Mustansyriah University
College of Engineering
Civil Engineering Department
$4^{th}$ Stage

# Reinforced Concrete-II

## Yield Line Analysis of Slabs

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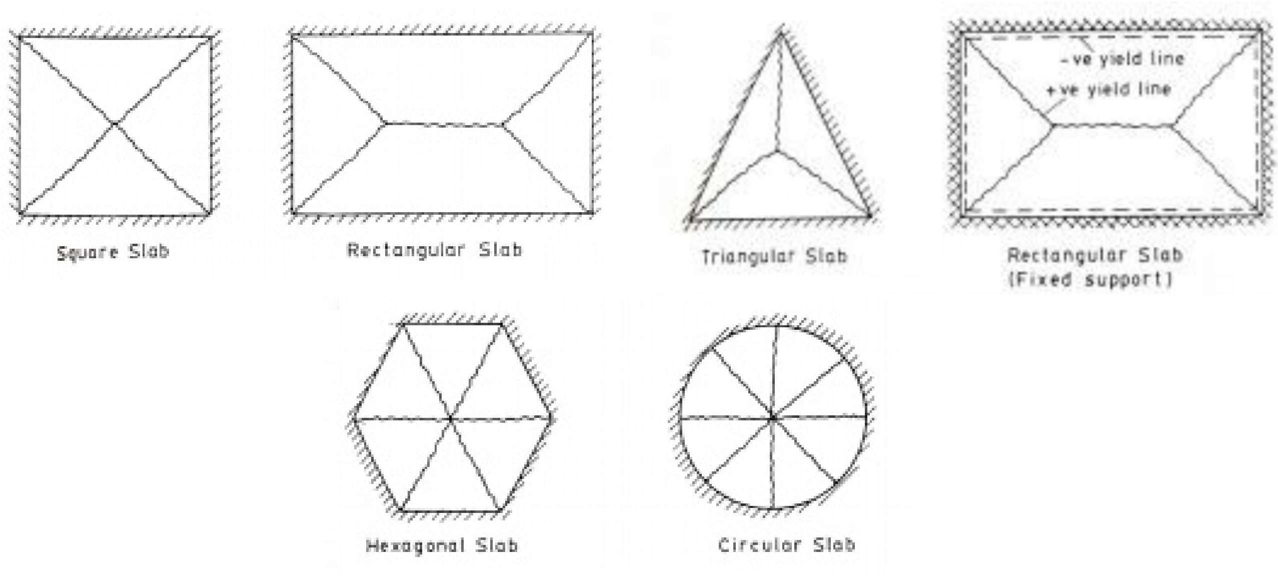
## 1-Yield Line Analysis of Slabs

A method of slab analysis, which permits the determination of failure moments in slabs of irregular, as well as rectangular shapes for a variety of support conditions and loading.




In a slab failing in flexure, the reinforcement will yield first in a region of high moment. When that occurs, this portion of the slab acts as a plastic hinge, only able to resist its hinging moment. When the load is increased further, the hinging region rotates plastically, and the moments due to additional loads are redistributed to adjacent sections, causing them to yield. The bands in which yielding has occurred are referred to as yield lines and divide the slab into a series of elastic plates. Eventually, enough yield lines exist to form a plastic mechanism in which the slab can deform plastically without an increase in the applied load.

In the yield-line method for slabs, the loads required to develop a plastic mechanism are compared directly to the plastic resistance (nominal strength) of the member.

### For the Hinged Slab (Simply Supported)

As load increase, $M_{max} \rightarrow M_{ult.}$

→Tension steel yields along line of $M_{max}$

→Curvature increase sharply,

→Deflection increase sharply,

→ "Plastic Deformation",

→ a "Hinge" form at "Yield Line",

→ "Plastic Hinge",

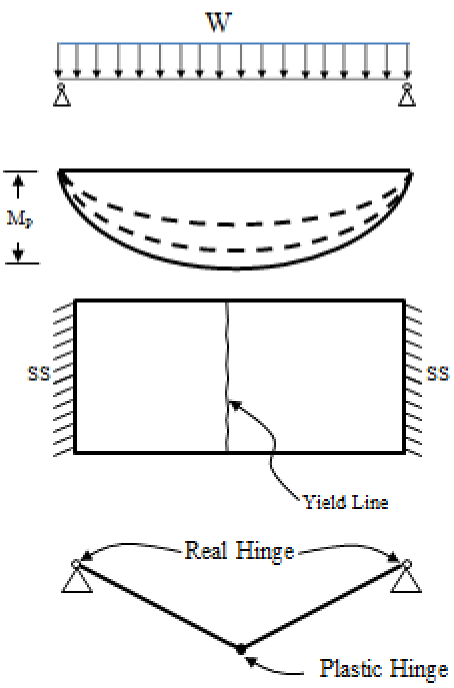
At plastic hinge, $M_P = M_n$ ,

A "Mechanism" forms (segment of the slab move),

→ Failure (Collapse).

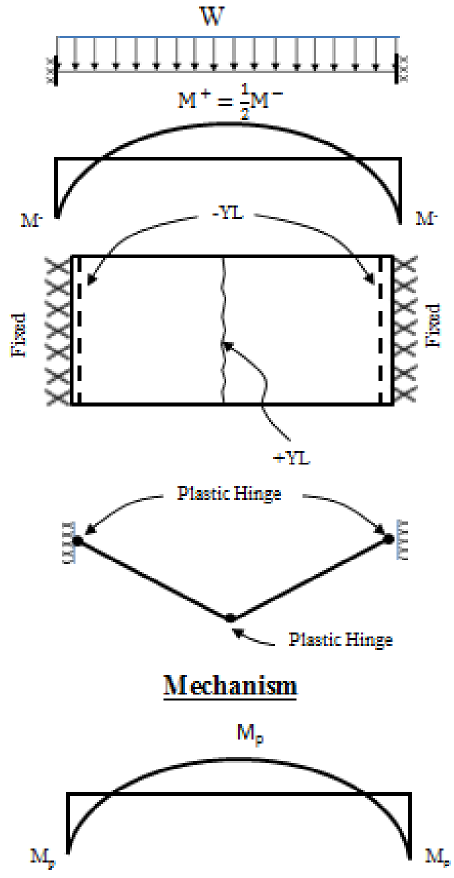
**Note**

$$M_n = \rho . b . d^2 . f_y \, (1 - 0.59 \rho \frac{f_y}{f_c'})$$




**Mechanism**

## For the Fixed Slab (Intermediate)




As load increase, $M^-_{max}$ and $M^+_{max} \rightarrow M_{ult.}$

→Tension steel yields at critical sections,

→Rotation occurs,

→At supports $M^-=M_p$ then "Two Plastic Hinge" forms,

→At mid span $M^+=M_p$ a third "Plastic Hinge" forms,

→ A "Mechanism" forms (segment of the slab move),

→Unstable,

→ Failure (Collapse).

## 2-Mechanism

The segment of the slab between the hinge and the supports are able to move without an increase in load (collapse).

## 3-Location of Yield Line

Yield Line location and orientation is evident in simple and fixed slab. For the other cases, the axis of rotation will be located along the lines of supports or over point supports (columns). The slab segments rotate as rigid bodies about these axes of rotation.

+YL= Associated with tension at BOTTOM of slab.

-YL= Associated with tension at TOP of slab.

## 4-Guide lines for drawing axes of rotation and yield lines

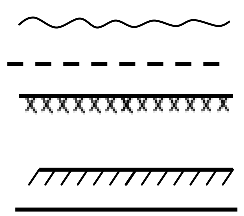
1-Yield lines are straight.

2-Axes of rotation lie along lines of support (the support line may be a real hinge or it may establish the location of a yield line which acts as a plastic hinge).

3-Axes of rotation pass over columns (supports).

4-A yield line (or its extend) passes through the intersection of the axes of rotation of adjacent slab segments.

## Notion



Positive Yield Line
Negative Yield Line
Fixed Support

Simple Support
Free Edge

## Isotropically Reinforced Slab

The resisting moment is the same along any line regardless of its location.

## Orthogonally Anisotropic (or orthotropic) Reinforced Slab

The resisting moments are different in two perpendicular directions.

## 5-Method of Analysis

There are two methods of analysis according to Yield Line:-

1-Virtual work method (Mechanism method).

2-Equilibrium method (Statical method).

## 6-Yield Line Analysis by the Virtual work method

There are two types of Virtual work:-

### 1-Translational Virtual work

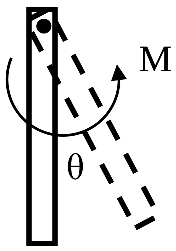
In this case, the displacement ($\Delta$) shall be in the same direction of force (P) and the work done is:-

$\boxed{\text{Work done} = P \times \Delta}$


### 2-Rotational Virtual work

In this case, the rotation ($\theta$) shall be in the same direction of moment (M) and the work done is:-

$\boxed{\text{Work done} = M \times \theta}$




**The external work done by the loads to cause a small Virtual deflection= The internal work done by the slab as rotates at the Yield Line.** Thus, a relation between the loads and the ultimate resisting moments of the slab is obtained.

$$\boxed{\textbf{External work } (W_E) = \textbf{Internal work } (W_I)}$$

**External work** ($W_E$) =Load x Displacement (Deflection)

→For concentrated load, ($W_E$)= $P x \delta$

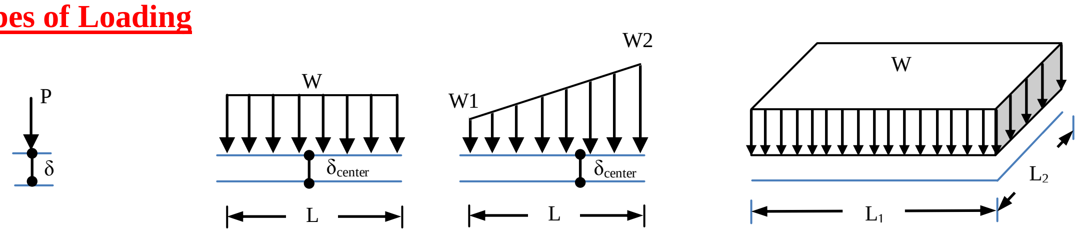
→For line load, ($W_E$)= $W x\ L\ x \delta_{center}$

→For uniformly distributed load, ($W_E$)= $W x$ Area x $\delta_{center}$

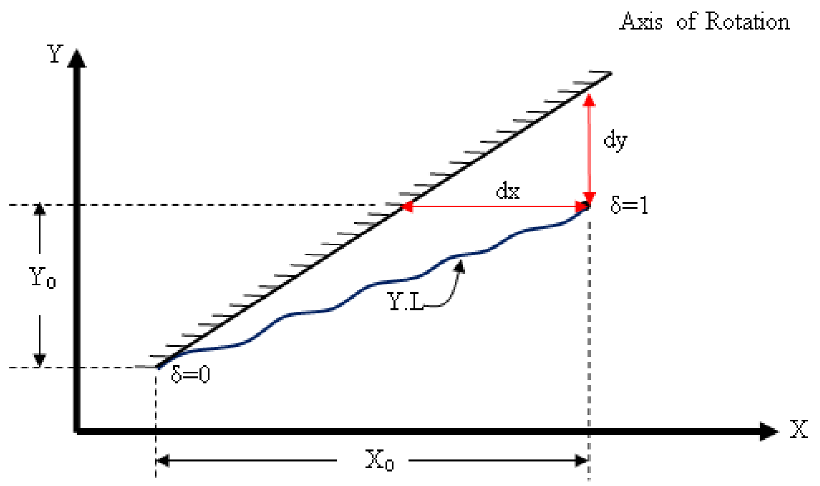
**Internal work** ($W_I$) =Moment x Rotation

$= m \times L \times \theta$

## 7-Types of Loading




## 8-Rotation about Inclined Axis




$Y_o$=Projection of Yield Line on Y-Axis.
$X_o$=Projection of Yield Line on X-Axis.
dy=Vertical distance measured from (δ) to Axis of Rotation.
dx=Horizontal distance measured from (δ) to Axis of Rotation.
δ=Deflection at any point lie along Yield Line.

**Internal work** $(W_I) = m*L_x*\theta_x + m*L_y*\theta_y = m*X_o*\frac{\delta}{dy} + m*Y_o*\frac{\delta}{dx}$

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