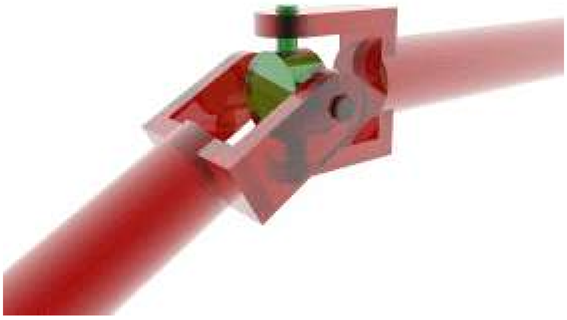
universal joint

1. introduction

If we look at the history of the universal joint, Robert Hooke is commonly known as the inventor of the 'Hooke's Joint' or 'Universal Joint'.



A universal joint (universal coupling, U-joint, Cardan joint, Spicer or Hardy Spicer joint, or Hooke's joint) is a joint or coupling connecting rigid rods whose axes are inclined to each other, and is commonly used in shafts that transmit rotary motion as shown in fig. 1. It consists of a pair of hinges located close together, oriented at 90° to each other, connected by a cross shaft. The universal joint is not a constant-velocity joint. In Europe the universal joint is often called the Cardano joint or Cardan shaft, after the Italian mathematician Gerolamo Cardano

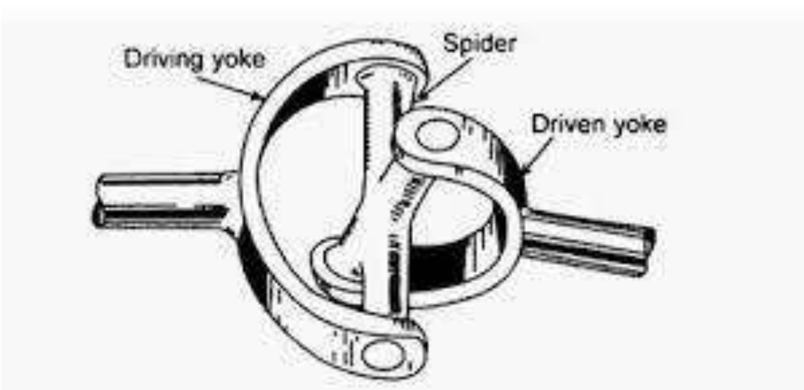


Fig. 1 simple universal joint

A Hooke's joint is used to connect two shafts, which are intersecting at a small angle, as shown in Fig. 2. The end of each shaft is forked to U-type and each fork provides two bearings for the arms of a cross. The arms of the cross are perpendicular to each other as shown in fig. 2

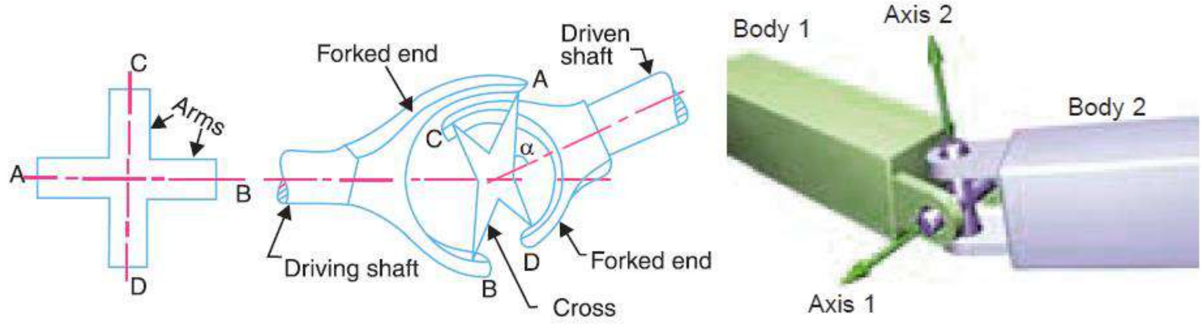


Fig. 2. Universal or Hooke's joint.

The motion is transmitted from the driving shaft to driven shaft through a cross. The inclination of the two shafts may be constant, but in actual practice it varies, when the motion is transmitted. The main application of the Universal or Hooke's joint is found in the transmission from the gear box to the differential or back axle of the automobiles. It is also used for transmission of power to different spindles of multiple drilling machine. It is also used as a knee joint in milling machines.

1.2 . Working of Universal Joint

In case of an automobile, the gearbox is rigidly mounted. Due to the action of the road springs, the position of the rear axle is constantly varied and the allowance is provided if the gearbox is mounted to the rear axle by a propeller shaft fig.3..

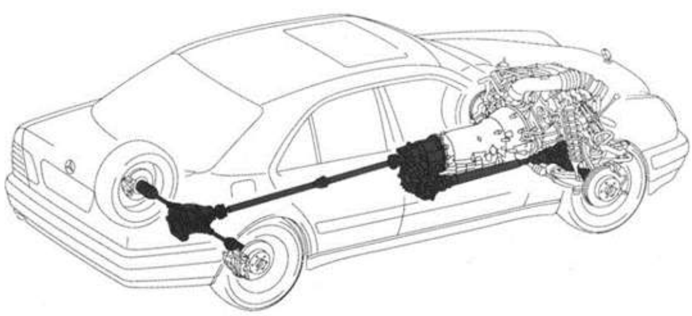


Fig. 3.Universal or Hooke's joint to transmission from the gear box to the differential or back axle of the automobiles.

1.3. Type of universals joints:

There are two types of universal joints, defined by their number of bending joints: Single joint: has only one bending aspect and is capable of operating at up to a 45-degree angle. Double joint: utilizes two bending joints, the double u-joint can operate at angles up to 90 degrees.

- Single joint

Single universal joints can compensate an angular offset of up to 45° between the input and output shaft as shown in fig 4. They typically operate at larger angles and higher torques than other types of couplings. They are also torsionally rigid and do not flex or wind up.

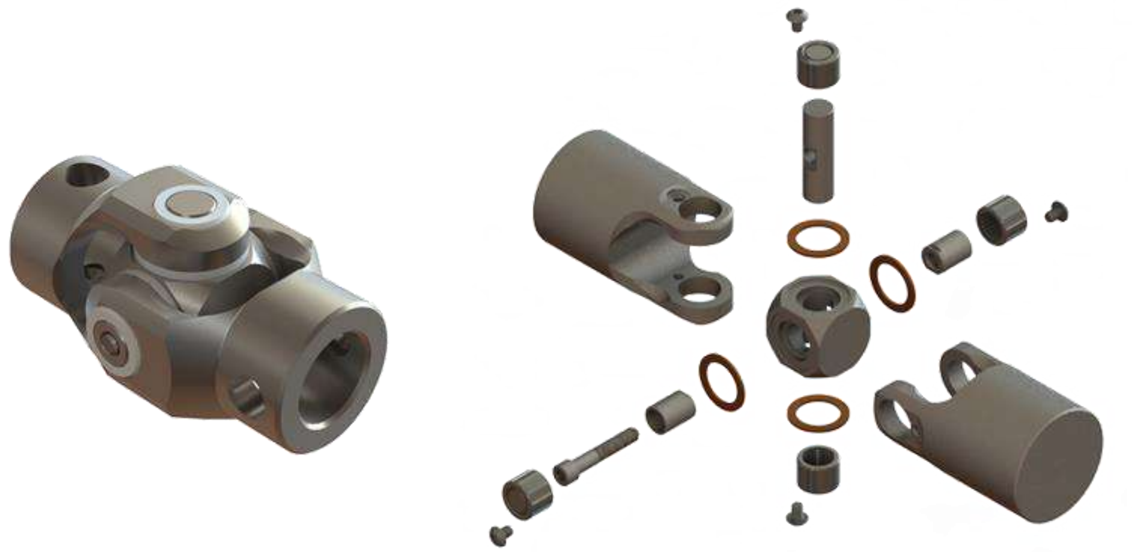


Fig 4. parts principals of Single joint

- Double joint

The Belden double universal joint provides the same reliability and service life as the single universal joint with a maximum combined working angle of 90° shown in figure. Double universal joints provide accurate positioning and flexibility under higher operating angles.

Fig 5. parts principals of Single joint

1.4. Materials Used for Universal Joints

Universal joints can be made from almost any material depending on the application.

Common materials used include stainless steel, steel, naval brass and other similar alloys to handle greater torque and temperature.

Plastics and thermoplastics are also used to create universal joints as they have greater rust and corrosion resistance as well as electrical and magnetic insulation in applications where this is required.

1.5. Applications of Universal Joint

Typical applications of universal joints include aircraft, appliances, control mechanisms, electronics, Instrumentation, medical and optical devices, ordnance, radio, sewing machines, textile machinery and tool drives.

1.6. Advantages and Disadvantages of Universal Joint

a) Advantages

- Universal coupling is more flexible than knuckle joint.
- It facilitates torque transmission between shafts which have angular misalignment.
- It is cheap and cost-effective.
- It is simple to be assembled and dismantled.
- Torque transmission efficiency is high.
- The joint permits angular displacements.

b) Disadvantages

Wear may occur if the joint is not properly lubricated.
Maintenance is often necessary to avoid wear.
Universal joint produces fluctuating motion
It does not support axial misalignment.

2. Kinematics

The universal (Cardan) joint suffers from one major problem: even when the input drive shaft axle rotates at a constant speed, the output drive shaft axle rotates at a variable speed, thus causing vibration and wear. is illustrated in Fig.6

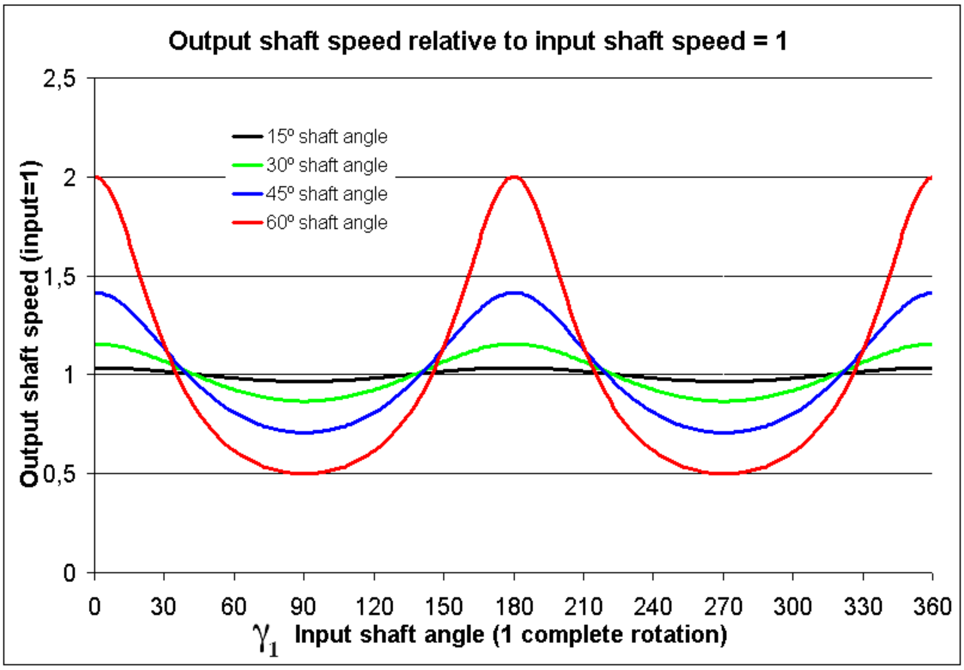


Fig. 6. .Angular (rotational) output shaft ω, versus rotation angle γ, for different bend angles β of the joint

3. Ratio of the Shafts Velocities

The top and front views connecting the two shafts by a universal joint are shown in Fig. 7. Let the initial position of the cross be such that both arms lie in the plane of the paper in front view, while the arm *AB* attached to the driving shaft lies in the plane containing the axes of the two shafts. Let the driving shaft rotates through an angle θ, so that the arm *AB* moves in a circle to a new position *A*1 *B*1 as shown in front view. A little consideration will show that the arm *CD* will also move in a circle of the same size. This circle when projected in the plane of paper appears to be an ellipse. Therefore the arm *CD* takes new position *C*1*D*1 on the ellipse, at an angle θ. But the true angle must be on the circular path. To find the true angle, project the point *C*1 horizontally to intersect the circle at *C*2. Therefore the angle *COC*2 (equal to φ) is the true angle turned by the driven shaft. Thus when the driving shaft turns through an angle θ, the driven shaft turns through an angle φ. It may be noted that it is not necessary that φ may be greater than θ or less than θ. At a particular point, it may be equal to θ.

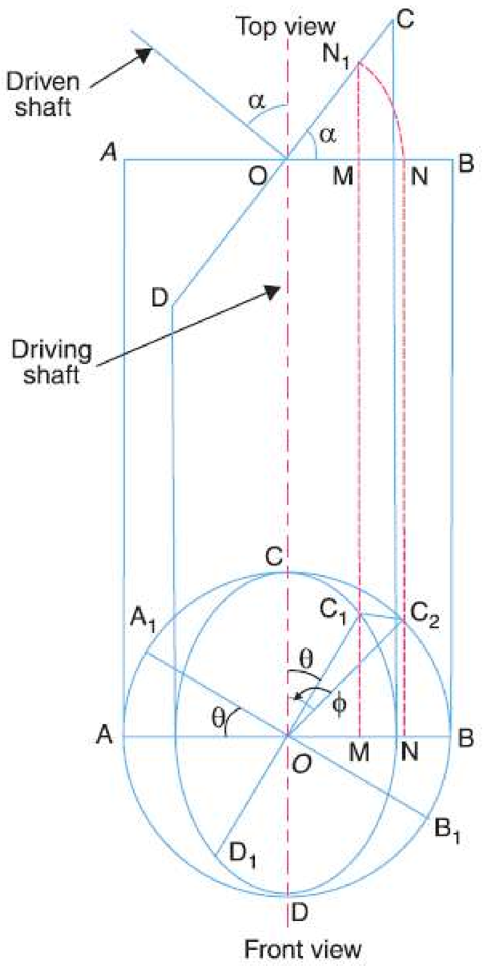


Fig. 7. *Polar diagram-salient features of driven shaft speed*

In triangle OC_1M, $\angle OC_1M = \theta$

$$\tan\theta = \frac{OM}{MC_1} \qquad \text{(i)}$$

and in triangle OC_2N, $\angle OC_2N = \varphi$

$$\tan\phi = \frac{ON}{NC_2} = \frac{ON}{MC_1}, NC2 = MC1 \qquad (ii)$$

Dividing equation **(*1*)** by **(*2*)**,

$$\frac{tan\theta}{tan\phi} = \frac{OM}{MC_1} \cdot \frac{MC_1}{ON} = \frac{OM}{ON}$$

But $OM = ON_1\, cos\,\alpha = ON\, cos\,\alpha\,, ON1 = ON\ (raduse\ for\ center\ o)$

$$\therefore \frac{tan\theta}{tan\phi} = \frac{ON\, cos\,\alpha}{ON} \xrightarrow{yields}$$

$$tan\theta = tan\phi \cdot \cos\alpha \qquad (iii)$$

Let ω = Angular velocity of the driving shaft = $d\theta / dt$

ω_1 = Angular velocity of the driven shaft = $d\phi / dt$

Differentiating both sides of equation **(*iii*)**,

$$\sec^2\theta\frac{d\theta}{dt} = \cos\alpha\cdot\sec^2\phi\frac{d\phi}{dt} - \sin\alpha\cdot\tan\phi\cdot\frac{d\alpha}{dt}, (\alpha = constant \xrightarrow{yields} \frac{d\alpha}{dt} = 0)$$

$$\sec^2\theta\cdot\frac{d\theta}{dt} = \cos\alpha\cdot\sec^2\phi\cdot\frac{d\phi}{dt} \xrightarrow{yields} \sec^2\theta\cdot\omega = \cos\alpha\cdot\sec^2\phi\cdot\omega_1$$

$$\frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha\cdot\sec^2\phi} = \frac{1}{\cos^2\theta\cos\alpha\cdot\sec^2\phi} \qquad (iv)$$

We know that:

$$\sec^2\phi = 1 + \tan^2\phi = 1 + \frac{\tan^2\theta}{\cos^2\alpha}, (from\ eq.\ iii)$$

$$\sec^2\phi = 1 + \frac{\sin^2\theta}{\cos^2\theta\cdot\cos^2\alpha} = \frac{\cos^2\theta\cdot\cos^2\alpha + \sin^2\theta}{\cos^2\theta\cdot\cos^2\alpha}$$

$$= \frac{\cos^2\theta\cdot(1 - \sin^2\alpha + \sin^2\theta}{\cos^2\theta\cdot\cos^2\alpha}$$

$$= \frac{\cos^2\theta - \cos^2\theta\cdot\sin^2\alpha + \sin^2\theta}{\cos^2\theta\cdot\cos^2\alpha} = \frac{1 - \cos^2\theta\cdot\sin^2\alpha}{\cos^2\theta\cdot\cos^2\alpha}$$

Substituting this value of $\sec^2\varphi$ in equation ***(iv)*,** we have veloity ratio,

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2\theta\cos\alpha\cdot}\cdot\frac{\cos^2\theta\cdot\cos^2\alpha}{1-\cos^2\theta\cdot\sin^2\alpha} = \frac{\cos\alpha}{1-\cos^2\theta\cdot\sin^2\alpha} \qquad (v)$$

Note:

If N = Speed of the driving shaft in r.p.m., and

N_1 = Speed of the driven shaft in r.p.m.

Then the equation **(*v*)** may also be written as

$$\frac{N_1}{N} = \frac{\cos\alpha}{1 - \cos^2\theta\cdot\sin^2\alpha}$$

3.1. Maximum and Minimum Speeds of Driven Shaft

We have discussed in the previous article that velocity ratio,

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1 - \cos^2\theta\cdot\sin^2\alpha}$$

$$\omega_1 = \frac{\omega\cdot\cos\alpha}{1-\cos^2\theta\cdot\sin^2\alpha} \qquad (vi)$$

The value of ω_1 will be maximum for a given value of α, if the denominator of equation **(*vi*)** is minimum. This will happen, when $\cos^2\theta = 1$, *i.e.* when θ = 0°, 180°, 360° etc.

∴ Maximum speed of the driven shaft,

$$\omega_{1max} = \frac{\omega\cdot\cos\alpha}{1-\cdot\sin^2\alpha} = \frac{\omega\cdot\cos\alpha}{\cdot\cos^2\alpha} = \frac{\omega}{\cos\alpha} \qquad (vii)$$

$$N_{1max} = \frac{N}{\cos\alpha}, (\text{where } N \text{ and } N1 \text{ are in r.p.m.})$$

Similarly, the value of ω_1 is minimum, if the denominator of equation ***(vi)*** is maximum. This will happen, when ($\cos^2\theta \,.\, \sin^2\alpha$) is maximum, or
$\cos^2\theta = 0$, *i.e.* when $\theta = 90°, 270°$ etc.
∴ Minimum speed of the driven shaft,

$$\omega_{1min} = \frac{\omega \cdot cos\alpha}{1 - cos^2\theta \cdot sin^2\alpha} = \omega \cdot cos\alpha$$

$$N_{1min} = \omega \cdot cos\alpha, (\text{where } N \text{ and } N1 \text{ are in r.p.m.})$$

3.2. Angular Acceleration of the Driven Shaft

We know that:

$$\omega_1 = \frac{\omega \cdot cos\alpha}{1 - cos^2\theta \cdot sin^2\alpha} = \omega \cdot cos\alpha \cdot (1 - cos^2\theta \cdot sin^2\alpha)^{-1}$$

Differentiating the above expression, we have the angular acceleration of the driven shaft,

$$\varepsilon = \frac{d\omega_1}{dt} = \omega \cdot cos\alpha \cdot [-1(1 - cos^2\theta \cdot sin^2\alpha)^{-2} \cdot (2 \cdot cos\theta \cdot \sin\theta \cdot \sin^2\alpha)] \cdot \frac{d\theta}{dt}$$

$$\varepsilon = \frac{-\omega^2 \cdot cos\alpha \cdot \sin 2 \cdot \theta \cdot sin^2\alpha}{(1 - cos^2\theta \cdot sin^2\alpha)^2}, (2 \cdot cos\theta \cdot sin\,\theta = sin2 \cdot \theta \; and \; \frac{d\theta}{dt} = \omega)$$

For ***angular acceleration to be maximum***, differentiate $d\omega_1 / dt$ with respect to θ and equate to zero. The result is approximated as:

$$\cos 2 \cdot \theta = \frac{\sin^2\alpha \cdot (2 - \cos^2 2\theta)}{2 - \sin^2\alpha}$$

Note: If the value of α is ***less than*** 30°, then cos 2 θ may approximately be written as:

$$\cos 2 \cdot \theta = \frac{\sin^2\alpha}{2 - \sin^2\alpha}$$

4. Maximum Fluctuation of Speed

We know that the maximum speed of the driven shaft,

$$\omega_{1max} = \frac{\omega}{cos\,\alpha}$$

and minimum speed of the driven shaft:

$$\omega_{1min} = \omega \cdot cos\alpha$$

∴ Maximum fluctuation of speed of the driven shaft:

$$q = \omega_{1max} - \omega_{1min} = \frac{\omega}{cos\,\alpha} - \omega \cdot cos\alpha = \omega \cdot (\frac{1}{cos\,\alpha} - cos\alpha)$$

$$q = \omega \cdot \left(\frac{1 - cos^2\alpha}{cos\,\alpha}\right) = \omega \cdot \left(\frac{sin^2\alpha}{cos\,\alpha}\right)$$

Since α is a small angle, therefore substituting cos α = 1, and sin α = α radians.
∴ Maximum fluctuation of speed:

$$q = \omega \cdot \alpha^2$$

Hence, ***the maximum fluctuation of speed of the driven shaft approximately varies as the square of the angle between the two shafts.***

5. Double Hooke's Joint

We have seen in the previous articles, that the velocity of the driven shaft is not constant, but varies from maximum to minimum values. In order to have a constant velocity ratio of the driving and driven shafts, an intermediate shaft with a Hooke's joint at each end as shown in Fig. 8, is used. This type of joint is known as double Hooke's joint.

Let the driving, intermediate and driven shafts, in the same time, rotate through angles θ, φ and γ from the position as discussed previously.

Now for shafts *A* and *B*,

$$\tan\theta = \tan\phi \cdot \cos\alpha \qquad (i)$$

and for shafts *B* and *C*,

$$\tan\gamma = \tan\phi \cdot \cos\alpha \qquad (ii)$$

From equations **(*i*)** and **(*ii*)**, we see that θ = γ or ωA = ωC.

$$\theta = \gamma \text{ or } \omega_A = \omega_C$$

This shows that the speed of the driving and driven shaft is constant. In other words, this joint gives a velocity ratio equal to unity, if

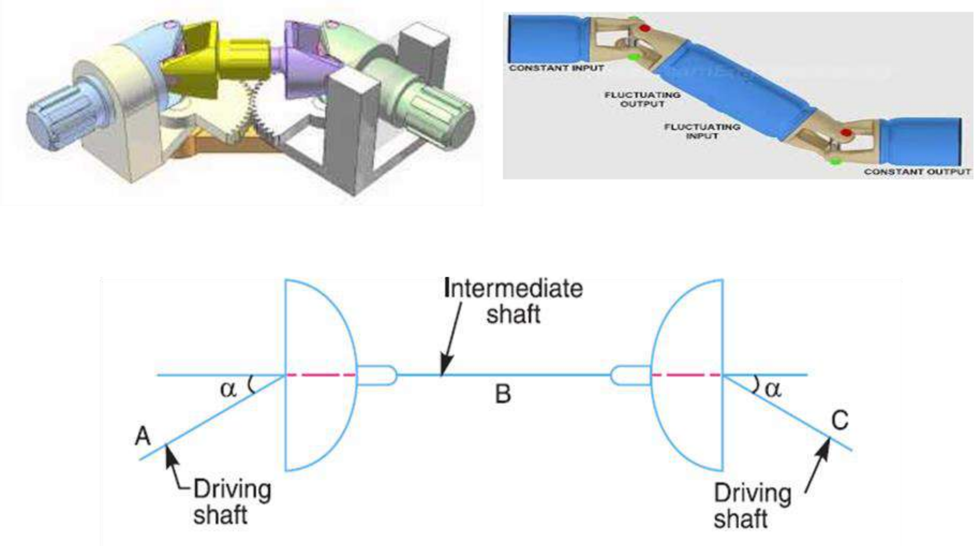


Fig.8.Double Hooke's joint.

Example 1

Two shafts with an included angle of 160° are connected by a Hooke's joint. The driving shaft runs at a uniform speed of 1500 r.p.m. The driven shaft carries a flywheel of mass 12 kg and 100 mm radius of gyration. Find the maximum angular acceleration of the driven shaft and the maximum torque required.

Solution.

Given: α = 180° – 160° = 20°; *N* = 1500 r.p.m.; *m* = 12 kg ; *k* = 100 mm = 0.1 m

We know that angular speed of the driving shaft,

$$\omega = 2\,\pi \times 1500 / 60 = 157 \text{ rad/s}$$

and mass moment of inertia of the driven shaft,

$$I = m.k2 = 12\,(0.1)^2 = 0.12 \text{ kg - m}^2$$

1. Maximum angular acceleration of the driven shaft:

$$\cos 2\cdot\theta = \frac{\sin^2\alpha}{2-\sin^2\alpha} = \cos 2\cdot\theta = \frac{\sin^2 20}{2-\sin^2 20} = 0.122 \xrightarrow{yields} 2\cdot\theta = 82.98$$

$$\theta = 41.49^o$$

$$\varepsilon = \frac{-\omega^2\cdot\ cos\alpha\cdot\sin 2\cdot\theta\cdot sin^2\,\alpha}{(1-cos^2\,\theta\cdot sin^2\,\alpha)^2} = \frac{157^2\cdot\ cos20\cdot\sin 82.98\cdot sin^2\,20}{(1-cos^2\,41.49\cdot sin^2\,20)^2}$$

$$\varepsilon = \frac{2689.2}{0.87} = 3080.29\,rad\ s^{-2}$$

2 Maximum torque required

We know that maximum torque required:

$$T_{max} = I\cdot\varepsilon = \ 0.12\ \cdot 3080.29 = 369.635\ Nm$$

Example. 2

Two shafts are connected by a Hooke's joint. The driving shaft revolves uniformly at 500 r.p.m. If the total permissible variation in speed of the driven shaft is not to exceed ± 6% of the mean speed, find the greatest permissible angle between the centre lines of the shafts.

Solution.

Given: *N* = 500 r.p.m. or ω = 2 π × 500 / 60 = 52.4 rad/s

Let α = Greatest permissible angle between the centre lines of the shafts.

Since the variation in speed of the driven shaft is ± 6% of the mean speed (*i.e.* speed of the driving speed), therefore total fluctuation of speed of the driven shaft,

$$q = 0.12 \cdot \omega = \omega \cdot \left(\frac{1 - cos^2\alpha}{cos\,\alpha}\right)$$

$$0.12 \cdot cos\,\alpha = 1 - cos^2\alpha \xrightarrow{yields} cos^2\alpha + 0.12 \cdot cos\,\alpha - 1 = 0$$

$$cos\,\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0.12 \pm \sqrt{0.12^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{-0.12 \pm 2.003}{2} = 0.9417$$

$$\alpha = 19.64^o$$

Example. 3.

Two shafts are connected by a universal joint. The driving shaft rotates at a uniform speed of 1200 r.p.m. Determine the greatest permissible angle between the shaft axes so that the total fluctuation of speed does not exceed 100 r.p.m. Also calculate the maximum and minimum speeds of the driven shaft.

Solution. Given: N = 1200 r.p.m.; q = 100 r.p.m.

a) Greatest permissible angle between the shaft axes:

Let: α = Greatest permissible angle between the shaft axes.

We know that total fluctuation of speed (q),

$$q = N \cdot \left(\frac{1 - cos^2\alpha}{cos\,\alpha}\right) \xrightarrow{yields} 100 = 1200 \cdot \left(\frac{1 - cos^2\alpha}{cos\,\alpha}\right)$$

$$\left(\frac{1 - cos^2\alpha}{cos\,\alpha}\right) = 0.083$$

$$cos^2\alpha + 0.083 \cdot cos\,\alpha - 1 = 0$$

$$cos\,\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0.083 \pm \sqrt{0.083^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1}$$

$$cos\,\alpha = \frac{-0.083 \pm 2.0017}{2} = 0.9593$$

$$\alpha = 16.39^O$$

b) Maximum and minimum speed of the driven shaft

We know that maximum speed of the driven shaft:

$$N_{1max} = \frac{N}{cos\,\alpha} = \frac{1200}{cos\,16.39} = 1250.83\,rpm$$

and minimum speed of the driven shaft,

$$N_{1min} = N \cdot cos\alpha = 1200 \cdot cos16.39 = 1151.23\ rpm$$