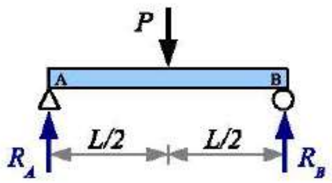
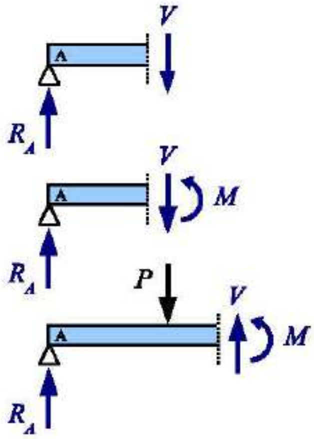
Shear Diagram

One Point Load

Imagine a simply-supported beam with a point load at the mid-span. Cut the beam to the left of the point load, and draw a free-body diagram of the beam segment. In a free-body diagram, forces must balance. Therefore, a downward force at the cut edge balances the support reaction R_A. We call this shear force V. It is a shear force because the force acts parallel to a surface (the cut edge of the beam).



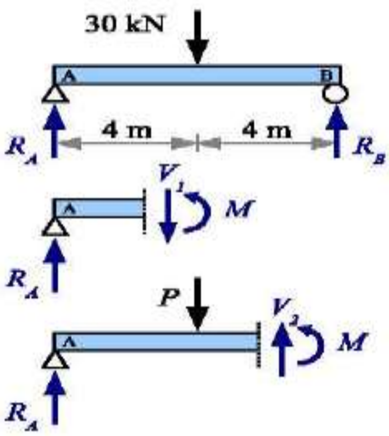


The forces R_A and V are in balance (equal in value; opposite in sign), but our segment wants to spin clockwise about point A. To counteract this tendency to spin, a moment M develops within the beam to prevent this rotation. The moment equals the shear force times its distance from point A.

Cut the beam to the right of the point load, and draw the free-body diagram. Since P is larger than R_A, force V points upwards.

Example 1:

Calculate the shear forces in this beam to the left and to the right of the 30 kN point load.



Solution The loading is symmetrical, so $R_A = R_B = \frac{P}{2} = \frac{30\,\text{kN}}{2} = 15\,\text{kN}$.

Use the sum of the forces to find V.

Between support A and point load P, $\uparrow + \ \sum F_y = 0 = R_A - V_1$.

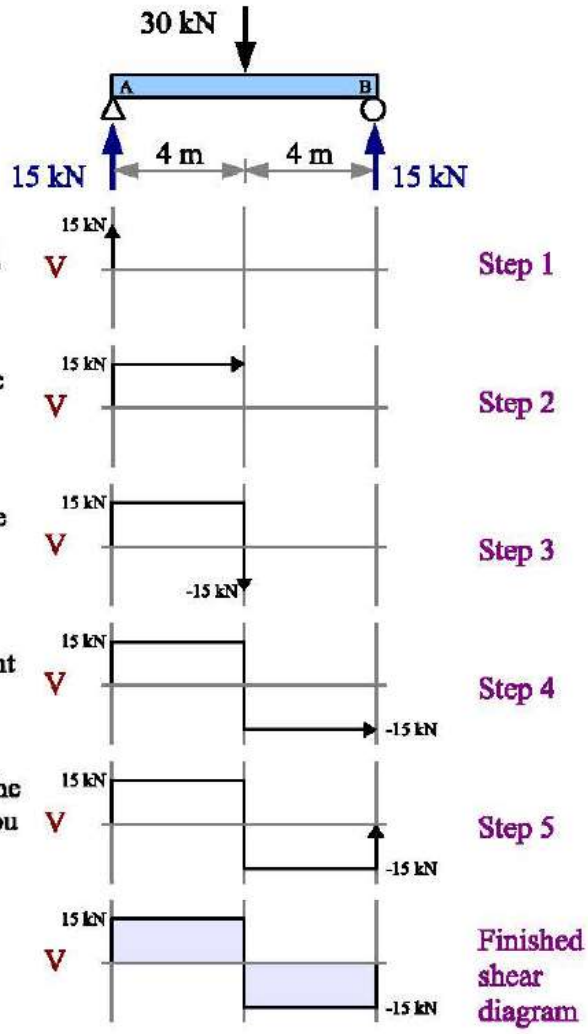
Solving for shear load, $V_1 = -R_A = -15\,\text{kN}$.

Between point load P and support B, $\uparrow + \ \sum F_y = 0 = R_A - P + V_2$.

Solving for shear load, $V_2 = -R_A + P = -15\,\text{kN} + 30\,\text{kN} = 15\,\text{kN}$.

We can sketch V as a function of location along the beam using a *Shear Diagram*. Draw vertical construction lines below the load diagram wherever the applied loads and reactions occur. Draw a horizontal construction line, indicating zero shear load. Next, draw the value of V along the length of the beam, as follows:

ملاحظة: في حالة الـ Point Load خط الـ Shear يكون مستقيم افقي.



Step 1 Starting at the left side of the shear diagram, go up 15 kN, because R_A is 15 kN upwards.

Step 2 There are no additional loads on the beam until you get to the midspan, so the shear value remains at 15 kN.

Step 3 The applied load at the midspan is 30 kN downwards, therefore the shear load is $15\,\text{kN} - 30\,\text{kN} = -15\,\text{kN}$.

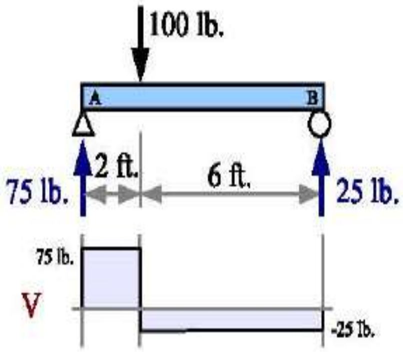
Step 4 There are no additional loads on the beam until you get to point B, so the shear value remains at -15kN.

Step 5 At point B, the reaction force $R_B = 15$ kN upwards, therefore the shear load is $-15\,\text{kN} + 15\,\text{kN} = 0$. If you don't get to 0, you know you made a mistake someplace.

Example 2:

Draw a complete shear diagram for a simply-supported 8 ft. beam with a 100 lb. point load 2 ft. to the right of point A.

Solution Use sum of the moments and sum of the forces to find the reaction forces R_A and R_B.



Starting the shear diagram at zero shear, go up $R_A = 75\text{lb.}$ at point A. There are no loads between point A and the applied point load, so the shear load does not change. Draw a horizontal line to the right, until you reach the point load. Draw a vertical line down 100 lb., reaching a value $V = -25\text{lb.}$ There are no loads between the point load and point B, so the shear load does not change. Draw a horizontal line to the right, until you reach point B. The reaction force at B is 25 lb. upwards, so draw a vertical line up 25 lb., reaching a value $V = 0$. Again we have two rectangles, but they are not symmetric; the beam carries three times as much shear load to the left of the point load than it does to the right.

Example 3:

Draw a complete shear diagram for a simply-supported 8 ft. beam with 100 lb.

Solution The loading is symmetrical, so the reaction forces equal half the total

Calculate the shear values as:

$V_1 = R_A = 150\,\text{lb.}$

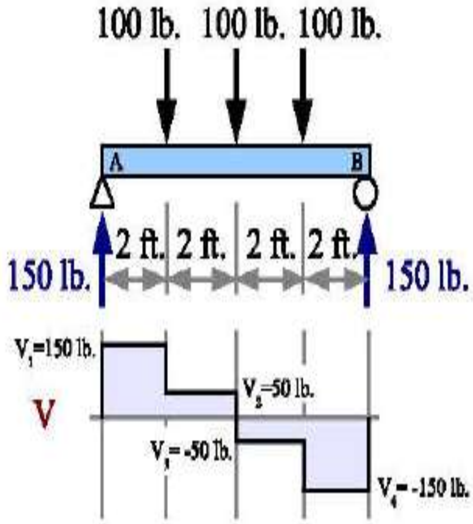
$V_2 = V_1 - 100\,\text{lb.} = 50\,\text{lb.}$

$V_3 = V_2 - 100\,\text{lb.} = -50\,\text{lb.}$

$V_4 = V_3 - 100\,\text{lb.} = -150\,\text{lb.}$

$V_5 = V_4 + R_B = -150\,\text{lb.} + 150\,\text{lb.} = 0$

In this problem, $|V|_{max} = 150\,\text{lb.}$

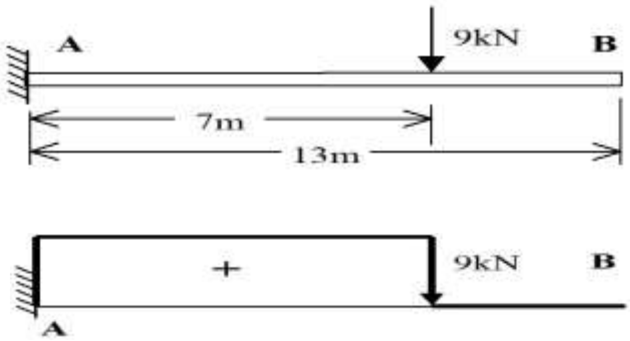


Example 4:

a) Calculate the support reactions;

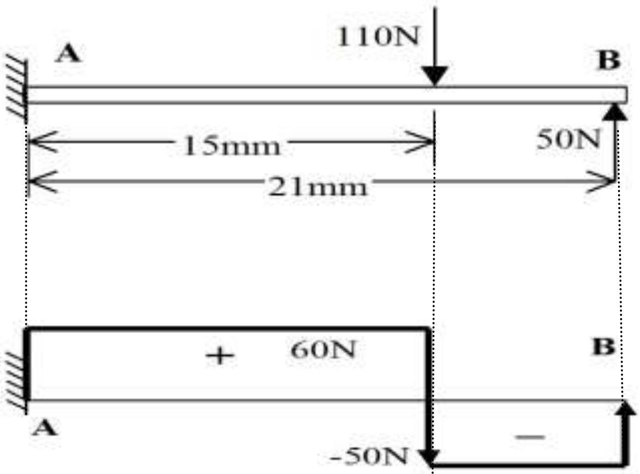
b) Draw the shear force diagram

$R_A = 9$ kN



Example 5:

$V_A = 110 - 50 = 60$ N

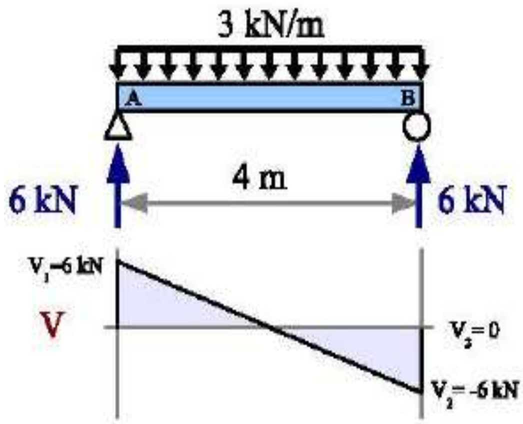


Shear diagram

Rectangular Uniform Load

ملاحظة: في حالة الـحمل المنتشر المستطيل (rectangular distributed load) خط الـ Shear يكون مستقيم مائل.

Example 1



Draw a complete shear diagram for a simply-supported 4 m beam with a uniformly distributed load of 3 kN/m.

Solution The loading is symmetrical, so the reaction forces equal half the total applied load. $R_A = R_B = \frac{W}{2} = \frac{wL}{2} = \frac{3\,\text{kN}\cdot 4\,\text{m}}{\text{m}\;\; 2} = 6\,\text{kN}$.

A complete shear diagram includes the values of the shear at locations of applied point loads and reaction forces.

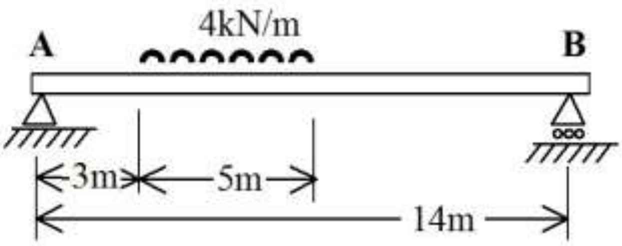
$V_1 = R_A = 6\,\text{kN}$

$V_2 = V_1 - \frac{3\,\text{kN}}{\text{m}} \underline{4\text{m}} = -6\,\text{kN}$

$V_3 = V_2 + R_B = -6\,\text{kN} + 6\,\text{kN} = 0$

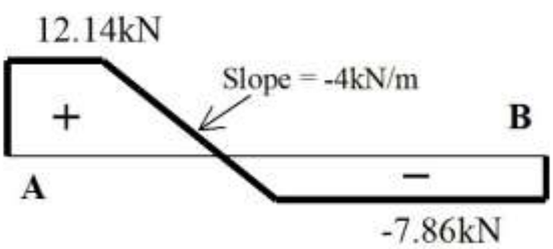
In this problem, $|V|_{max} = 6\,\text{kN}$

Example 2:



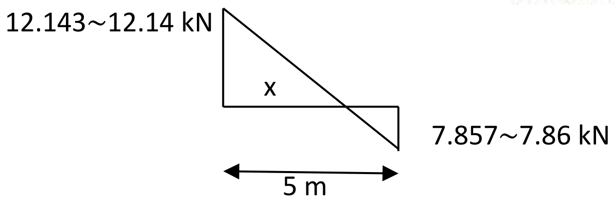
V_B = (5×4×(3+5/2))/14 = 7.857 kN

V_A = 4×5 – 7.857 = 12.143 kN



How to find slope?

من تشابه المثلثات نوجد قيمة x. ثم نجد الميل من حاصل قسمة المقابل على المجاور.



$$\frac{7.857}{12.143} = \frac{5-x}{x}$$

7.857*x=12.143(5-x)

X=3.03575 m

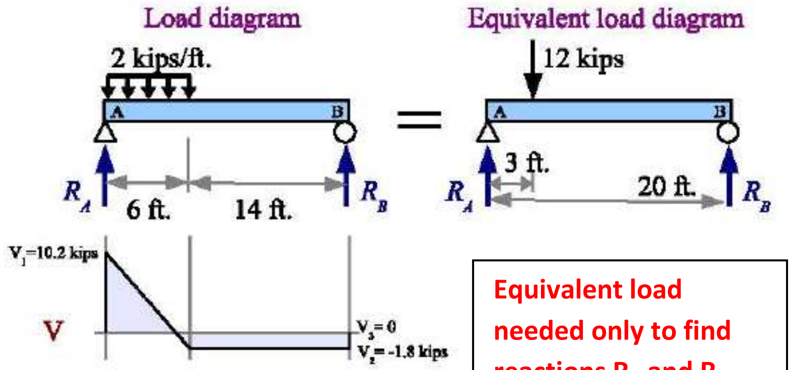
Slope=$\frac{12.143}{3.03575} = \frac{4kN}{m}$ $Descent$ therfore $\boldsymbol{minus}$

Example 3

Draw a complete shear diagram for a simply-supported 20 ft. beam which has a uniform distributed load of 2 kips/ft. running from the left end for 6 feet.

Solution Draw an equivalent load diagram, placing the equivalent load at the centroid of the distributed load. Use the equivalent load diagram to find the reaction forces.

The distributed load runs for 6 ft., so the location of the equivalent load is 3 ft. from the left end of the beam.



Equivalent load needed only to find reactions R_A and R_B. With experience NO need to do it

The equivalent load $W = wL = \frac{2\,\text{kips}\cdot 6\,\text{ft.}}{\text{ft.}} = 12\,\text{kips}$.

The moment about point A is ↶+ $\sum M_A = 0 = -12\,\text{kips}\cdot 3\,\text{ft.} + R_B \cdot 20\,\text{ft.}$.

Rewrite the equation to find the reaction force $R_B = \frac{12\,\text{kips}\cdot 3\,\text{ft.}}{20\,\text{ft.}} = 1.8\,\text{kips}$.

Sum of the forces ↑+ $\sum F_y = 0 = R_A - 12\,\text{kips} + 1.8\,\text{kips}$. Solve for the reaction force $R_A = 12\,\text{kips} - 1.8\,\text{kips} = 10.2\,\text{kips}$.

Draw construction lines down from the original load diagram wherever a point load or reaction exists, and wherever a distributed load starts or stops. Calculate the shear loads at these points.

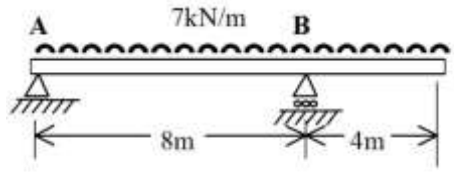
$V_1 = R_A = 10.2\,\text{kips}$

$V_2 = V_1 - \frac{2\,\text{kips}}{\text{ft.}}\,\frac{6\,\text{ft.}}{} = -1.8\,\text{kips}$

$V_3 = V_2 + R_B = -1.8\,\text{kips} + 1.8\,\text{kips} = 0$

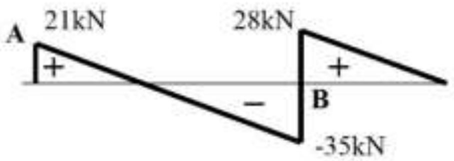
In this problem, $|V|_{max} = 10.2\,\text{kips}$

Example 4:



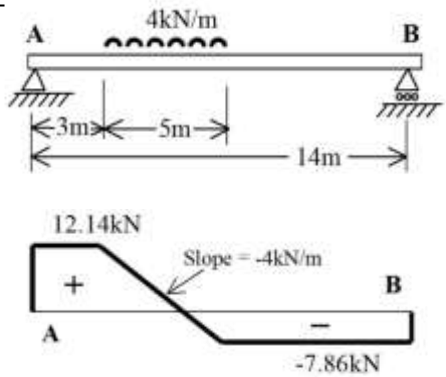
V_A = (8×7×4 – 4×7×2)/8 = 21.0 kN

V_B = 12×7 – 21.0 = 63.0 kN



H.W: Find the Slope of the Line?

Example 5:



V_B = (5×4×(3+5/2))/14 = 7.857 kN

V_A = 4×5 – 7.857 = 12.143 kN

Shear diagram

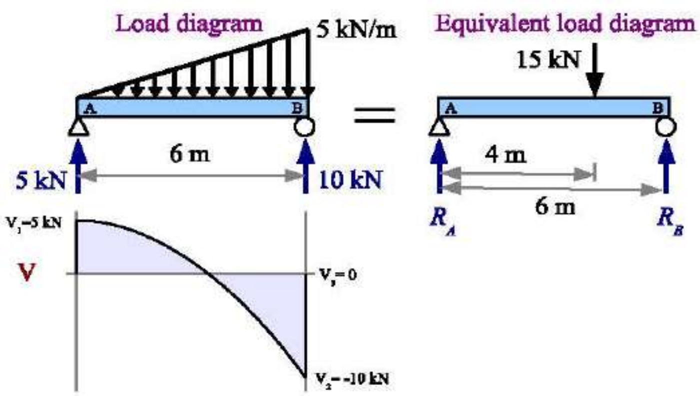
Triangle Uniform Load

ملاحظة: في حالة الحمل المنتشر المثلث (Triangle distributed load) خط الـ Shear يكون Parabola Curve.

Example 1:

Draw a complete shear diagram for a simply-supported 6 m beam which has a wedge-shaped nonuniformly distributed load of 0 kN/m at the left end of the beam to 5 kN/m at the right end of the beam.

Solution Draw an equivalent load diagram, placing the equivalent load at the centroid of the distributed load. The centroid of a triangle lies two-thirds of the distance from the point of the triangle. Use the equivalent load diagram to find the reaction forces.



The centroid is located at $x=\frac{2}{3}L=\frac{2\cdot 6\,\text{m}}{3}=4\,\text{m}$, measuring from the left end of the beam. The equivalent load is the average of the minimum and maximum distributed loads times the length of the distributed load: $W=\frac{(0+5)\,\text{kN}}{2\quad\text{m}}\,6\,\text{m}=15\,\text{kN}$.

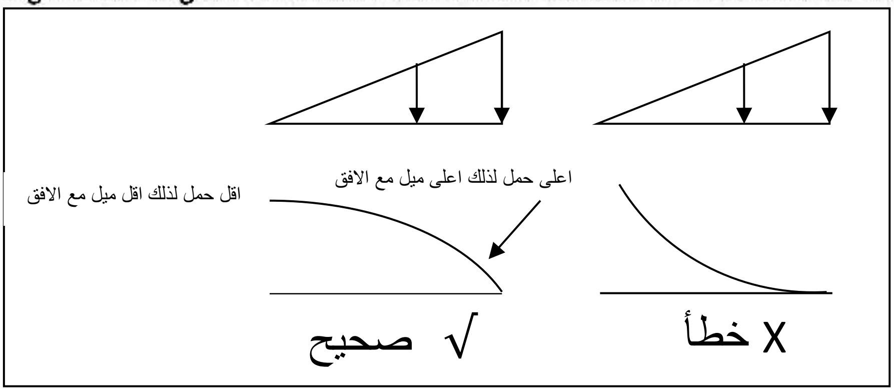
The moment about point A is $\sum M_A=0=-15\,\text{kN}\cdot 4\,\text{m}+R_B\cdot 6\,\text{m}$. Rewrite the equation to find the reaction force $R_B=\frac{15\,\text{kN}\cdot 4\,\text{m}}{6\,\text{m}}=10\,\text{kN}$. Sum of the forces $\uparrow+$ $\sum F_y=0=R_A-15\,\text{kN}+10\,\text{kN}$. Solve for $R_A=15\,\text{kN}-10\,\text{kN}=5\,\text{kN}$.

Draw construction lines down from the original load diagram at the two reaction forces. Calculate the shear loads at these points.

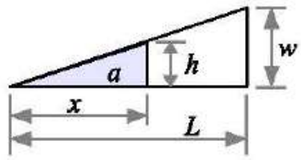
$V_1=R_A=5\,\text{kN}$

$V_2=V_1-15\,\text{kN}=-10\,\text{kN}$

$V_3=V_2+R_B=-15\,\text{kN}+15\,\text{kN}=0$



The shear line crosses the axis where the area of the distributed load equals R_A. The area of the distributed load is one half the base times the height of the triangle, or $a=\frac{xh}{2}$. By similar triangles, the height of the little triangle $h=\frac{x}{L}w$, so $a=\frac{x^2 w}{2L}=R_A$. Solving, $x=\sqrt{\frac{2LR_A}{w}}=\sqrt{\frac{2 \cdot 6\text{ m} \cdot 5\text{ kN}}{}\frac{\text{m}}{5\text{ kN}}}=3.46\text{ m}$. The shear line crosses the axis 3.46 m from the left end of the beam.



Look at the shear diagrams, and you can see that point loads create rectangles, uniform distributed loads create triangles, and wedge-shaped (triangular) distributed loads create parabolas.

Example 2:

1- from ∑M@A=0 and ∑Fy=0⇒

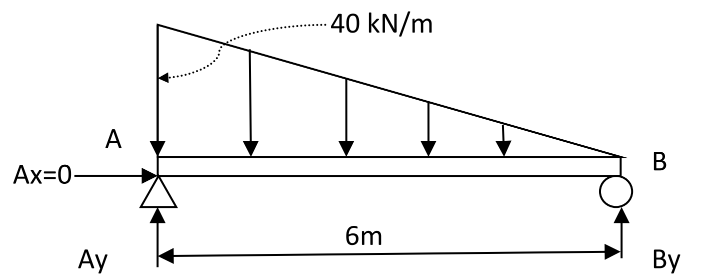
Ay=80 kN, By=40 kN

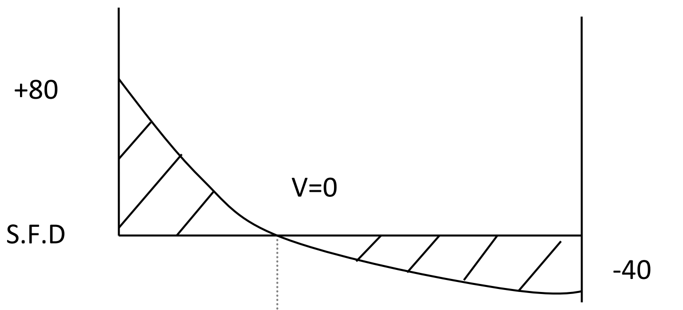
2- Draw shear Force Diagram.

3- Find x, and y

Q/how to find x and y @V=0?

Answer:

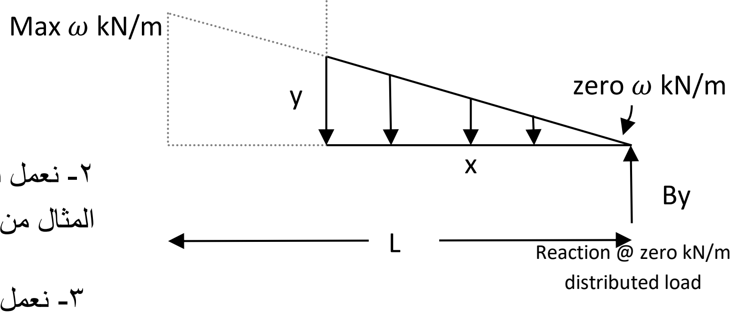




١- نعمل تشابه مثلثات

$\frac{\omega}{L}=\frac{y}{x} \quad \Rightarrow \quad \frac{40\,kN/m}{6\text{ m}}=\frac{y}{x}$

y=6.66 x



٢- نعمل قطع مسافة xعن الـ ω zero الى حد يكون فيه الـ shear =٠ وفي هذا المثال من جهة اليمين عندما يكون مقدار الـ reaction عند الـ support = ٤٠ kN

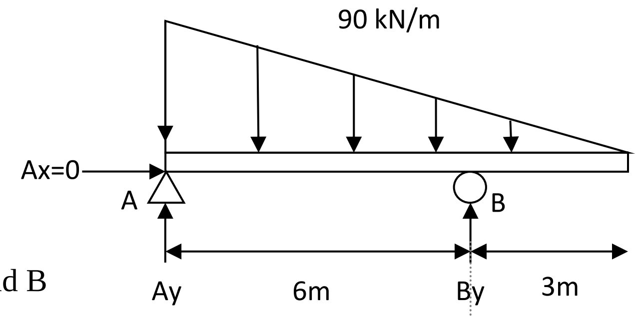
٣- نعمل ∑Fy=0 لهذا الجزء. لذلك فان قيمة الحمل المثلث = قيمة By.

$40\text{kN}=\frac{1}{2}*x*y \quad \Rightarrow \quad 2*40=x*6.66x \quad \Rightarrow x^2=12.01 \quad \Rightarrow \quad x=3.465\text{m}$

$\therefore y=6.66*3.465=23.09\ kn/m$

ملاحظة: ان مركز الـ equivalent triangle load يبعد ثلث المسافة عن الـ Max ω وهي ٢م في هذا المثال ولا تمثل موقع القص عندما يكون صفر لذلك نحن نقوم بحساب قيمة x

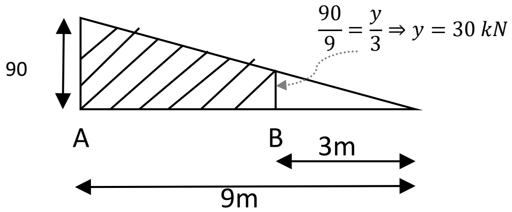
Example 3: Draw shear diagram and find x and y?

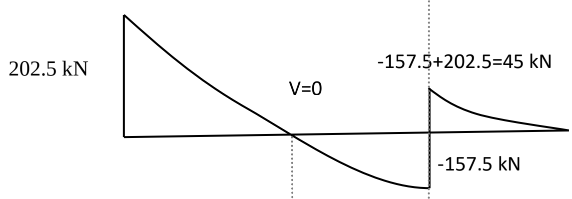


Ay= By=202.5 kN↑

Q/what is the shear value at B?

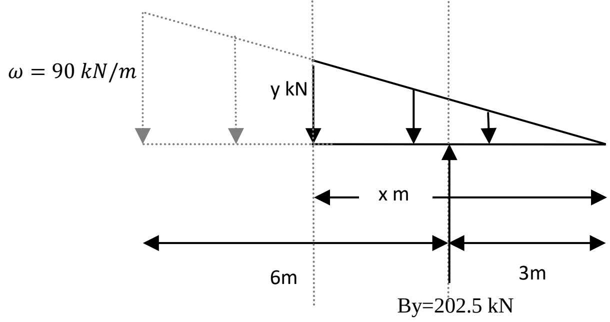
Answer: find the Shaded Trapezoidal area between A and B





Trapezoidal area between A and B=$\frac{30+90}{2}\times 6$=360 kN↓

↑∑Fy=0 ⇒+202.5-360=-157.5 kN shear value at B.



Q/ find x and y?

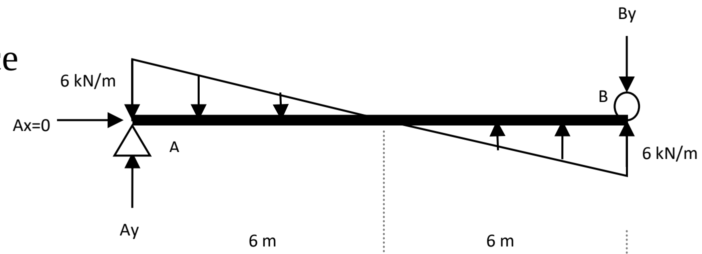
Answer: $\frac{\omega}{L}=\frac{y}{x}\Rightarrow y=10x$

Area of triangle=$\frac{1}{2}*y*x$

↑∑Fy=0 ⇒By=$\frac{1}{2}*y*x=202.5\ Kn\ \Rightarrow$

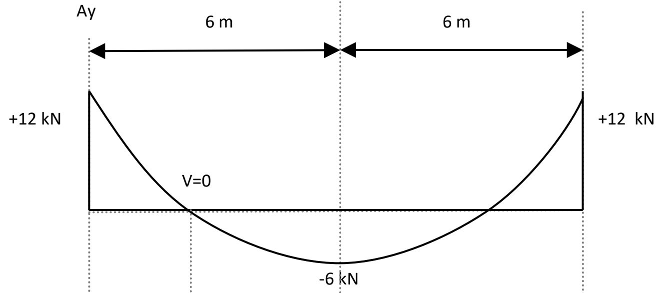
$\therefore x=6.36\ m\ and\ y=63.6\ kN$

Example 4: Draw S.F.D and find the distance from point A where the shear V=0. For the simply supported beam shown?



Answer: Ay =12 kN↑ , and By=12 kN↓

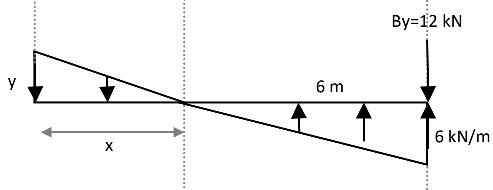
Shear @ $midspan =$



↑∑Fy=0⇒Ay-triangle area=+12-$\frac{6*6}{2}=-6\ kN$

$\frac{\omega}{6m}=\frac{y}{x}\quad\Rightarrow y=x$

↑∑Fy=0⇒$-\frac{1}{2}*x*y+\frac{1}{2}*6m*6kN/m-12kN=0$



x=3.464 m and y=3.464 kN

the distance from point A where the shear V=0 is 12-6-3.464=2.536 m

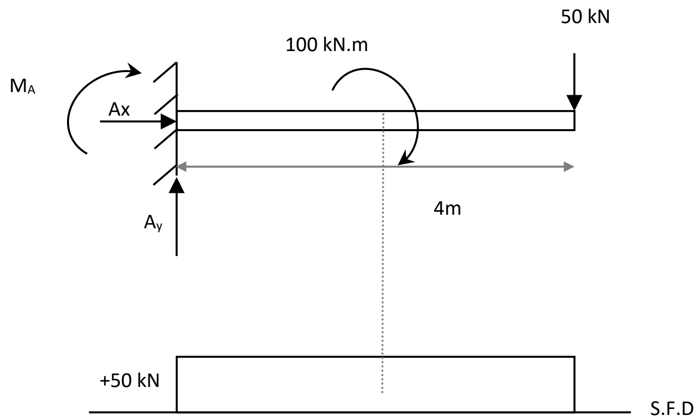
Shear diagram

Effect of a moment and Inclined Forces on S.F.D

ملاحظة: لا يتاثر مخطط القص Shear Force Diagram بالعزم المسلط على الـ beam. ولا يتأثر بالقوة المحورية Axial

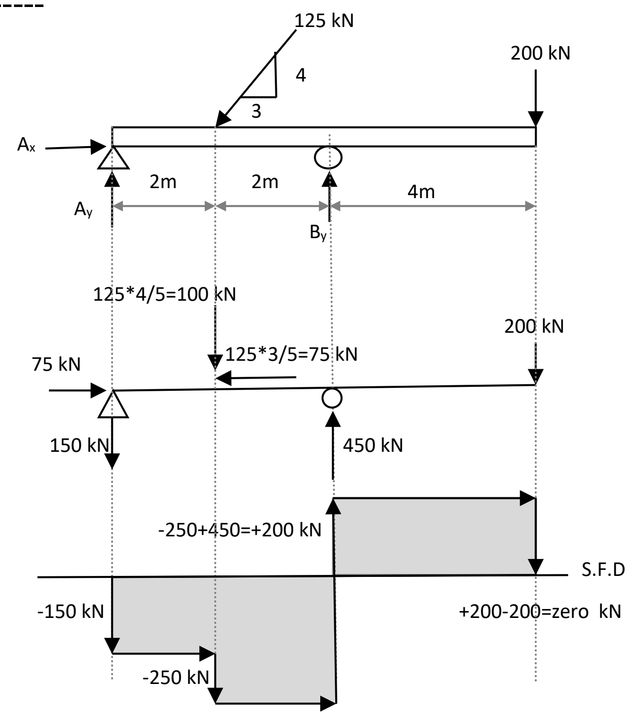
Example 1:

M_A=300 kN.m ↺ , A_y=50 kN↑ , A_x=0



Example 2:

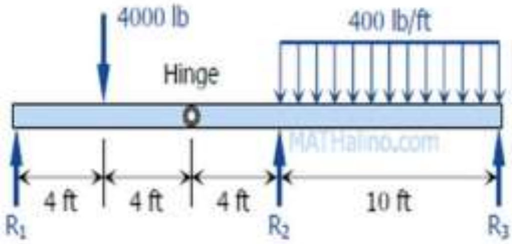
B_y=450 kN↑ , A_y=150 kN↓ , A_x=75 kN→

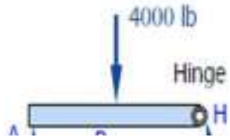


Shear diagram

Effect of a frictionless hinge (internal hinge) on S.F.D

Example 1:





$\Sigma M_H = 0$

$8R_1 = 4000(4)$

$R_1 = 2000$ lb

To draw the Shear Diagram

1. $V_A = 0$
2. $V_B = 2000$ lb

   $V_{B2} = 2000 - 4000 = -2000$ lb
3. $V_H = -2000$ lb
4. $V_C = -2000$ lb

   $V_C = -2000 + 4800 = 2800$ lb
5. $V_D = 2800 - 400(10) = -1200$ lb
6. Location of zero shear:

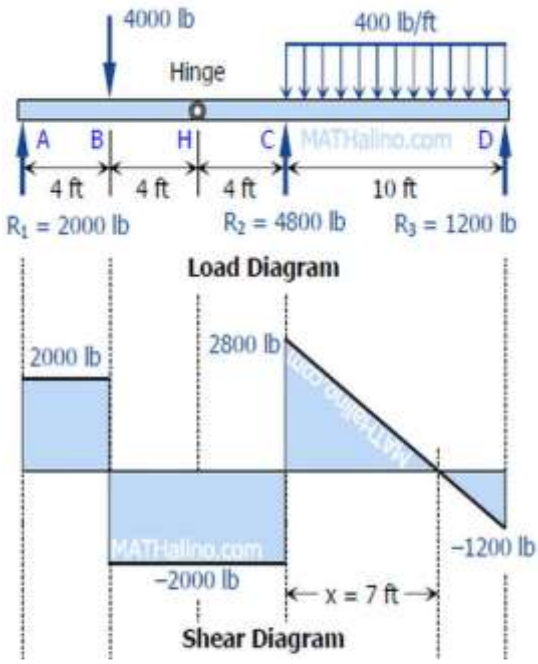
   x / 2800 = (10 - x) / 1200

   1200x = 28000 - 2800x

   x = 7 ft

$)(14) + 400(10)(5)$

$00) = 400(10)(9)$



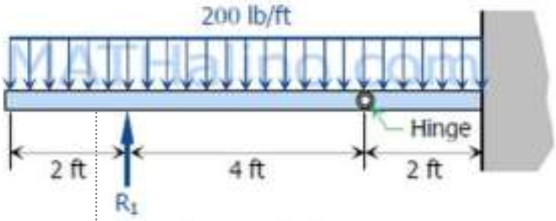
ملاحظة: ١- بوجود hinge داخلي يجب فصل الـ beam في منطقة الـ hinge لايجاد الـ reactions ونختار الجهة التي تحتوي reaction واحد لتسهيل الحل.

٢- توجد قوتين في المفصل hinge احدهما افقية والاخرى عمودية, ولعدم وجود قوة افقية في هذا المثال فلم تحتسب.

٣- لا يتاثر مخطط القص Shear diagram بوجود Hinge داخلي.

٤- يمكن تسليط اي حمل على المفصل الداخلي hinge.

Example 2:



From the FBD of the section to the left of hinge

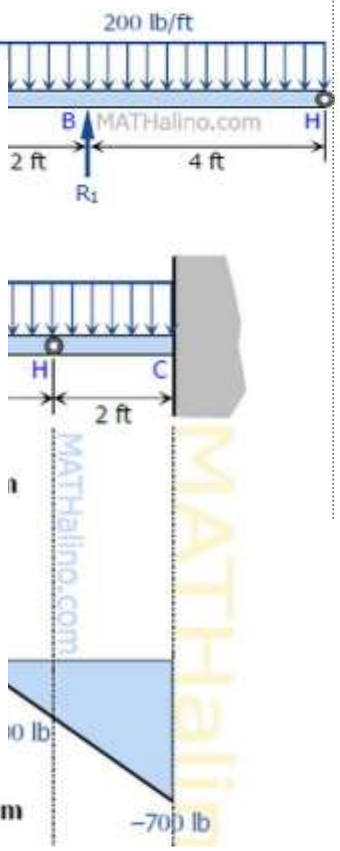
$\Sigma M_H = 0$

$4R_1 = 200(6)(3)$

$R_1 = 900\text{ lb}$

To draw the Shear Diagram

1. $V_A = 0$
2. $V_B = V_A$ + Area in load diagram
   $V_B = 0 - 200(2) = -400$ lb
   $V_{B2} = V_B + R_1 = -400 + 900 = 500$ lb
3. $V_H = V_{B2}$ + Area in load diagram
   $V_H = 500 - 200(4) = -300$ lb
4. $V_C = V_H$ + Area in load diagram
   $V_C = -300 - 200(2) = -700$ lb
5. Location of zero shear:
   $x / 500 = (4 - x) / 300$
   $300x = 2000 - 500x$
   $x = 2.5$ ft



Shear Diagram