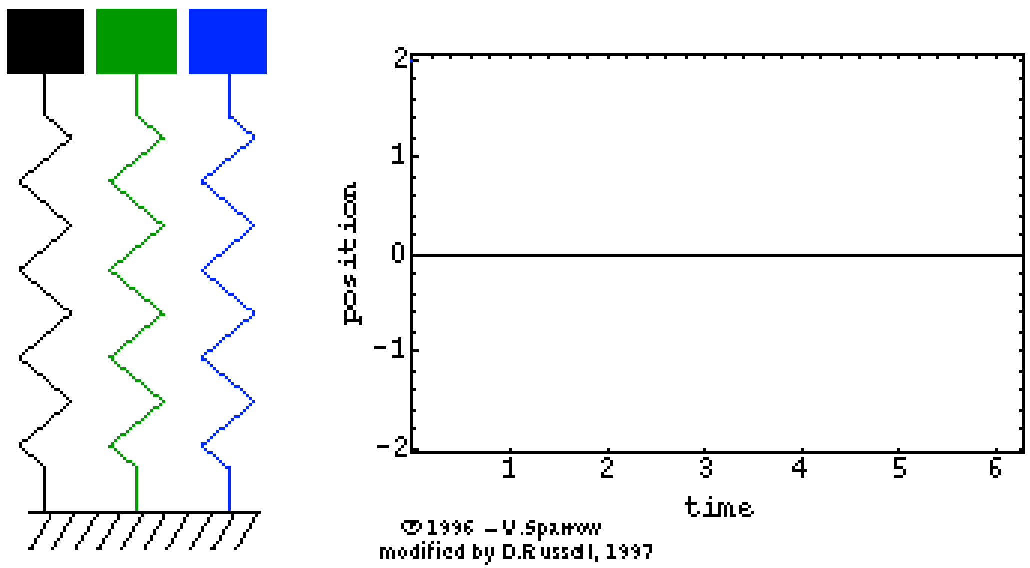
Introduction to Mechanical Vibrations

Dr. Muhanad Nazer Mustafa

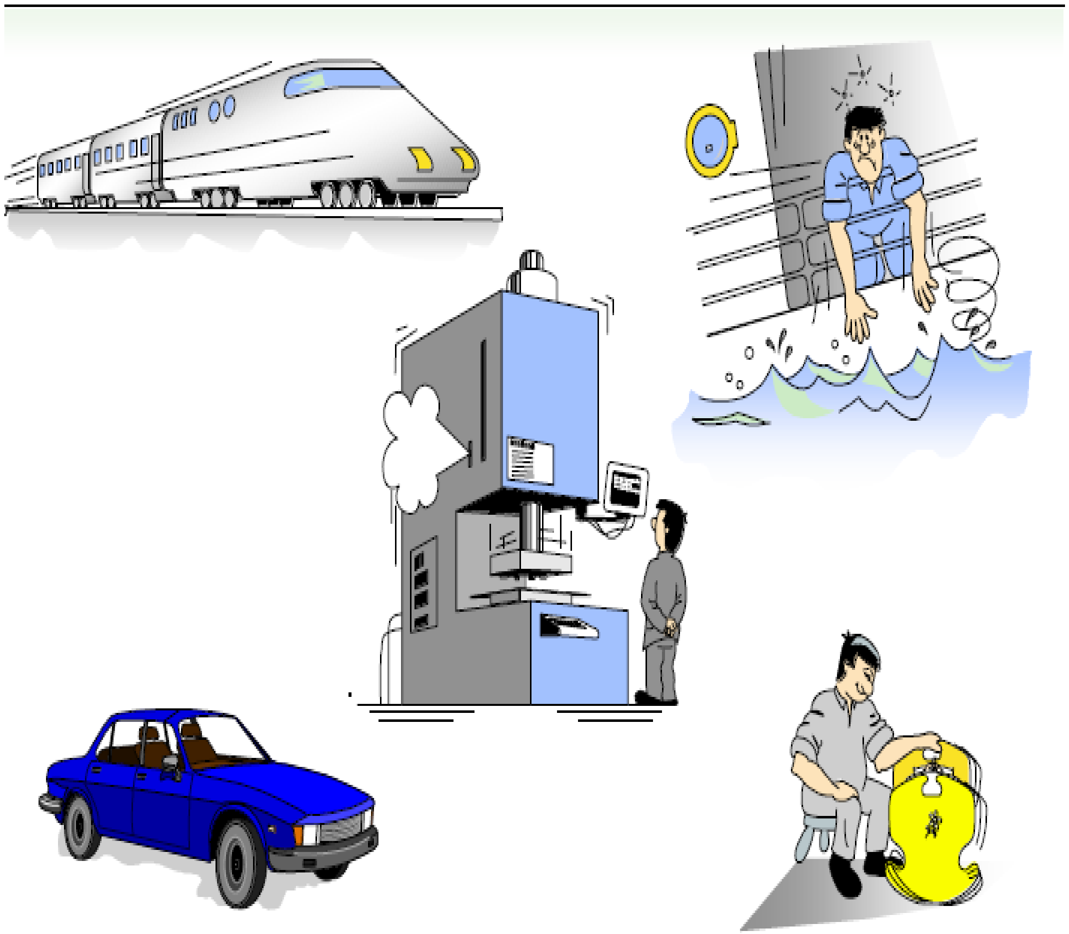
Lec. Ahmed Ali Farhan

What is vibration?

- Vibrations are oscillations of a system about an equilbrium position.



Vibration…

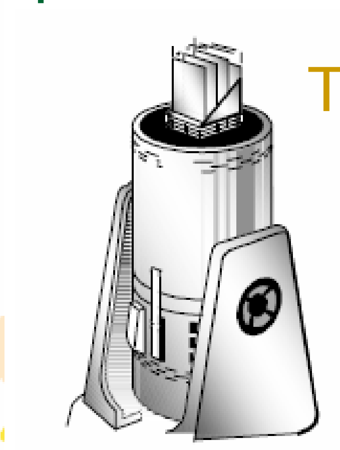


It is also an everyday phenomenon we meet on everyday life

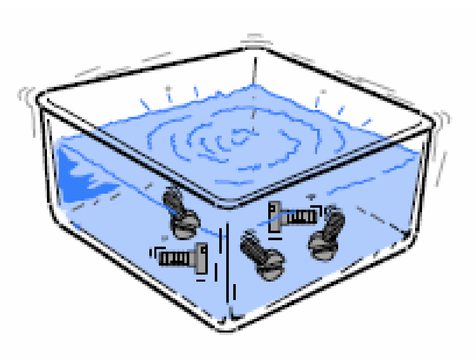
Vibration …

Useful Vibration

Compressor



Testing

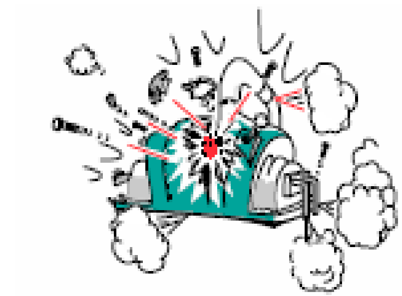


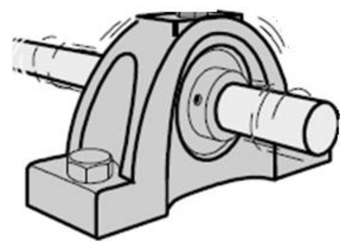
Ultrasonic cleaning

Harmful vibration

Noise

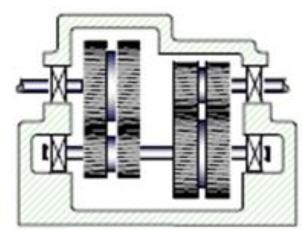
Destruction



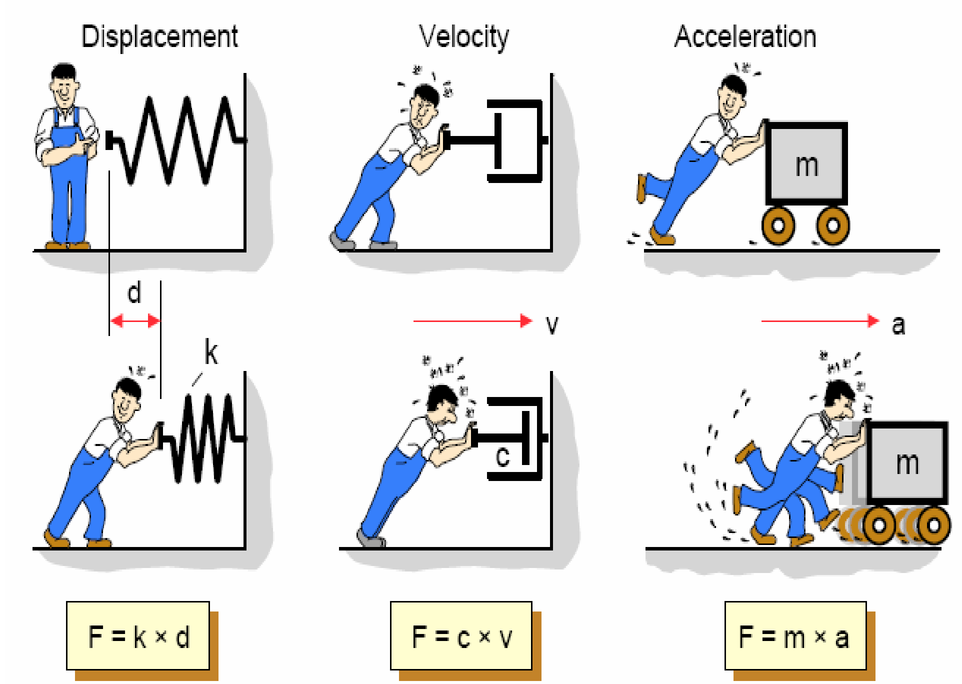


Wear

Fatigue



Vibration parameters



All mechanical systems can be modeled by containing three basic components:

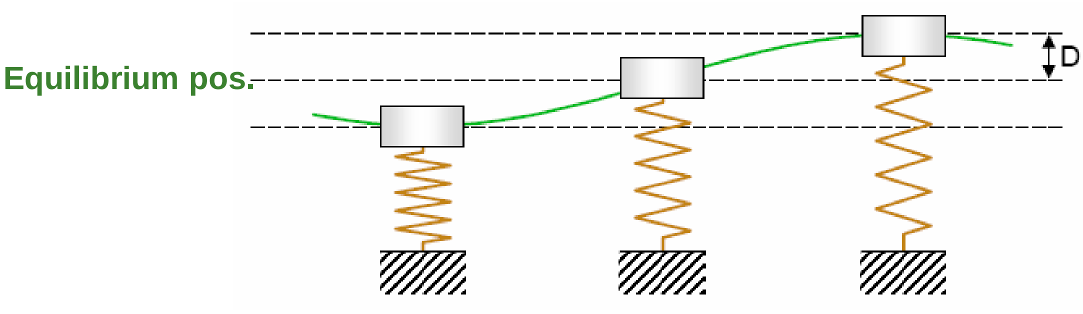
spring, damper, mass

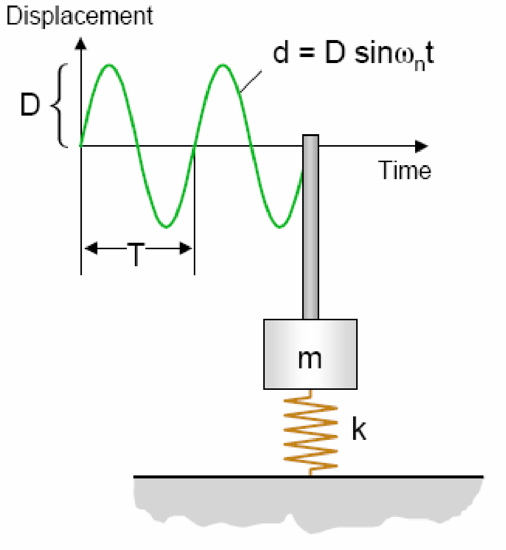
When these components are subjected to *constant* force, they react with a *constant*

displacement, velocity and acceleration

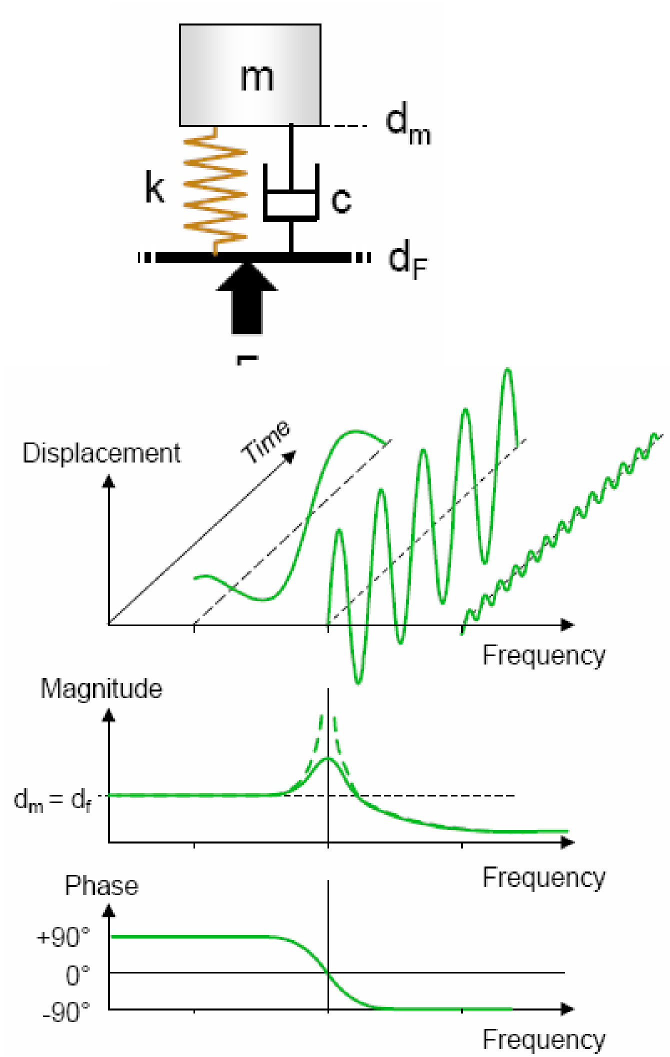
Free vibration

- When a system is initially disturbed by a displacement, velocity or acceleration, the system begins to vibrate with a constant amplitude and frequency depend on its stiffness and mass.
- This frequency is called as **natural frequency**, and the form of the vibration is called as **mode shapes**





Forced Vibration



If an external force applied to a system, the system will follow the force with the same frequency.

However, when the force frequency is increased to the system's natural frequency, amplitudes will dangerously increase in this region. This phenomenon called as **"Resonance"**

Watch these … YouTube

Bridge collapse:

http://www.youtube.com/watch?v=j-zczJXSxnw

Hellicopter resonance:
http://www.youtube.com/watch?v=0FeXjhUEXlc

Resonance vibration test:

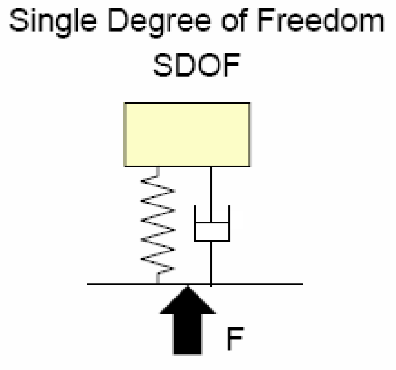
http://www.youtube.com/watch?v=LV_UuzEznHs

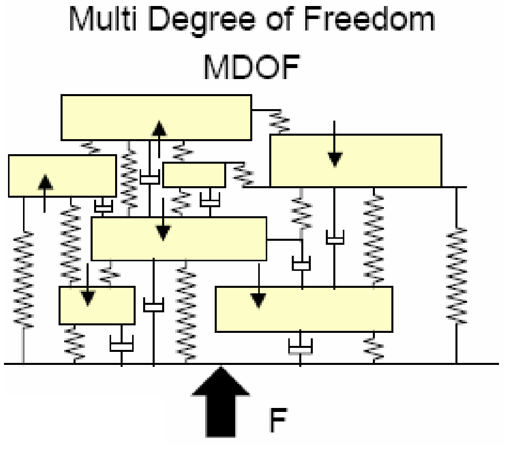
Flutter (Aeordynamically induced vibration) :

http://www.youtube.com/watch?v=OhwLojNerMU

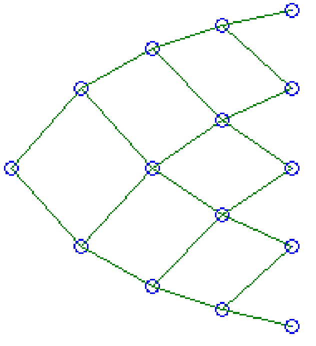
Modelling of vibrating systems

Lumped (Rigid) Modelling

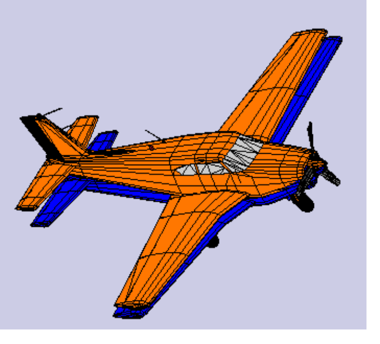




Numerical Modelling



Element-based methods (FEM, BEM)



Statistical and Energy-based methods

(SEA, EFA, etc.)

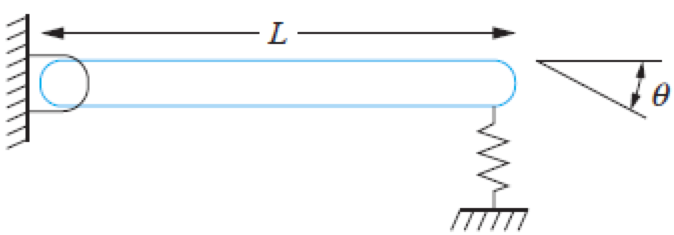
Degree of Freedom (DOF)

- Mathematical modeling of a physical system requires the selection of a set of variables that describes the behavior of the system.

- The number of *degrees of freedom* for a system is the number of kinematically independent variables necessary to completely describe the motion of every particle in the system

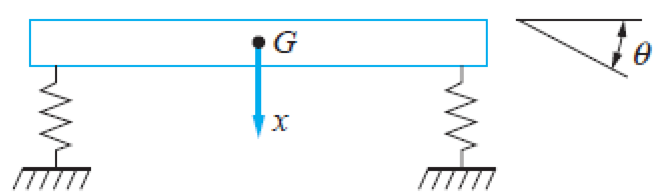
DOF=1

Single degree of freedom (SDOF)



DOF=2

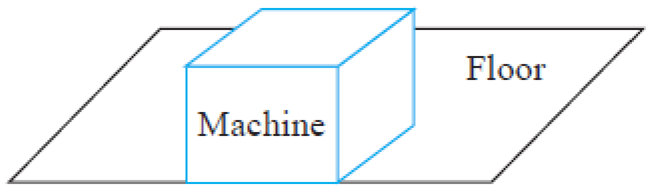
Multi degree of freedom (MDOF)



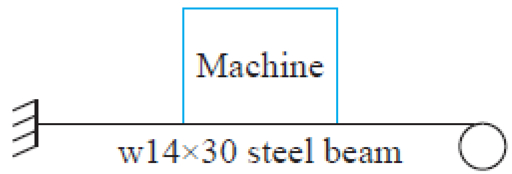
Equivalent model of systems

Example 1:

Example 2:



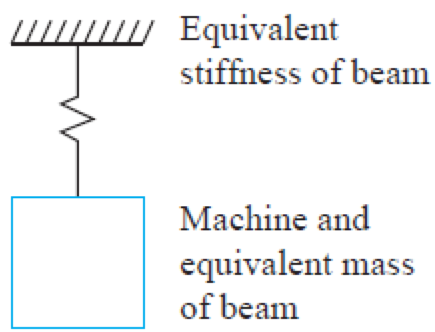
(a)



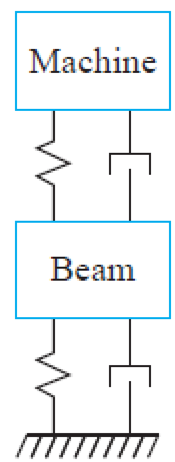
(b)

SDOF

DOF=1



(c)



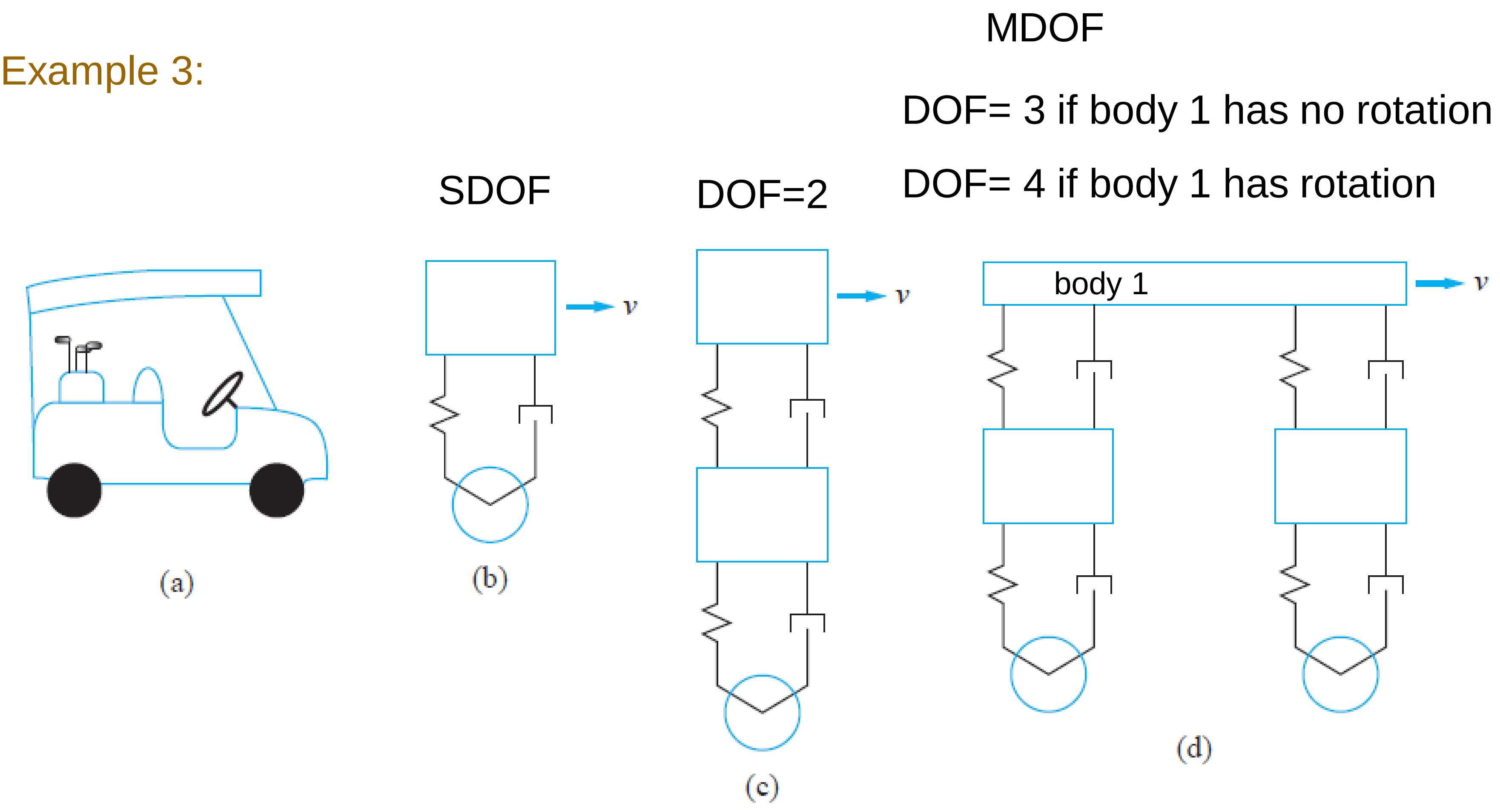
(d)

MDOF

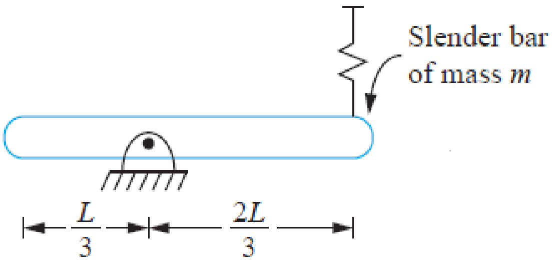
DOF=2

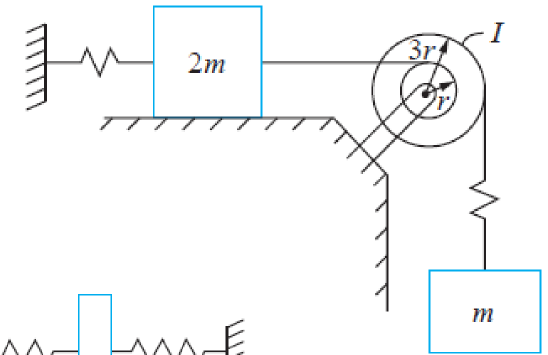
Equivalent model of systems

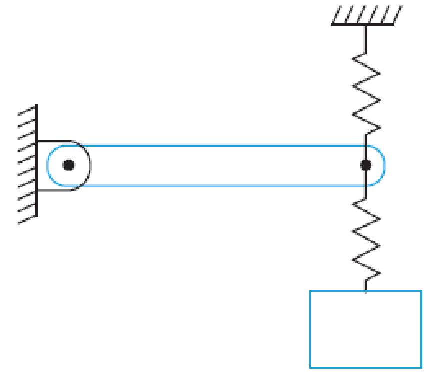
Example 3:

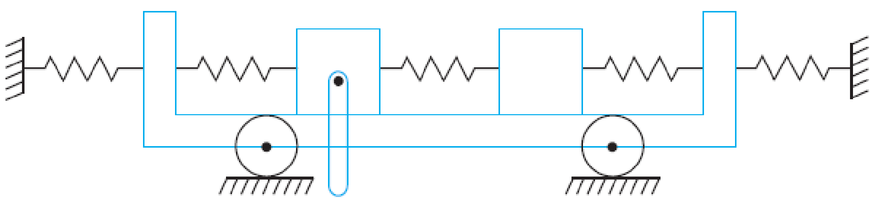


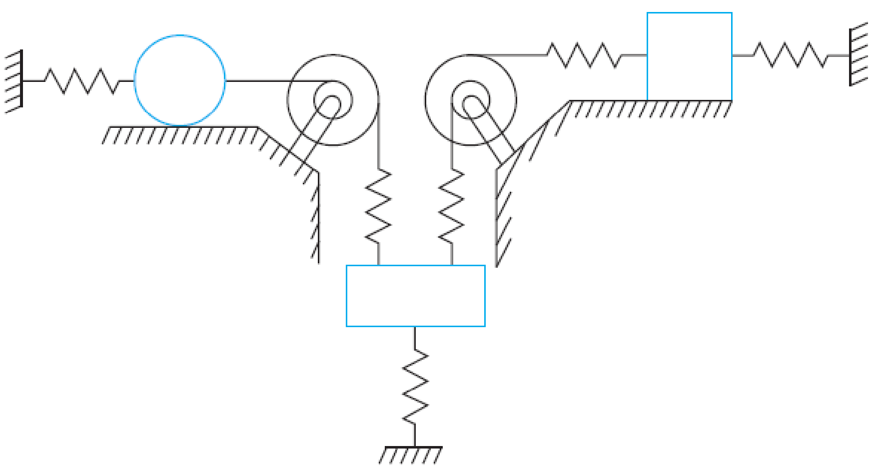
What are their DOFs?

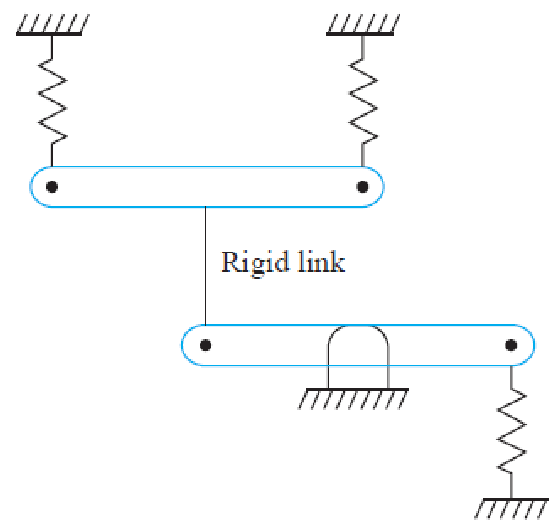












Do it yourself !

Do it yourself !

SDOF systems

- Helical springs

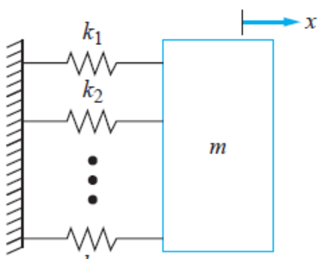
Shear stress: $\tau_{\max} = \dfrac{FrD}{2J} = \dfrac{16Fr}{\pi D^3}$

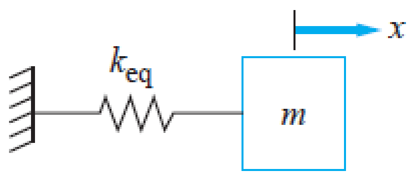
Stiffness coefficient: $k = \dfrac{GD^4}{64Nr^3}$

*F: Force, D: D*iameter, *G:* Shear modulus of the rod,
N: Number of turns, *r* : Radius

- Springs in combinations:

Parallel combination

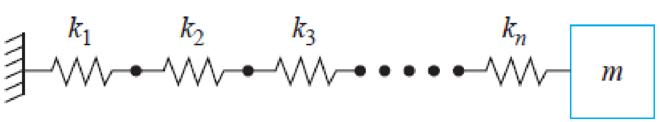




$$F = k_1x + k_2x + \cdots + k_nx = \left(\sum_{i=1}^{n} k_i\right)x$$

$$k_{eq} = \sum_{i=1}^{n} k_i$$

Series combination

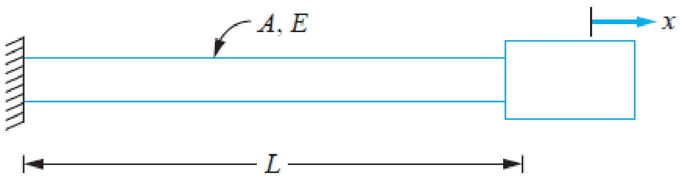


$$x = x_1 + x_2 + \cdots + x_n = \sum_{i=1}^{n} x_i$$

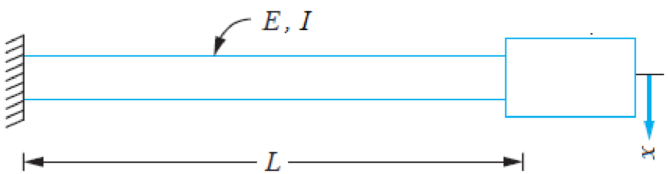
$$x = \sum_{i=1}^{n} \frac{F}{k_i} \qquad k_{eq} = \frac{1}{\sum_{i=1}^{n} \frac{1}{k_i}}$$

Elastic elements as springs

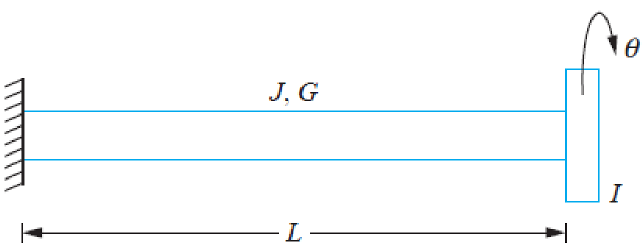
System	Stiffness Coeff.	SDOF Model



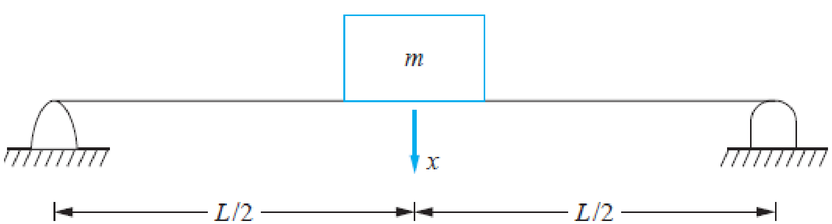
$$k = \frac{AE}{L}$$



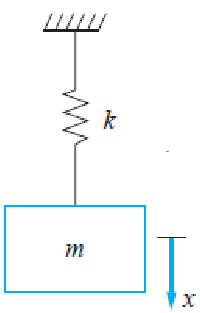
$$k = \frac{3EI}{L^3}$$



$$k = \frac{JG}{L}$$

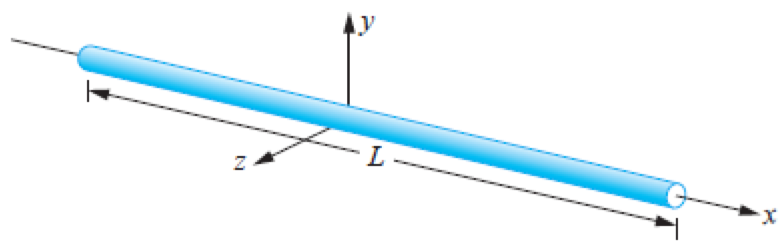


$$k = \frac{48EI}{L^3}$$



Moment of Inertia

Slender rod

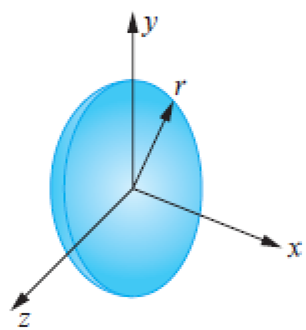


$$\bar{I}_x \approx 0$$

$$\bar{I}_y = \frac{1}{12} m L^2$$

$$\boxed{\bar{I}_z = \frac{1}{12} m L^2}$$

Thin disk

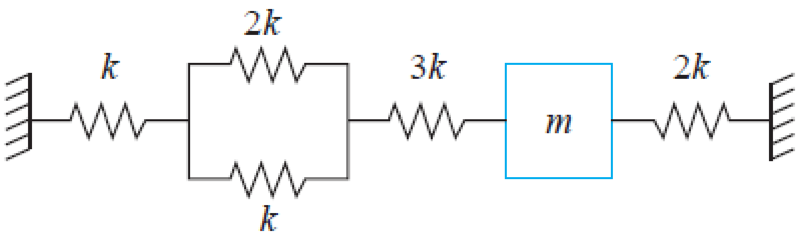


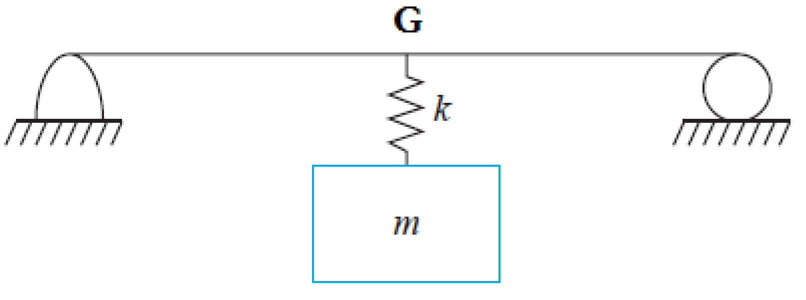
$$\boxed{\bar{I}_x = \frac{1}{2} m r^2}$$

$$\bar{I}_y = \frac{1}{4} m r^2$$

$$\bar{I}_z = \frac{1}{4} m r^2$$

What are the equivalent stiffnesses?





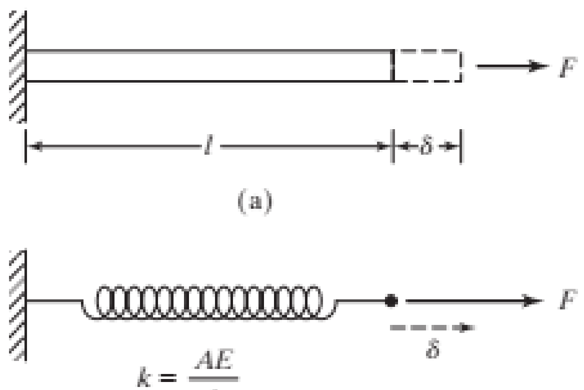
😊 Do it yourself !

😊 Do it yourself !

Example

EXAMPLE 1.3 Spring Constant of a Rod

Find the equivalent spring constant of a uniform rod of length l, cross-sectional area A, and Young's modulus E subjected to an axial tensile (or compressive) force F as shown in Fig. 1.24(a).



Solution: The elongation (or shortening) δ of the rod under the axial tensile (or compressive) force F can be expressed as

$$\delta = \frac{\delta}{l} l = \varepsilon l = \frac{\sigma}{E} l = \frac{Fl}{AE} \tag{E.1}$$

where $\varepsilon = \dfrac{\text{change in length}}{\text{original length}} = \dfrac{\delta}{l}$ is the strain and $\sigma = \dfrac{\text{force}}{\text{area}} = \dfrac{F}{A}$ is the stress induced in the rod. Using the definition of the spring constant k, we obtain from Eq. (E.1):

$$k = \frac{\text{force applied}}{\text{resulting deflection}} = \frac{F}{\delta} = \frac{AE}{l} \tag{E.2}$$

The significance of the equivalent spring constant of the rod is shown in Fig. 1.24(b).

☺ Do it yourself ! ☺ Do it yourself !