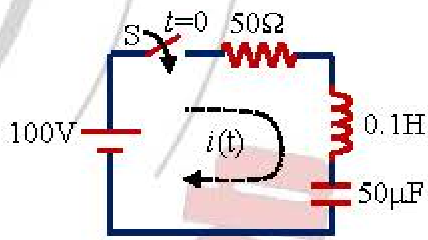
ω_d = damping radian frequency (rad/sec) = $\sqrt{\omega_0^2 - \alpha^2} = j\beta$

β = natural radian frequency (rad/sec)

<u>Example:</u> For the circuit shown, when the switch closed at t=0 and by assuming zero initial charge on the capacitor terminals. Find:

a. The transient current equation w.r.t. time.

b. The value of "C" that makes the circuit as critically-damping.



Solution:

a. $\alpha = \frac{R}{2L} = \frac{50}{2*0.1} = 250$

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1*50*10^{-6}}} = 447.214 rad/sec$

Since $\alpha < \omega_0$ we conclude that the response is under-damped. This is also evident from the fact that the roots are complex and negative. The roots are:

$s_1 = -\alpha + j\sqrt{\omega_0^2 - \alpha^2} = -250 + j\sqrt{(447.214)^2 - 250^2} = -250 + j371$

$s_2 = -\alpha - j\sqrt{\omega_0^2 - \alpha^2} = -250 - j\sqrt{(447.214)^2 - 250^2} = -250 - j371$

$\therefore\ i(t) = e^{-250t}(K_1 cos371t + K_2 sin371t)$

$i_0 = 0 = i(0) = e^0 K_1 cos0 \quad \Rightarrow \quad K_1 = 0$

$or \quad i(t) = K_2 e^{-250t} sin371t$

$L\frac{di(t)}{dt}\Big|_{t=0} = V \quad \Rightarrow \quad \frac{di(t)}{dt}\Big|_{t=0} = \frac{100}{0.1} = 1000 = 371K_2 e^0 cos0 \quad \Rightarrow \quad K_2 = 2.7$

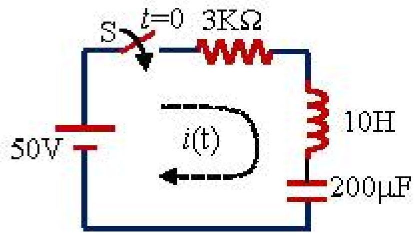
$\therefore\ i(t) = 2.7e^{-250t} sin371t \quad Amp. \quad t \geq 0$

b. In critical-damping $\beta = 0$

$(\frac{R}{2L})^2 = \frac{1}{LC} \quad \Rightarrow \quad C = \frac{1}{0.1*50*10^{-6}} * (\frac{2*0.1}{50})^2 = 3.2F$

Example: For the circuit shown, when the switch closed at t=0 and by assuming zero initial charge on the capacitor terminals. Find:

a. The transient current equation w.r.t. time.
b. Maximum current.
c. Natural frequency.
d. The value of "C" that makes the circuit as critically-damping.



Solution:

a. $\alpha = \frac{R}{2L} = \frac{3*10^3}{2*10} = 150$

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10*200*10^{-6}}} = 22.36 rad/sec$

Since $\alpha > \omega_0$ we conclude that the response is over-damped. This is also evident from the fact that the roots are real and negative. The roots are:

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -150 + \sqrt{150^2 - (22.36)^2} = -1.676$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -150 - \sqrt{150^2 - (22.36)^2} = -298.3$

$\therefore i(t) = K_1 e^{-1.676t} + K_2 e^{-298.3t}$

$i_0 = 0 = i(0) = K_1 e^0 + K_2 e^0 \quad \Rightarrow \quad K_1 = -K_2 \ldots (1)$

$L\frac{di(t)}{dt}\Big|_{t=0} = V \quad \Rightarrow \quad \frac{di(t)}{dt}\Big|_{t=0} = \frac{50}{10} = 5 = -1.676K_1 - 298.3K_2 \ldots (2)$

Solve equations "1" & "2" yields: K_1=0.0168 & K_2=-0.0168

$\therefore i(t) = 16.8(e^{-1.676t} - e^{-298.3t}) \ mAmp. \quad t \geq 0$

b. $\frac{di(t)}{dt} = 0 \quad \Rightarrow \quad t = 0.0175 sec \quad \Rightarrow \quad i_{max}(0.0175) = 0.0161\ Amp.$

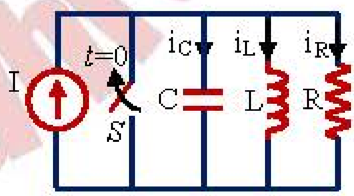
c. $\therefore \beta = \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}} = \sqrt{(\frac{3*10^3}{2*10})^2 - \frac{1}{10*200*10^{-6}}} = 148.3\ rad/sec$

$f_n = \frac{\beta}{2\pi} = 23.6\ Hz$

d. In critical-damping $\beta = 0$

$(\frac{R}{2L})^2 = \frac{1}{LC} \quad \Rightarrow \quad C = \frac{1}{10} * (\frac{2*10}{3*10^3})^2 = 4.44\mu F$

12.4 Transient Currents for Parallel Circuits:



Parallel RLC circuits find many practical applications, notably in communications networks and filter designs. Assume initial inductor current I_0 and initial capacitor voltage V_0,

$i(0) = I_0 = \frac{1}{L}\int_{\infty}^{0} v(t)dt$ and $v(0) = V_0$

Since the three elements are in parallel, they have the same voltage v across them. According to passive sign convention, the current is entering each element; that is, the current through each element is leaving the top node. Thus, applying KCL at the top node gives:

$0 = I_R + I_L + I_C = \frac{v}{R} + \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau + C\frac{dv}{dt} \quad at \quad t < 0$

Differentiating both sides with respect to t and dividing by C results in,

$\frac{d^2v}{d^2t} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$

The roots of the characteristic equation are:

$\therefore\ s_{12} = \frac{-1}{2RC} \mp \sqrt{(\frac{1}{2RC})^2 - \frac{1}{LC}}$

A more compact way of expressing the roots is

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \quad and \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

where $= \frac{1}{2RC}$, $\omega_0 = \frac{1}{\sqrt{LC}}$ and $\beta = \sqrt{(\frac{1}{2RC})^2 - \frac{1}{LC}} = \sqrt{\alpha^2 - \omega_0^2}$

Depending on the value of "β" there are three cases:

1- **Over-Damping:** $\alpha > \omega_0 \Rightarrow$ s_1 and s_2 are real, negative and unequal roots.

$s_1 = -\alpha + \beta$ and $s_2 = -\alpha - \beta$

$\therefore\ v(t) = K_1 e^{(-\alpha+\beta)t} + K_2 e^{(-\alpha-\beta)t} = e^{-\alpha t}(K_1 e^{\beta t} + K_2 e^{-\beta t})$

2- **Critical-Damping:** $\alpha = \omega_0 \Rightarrow$ s_1 and s_2 are real, negative and equal roots.

$\beta = 0 \quad \Rightarrow \quad s_1 = s_2 = -\alpha$

$\therefore\ v(t) = e^{-\alpha t}(K_1 + K_2 t)$

3- **Oscillatory (Under)-Damping:** $\alpha < \omega_0 \Rightarrow$ s_1 and s_2 are unequal and complex conjugate roots.

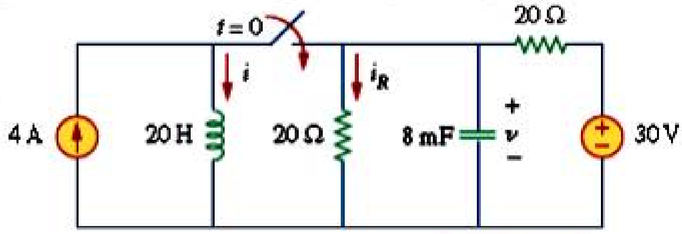
$s_1 = -\alpha + j\beta = -\alpha + \omega_d$

$s_2 = -\alpha - j\beta = -\alpha - \omega_d$

$\therefore\ v(t) = e^{-\alpha t}(K_1 cos\beta t + K_2 sin\beta t)$

where $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = j\beta$ is damping radian frequency (rad/sec).

Example: For the circuit shown with zero initial charge, Find the transient currents $i(t)$ & $i_R(t)$ for $t \geq 0$, when the switch closed at $t=0$.



Solution:

For t<0 the switch is open, and the circuit is partitioned into two independent sub-circuits. The 4-A current flows through the inductor, so that: $i(0) = 4A$.

The capacitor acts like an open circuit and the voltage across it is the same as the voltage across the 20Ω resistor connected in parallel with it. By voltage division, the initial capacitor voltage is

$v(0) = 30 * \frac{20}{20+20} = 15V$

For t>0 the switch is closed, and we have a parallel RLC circuit with a current source. The voltage source is zero which means it acts like a short-circuit. The two 20Ω resistors are now in parallel. They are combined to give $= 20//20 = 10\Omega$. The characteristic roots are determined as follows:

$\alpha = \frac{1}{2RC} = \frac{1}{2\times10\times8\times10^{-3}} = 6.25$

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20\times8\times10^{-3}}} = 2.5\ rad/sec$

$s_1 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -6.25 \pm \sqrt{39.0625 - 6.25} = -6.25 \pm 5.7282$

$\therefore\ s_1 = -11.978, \quad s_2 = -0.5218$

Since $\alpha > \omega_0$ so we have the over-damped case. Hence,

$i(t) = K_1 e^{-11.978t} + K_2 e^{-0.5218t}$

$i(0) = 4 = K_1 + K_2 \quad \Rightarrow \quad K_1 = 4 - K_2$

$\frac{di(0)}{dt} = \frac{v(0)}{L} \quad \Rightarrow \quad -11.978K_1 - 0.5218K_2 = \frac{15}{20} = 0.75$

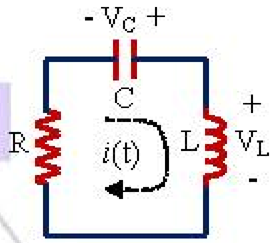
Solving the above equations for K_1 and K_2 yields:

$K_1 = -0.247$ and $K_2 = 4.247$

$\therefore i(t) = -0.247e^{-11.978t} + 4.247e^{-0.5218t} Amp.\ t \geq 0$

$i_R(t) = \frac{v(t)}{20} = \frac{L}{20}\frac{di}{dt} = 2.959e^{-11.978t} - 2.216e^{-0.5218t} Amp.\ t \geq 0$

12.5 Source Free Circuits:



A source-free RLC circuit occurs when its dc source is suddenly disconnected. The energy already stored in the capacitor is released to the resistors. Also, the energy already stored in the inductor is released to the resistors.

a. Series RLC: The circuit is being excited by the energy initially stored in the capacitor and inductor. The energy is represented by the initial capacitor voltage V_0 and initial inductor current I_0. Thus, at $t=0$,

$v_C(0) = \frac{1}{C}\int_{-\infty}^{0} i dt = V_0$ and $i(0) = I_0$

Apply KVL:

$V_R + V_L + V_C = \mathrm{Ri} + \mathrm{L}\frac{\mathrm{di}}{\mathrm{dt}} + \frac{1}{\mathrm{C}}\int_{-\infty}^{\mathrm{t}} \mathrm{i}(\tau)\mathrm{d}\tau = 0$

To eliminate the integral, we differentiate both sides with respect to t and rearrange terms. We get

$\frac{\mathrm{d^2i}}{\mathrm{d^2t}} + \frac{\mathrm{R}}{\mathrm{L}}\frac{\mathrm{di}}{\mathrm{dt}} + \frac{1}{\mathrm{LC}}\mathrm{i} = 0$

$\therefore s_{12} = \frac{-R}{2L} \mp \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}}$

where $\alpha = \frac{R}{2L}$ and $\omega_0 = \frac{1}{\sqrt{LC}}$

$\beta = \sqrt{(\frac{R}{2L})^2 - \frac{1}{LC}} \quad \Rightarrow \quad \omega_d = j\beta = \sqrt{\omega_0^2 - \alpha^2}$

Depending on the value of "β" there are three cases for i:

1- <u>Over-Damping (Natural response):</u> $\alpha > \omega_0 \quad \Rightarrow$ s_1 and s_2 are real and unequal roots.

$s_1 = -\alpha + \beta$ and $s_2 = -\alpha - \beta$

$\therefore i = K_1 e^{(-\alpha+\beta)t} + K_2 e^{(-\alpha-\beta)t} = e^{-\alpha t}(K_1 e^{\beta t} + K_2 e^{-\beta t})$

2- <u>Critical-Damping:</u> $\alpha = \omega_0 \Rightarrow$ s_1 and s_2 are equal roots.

$\beta = 0 \ , \ s_1 = s_2 = -\alpha$

$\therefore i = e^{-\alpha t}(K_1 + K_2 t)$

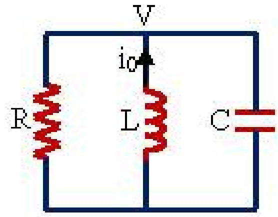
3- <u>Oscillatory (Under)-Damping:</u> $\alpha < \omega_0 \Rightarrow$ s_1 and s_2 are complex conjugate roots.

$s_1 = -\alpha + j\beta = -\alpha + \omega_d$

$s_2 = -\alpha - j\beta = -\alpha - \omega_d$

$\therefore i = e^{-\alpha t}(K_1 cos\omega_d t + K_2 sin\omega_d t)$

b. Parallel RLC:



Consider the parallel RLC circuit shown in figure beside Assume initial inductor current I_0 and initial capacitor voltage V_0,

$i(0) = I_0 = \frac{1}{L}\int_{\infty}^{0} v(t)\,dt$ and $v(0) = V_0$

Apply KCL:

$$I_R + I_L + I_C - i(t_0) = \frac{v}{R} + \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau + C\frac{dv}{dt} = 0$$

Taking the derivative of both sides with respect to t and dividing by C results in

$$\frac{d^2v}{d^2t} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

$$\therefore s_{12} = \frac{-1}{2RC} \mp \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

where $\alpha = \frac{1}{2RC}$ and $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\therefore \beta = \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \quad \Rightarrow \quad \omega_d = j\beta = \sqrt{\omega_0^2 - \alpha^2}$$

Depending on the value of "β" there are three cases:

1- <u>Over-Damping (Natural response):</u> $\alpha > \omega_0 \;\Rightarrow\; s_1$ and s_2 are real and unequal roots.

$s_1 = -\alpha + \beta$ and $s_2 = -\alpha - \beta$

$\therefore v(t) = K_1 e^{(-\alpha+\beta)t} + K_2 e^{(-\alpha-\beta)t} = e^{-\alpha t}(K_1 e^{\beta t} + K_2 e^{-\beta t})$

2- <u>Critical-Damping:</u> $\alpha = \omega_0 \;\Rightarrow\; s_1$ and s_2 are equal roots.

$\beta = 0$, $s_1 = s_2 = -\alpha$

$\therefore v(t) = e^{-\alpha t}(K_1 + K_2 t)$

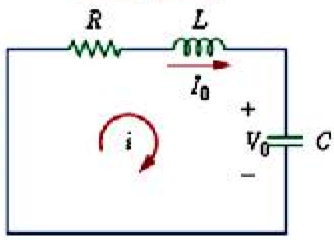
3- <u>Oscillatory (Under)-Damping:</u> $\alpha < \omega_0 \;\Rightarrow s_1$ and s_2 are complex conjugate roots.

$s_1 = -\alpha + j\beta = -\alpha + \omega_d$

$s_2 = -\alpha - j\beta = -\alpha - \omega_d$

$\therefore v(t) = e^{-\alpha t}(K_1 cos\omega_d t + K_2 sin\omega_d t)$

<u>**Example:**</u> In figure shown beside, R=40Ω, L=4H and C=0.25F. Calculate the characteristic roots of the circuit. Is the natural response over-damped, under-damped, or critically damped? Then write the current equation.



Solution: We first calculate

$\alpha = \frac{R}{2L} = \frac{40}{2\times4} = 5$ and $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\times0.25}} = 1\; rad/sec$

Since $\alpha > \omega_0$ we conclude that the response is over-damped. This is also evident from the fact that the roots are real and negative.

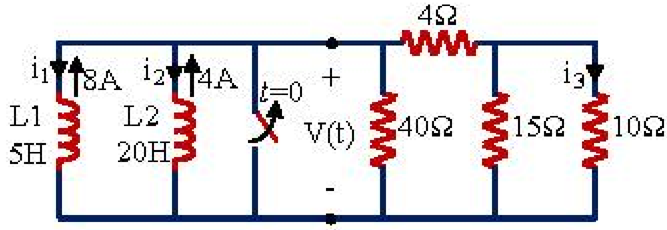
The roots are:

$s_{12} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5 \pm \sqrt{25 - 1} \quad \Rightarrow \quad s_1 = -0.101 \;\; and \;\; s_2 = -9.899$

$\therefore i(t) = K_1 e^{-0.101t} + K_2 e^{-9.899t}\; Amp.\; t \geq 0$

$V_C = \frac{1}{C}\int_0^t i dt = 4\left(\frac{K_1}{-0.101}e^{-0.101t} + \frac{K_2}{-9.899}e^{-9.899t}\right)_0^t \quad Volt \quad t \geq 0$

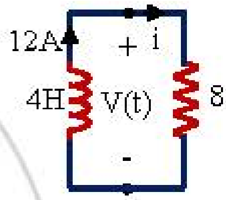
Example: Refer to the circuit shown, the initial currents of L_1 and L_2 have been established by sources not shown. The switch opens at t=0.



1. Find the transient currents i_1, i_2 and i_3.
2. Calculate the initial energy stored in L_1 and L_2.
3. Determine energy trapped in L_1 and L_2 as $t \to \infty$.
4. Show that the total energy delivered to resistors=difference between "2" and "3".

Solution:

1. $L_p = 5//20 = 4H$ and $R_p = 40//(4 + (15//10)) = 8\Omega$



$TC = Time\ Constant = \frac{L_P}{R_P} = \frac{4}{8} = \frac{1}{2} sec$

$\therefore\ i(t) = 12e^{-2t} \quad Amp. \quad t \geq 0$

$V(t) = R_p i(t) = 8 * 12e^{-2t} = 96e^{-2t} \quad Volt \quad t \geq 0$

$i_1 = \frac{1}{5}\int_0^t 96e^{-2t}dt - 8 = 1.6 - 9.6e^{-2t} \quad Amp. \quad t \geq 0$

$i_2 = \frac{1}{20}\int_0^t 96e^{-2t}dt - 4 = -1.6 - 2.4e^{-2t} \quad Amp. \quad t \geq 0$

$i_3 = \frac{V(t)}{10} * \frac{10//15}{(10//15)+4} = 5.76e^{-2t} \quad Amp. \quad t \geq 0$

2. Initial energy stored in L_1 and L_2 is:

$W = \frac{1}{2} * 5 * (8)^2 + \frac{1}{2} * 20 * (4)^2 = 320 Jouls$

3. As $t \to \infty$ $\quad i_1 \to 1.6A \quad i_2 \to -1.6A$

$W = \frac{1}{2} * 5 * (1.6)^2 + \frac{1}{2} * 20 * (-1.6)^2 = 32\ Jouls$

4. $P = i^2(t)R_p = (12e^{-2t})^2 * 8 = 1152e^{-4t} \quad Watt$

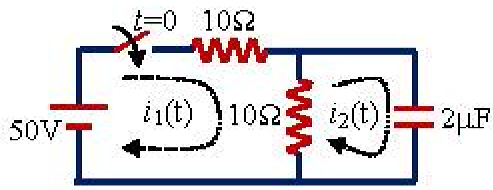
total energy delivered to resistors is:

$W = \int_0^\infty Pdt = \int_0^\infty 1152e^{-4t}dt = 1152\frac{e^{-4t}}{-4}\Big|_0^\infty = 288\ Jouls = 320 - 32$

12.6 General Second-Order Circuits:

Now that we have mastered series and parallel RLC circuits, we are prepared to apply the ideas to any second-order circuit having one or more independent sources with constant values. Although the series and parallel RLC circuits are the second-order circuits of greatest interest study. Given a second-order circuit, we determine its step response (which may be voltage or current) depending on the circuit condition.

Example: For the circuit shown with zero initial charge, Find the transient currents i_1 & i_2 and the transient voltage V_C across the capacitor, when the switch closed at t=0.



Solution:

Loop1: $20i_1 - 10i_2 = 50$ D.B.S

$\therefore\ 2\frac{di_1}{dt} = \frac{di_2}{dt}$ ... (1)

Loop2: $-10i_1 + 10i_2 + \frac{10^6}{2}\int i_2 dt = 0$ D.B.S

$\therefore\ -\frac{di_1}{dt} + \frac{di_2}{dt} 5 * 10^4 i_2 = 0$ ... (2)

Substitute equation "1" in "2" get:

$\frac{1}{2}\frac{di_2}{dt} + 5 * 10^4 i_2 = 0 \Rightarrow i_2(t) = Ke^{-10^5 t}$ Amp.

$i_{20} = \frac{50}{10} = i_2(0) = Ke^0 \Rightarrow K = 5$

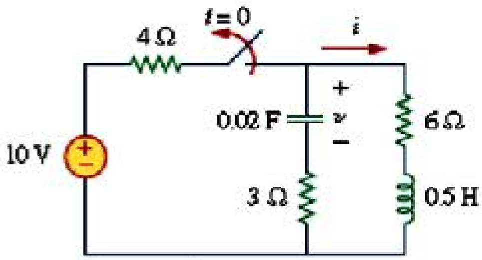
$\therefore\ i_2(t) = 5e^{-10^5 t}$ Amp. $t \geq 0$

Substitute i_2 in equation "1" get:

$i_1(t) = 2.5 + 2.5e^{-10^5 t}$ Amp. $t \geq 0$

$V_C = \frac{1}{C}\int i_2 dt = \frac{10^6}{2}\int_0^t 5e^{-10^5 t} dt = 25\left(1 - e^{-10^5 t}\right)$ Volt $t \geq 0$

Example: For the circuit shown with zero initial charge, Find the transient currents $i(t)$ and the transient voltage across the capacitor, when the switch opened at t=0. Assume that the circuit has reached steady state at t<0.



Solution:

For t<0, the switch is closed. The capacitor acts like an open circuit while the inductor acts like a shunted circuit. Thus, at t=0,

$i(0) = \frac{10}{4+6} = 1A$ and $v(0) = 6i(0) = 6V = 10 \times \frac{6}{4+6}$

For t>0 the switch is opened and the voltage source is disconnected, which is a source free series RLC circuit. Notice that the 3Ω and 6Ω resistors, which are in series when the switch is opened, have been combined to give 9Ω. The roots are calculated as follows:

$\alpha = \frac{R}{2L} = \frac{9}{2\times 0.5} = 9$ and $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5\times 0.02}} = 10\ rad/sec$

$s_{12} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -9 \pm \sqrt{81 - 100} = -9 \pm j4.359$

Hence, the response is under-damped ($\alpha < \omega_0$); that is,

$\therefore\ i(t) = e^{-9t}(K_1 cos4.359t + K_2 sin4.359t)$

We now obtain K_1 and K_2 using the initial conditions. At t=0,

$i(0) = 1 = K_1$

$\left.\frac{di}{dt}\right|_{t=0} = -\frac{1}{L}[Ri(0) + v(0)] = -2[9 \times 1 - 6] = -6A/s$

Note that $v(0) = V_0 = -6V$ is used, because the polarity of v is opposite to the direction of I_0. Taking the derivative of i(t),

$\left.\frac{di}{dt}\right|_{t=0} = -9e^{-9t}(K_1 cos4.359t + K_2 sin4.359t) + 4.359e^{-9t}(-K_1 sin4.359t +$

$K2cos4.359t = -9K1 + 0 + 4.359 - 0 + K2 = -6$

Using $K_1 = 1$ then $K_2 = 0.6882$

$\therefore\ i(t) = e^{-9t}(cos4.359t + 0.6882 sin4.359t) Amp.\quad t \geq 0$