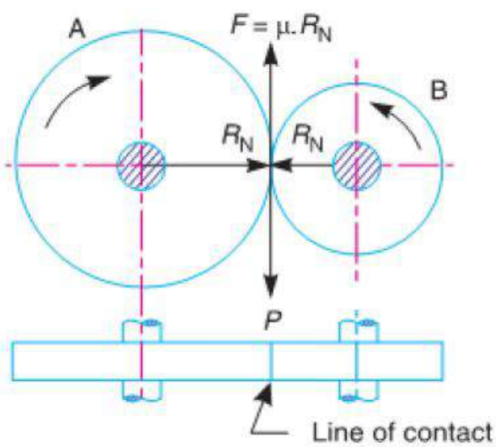
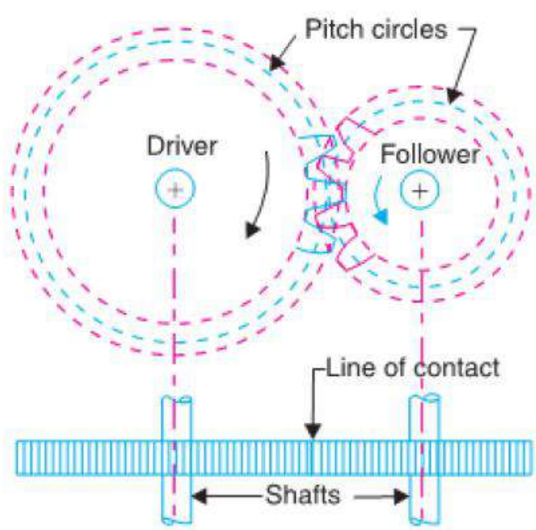
8. Gears.

8.1. Introduction

We have discussed in the previous chapter, that the slipping of a belt or rope is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slipping is to reduce the velocity ratio of the system. In precision machines, in which a definite velocity ratio is of importance (as in watch mechanism), the only positive drive is by means of gears or toothed wheels. A gear drive is also provided, when the distance between the driver and the follower is very small. The motion and power transmitted by gears is kinematically equivalent to that transmitted by friction wheels or discs. In order to understand how the motion can be transmitted by two toothed wheels, consider two plain circular wheels A and B mounted on shafts, having sufficient rough surfaces and pressing against each other as shown in the Fig.



(*a*) Friction wheels.



(*b*) Toothed wheels.

8.2. Advantages and Disadvantages of Gear Drive

The following are the advantages and disadvantages of the gear drive as compared to belt, rope and chain drives:

a) Advantages

1. It transmits exact velocity ratio.

2. It may be used to transmit large power.

3. It has high efficiency.

4. It has reliable service.

5. It has compact layout.

b) Disadvantages

1. The manufacture of gears require special tools and equipment.

2. The error in cutting teeth may cause vibrations and noise during operation.

8.3. Classification of Toothed Wheels

The gears or toothed wheels may be classified as follows:

8.3.1 According to the position of axes of the shafts. The axes of the two shafts between which the motion is to be transmitted, may be

(a) Parallel,

(b) Intersecting

(c) Non-intersecting and non-parallel **(spiral gear)**

a) parallel toothed gear

b) Intersecting tooth gear

c) Non-intersecting and non-parallel (spiral gear)

8.3.2. According to the peripheral velocity of the gears. The gears, according to the peripheral velocity of the gears may be classified as:

(a) Low velocity,

(b) Medium velocity,

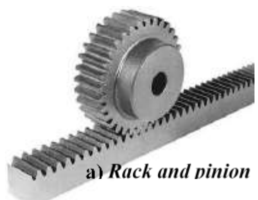
(c) High velocity

8.2.3According to the type of gearing. The gears, according to the type of gearing may be classified as:

(a) External gearing,
(b) Internal gearing,
(c) Rack and pinion

a) Internal gearing

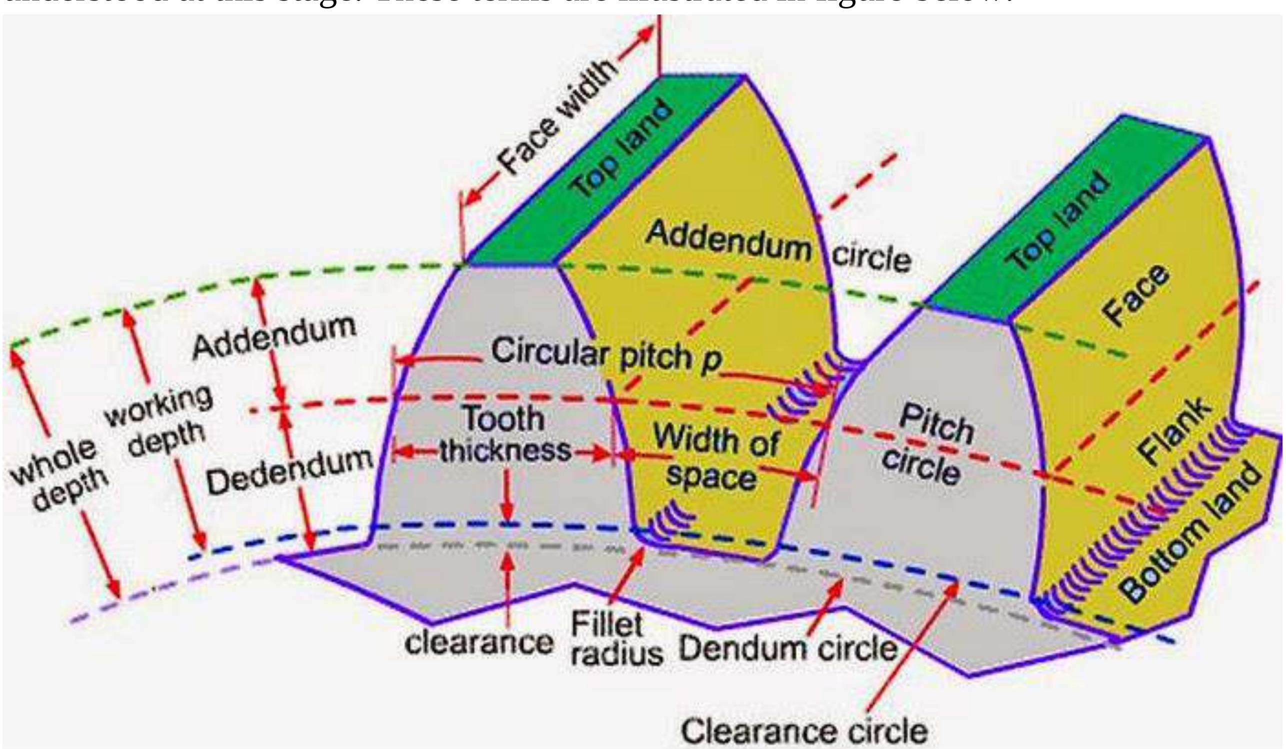
a) External gearing



a) *Rack and pinion*

8.4. Term used in Gears:

The following terms, which will be mostly used in this chapter, should be clearly understood at this stage. These terms are illustrated in figure below:



1) Pitch circle.

It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2) *Addendum*.

It is the radial distance of a tooth from the pitch circle to the top of the tooth.

3) *Dedendum*.

It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

4) ***Addendum circle***.

It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

5)Dedendum circle.

It is the circle drawn through the bottom of the teeth. It is also called root circle.

6)Circular pitch. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c. Mathematically:

Circular pitch:

$$p_c = \frac{\pi \cdot D}{T}$$

Where: D = Diameter of the pitch circle, and; T = Number of teeth on the wheel

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively, then for them to mesh correctly:

$$p_c = \frac{\pi \cdot D_1}{T_1} = \frac{\pi \cdot D_2}{T_2} \xrightarrow{yields} \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

7) Diametral pitch. It is the ratio of number of teeth to the pitch circle diameter in millimeters. It is denoted by p_d . Mathematically:

$$p_d = \frac{T}{D} = \frac{\pi}{p_c}$$

8)Clearance.

It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle***.

9) Total depth.

It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.

10) ***Tooth thickness***.

It is the width of the tooth measured along the pitch circle

11) ***Tooth space.***

It is the width of space between the two adjacent teeth measured along the pitch circle.

Module. It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m. Mathematically

$$m = \frac{d}{T}$$

8.5. Gear Materials

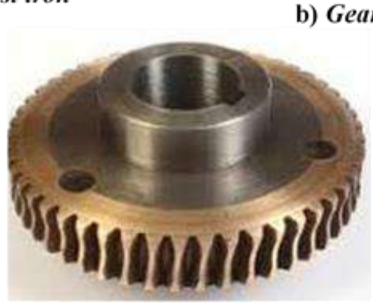
The material used for the manufacture of gears depends upon the strength and service conditions like wear, noise etc. The gears may be manufactured from metallic or non-metallic materials. The metallic gears with cut teeth are commercially obtainable in cast iron, steel and bronze. The nonmetallic materials like wood, raw hide, compressed paper and synthetic resins like nylon are used for gears, especially for reducing noise.

The cast iron is widely used for the manufacture of gears due to its good wearing properties, excellent machinability and ease of producing complicated shapes by casting method. The cast iron gears with cut teeth may be employed, where smooth action is not important. The steel is used for high strength gears and steel may be plain carbon steel or alloy steel. The steel gears are usually heat treated in order to combine properly the toughness and tooth hardness.

The phosphor bronze is widely used for worm gears in order to reduce wear of the worms which will be excessive with cast iron or steel.

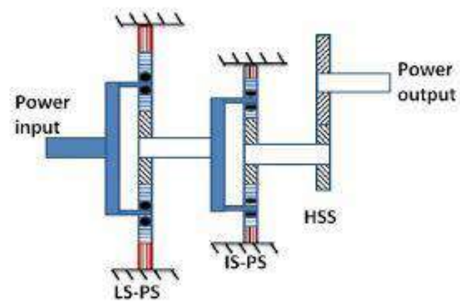

a) *Gear made of cast iron*


b) *Gear made of still*


c) *Gear made of phosphor bronze*

8.6. Basic gear box theory

Consider a simple schematic of a gear box with an input and output shaft



8.6.1. Gear box ratio

The ratio of the gear box is defined as:

$$G.R. = \frac{INPUT\ SPEED}{OUTPUT\ SPEED} = \frac{n_1}{n_2}$$

N is usually in rev/min but ratio is the same whatever units of speed are used. If angular velocity is used then:

$$G.R. = \frac{INPUT\ SPEED}{OUTPUT\ SPEED} = \frac{\omega_1}{\omega_2}$$

8.6.2. Torque and efficiency

The power transmitted by a torque $\boldsymbol{T_q}$Nm applied to a shaft rotating at N rev/min is given by:

$$S.P. = \frac{2 \cdot \pi \cdot n \cdot T_q}{60}$$

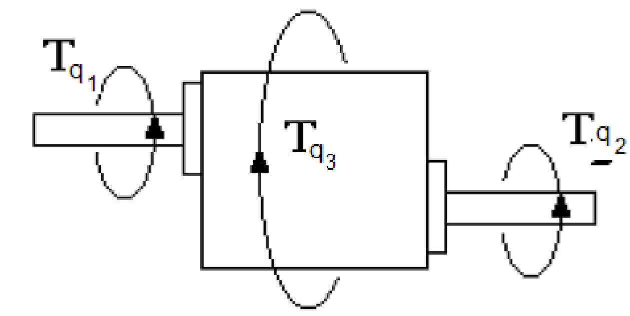
In an ideal gear box. the input and output powers are the same so

$$\frac{2 \cdot \pi \cdot n_1 \cdot T_{q_1}}{60} = \frac{2 \cdot \pi \cdot n_2 \cdot T_{q_2}}{60} \xrightarrow{yields} n_1 \cdot T_{q_1} = n_2 \cdot T_{q_2}; or \frac{T_{q_2}}{T_{q_1}} = \frac{n_1}{n_2}$$

It follows that if the speed is reduced, the torque is increased and vice versa. In a real gear box. power is lost through friction and the power output is smaller than the power input. The efficiency is defined as:

$$\eta = \frac{power\ output}{power\ input} = \frac{P_{out}}{P_{in}} = \frac{\frac{2 \cdot \pi \cdot n_2 \cdot T_{q_2}}{60}}{\frac{2 \cdot \pi \cdot n_1 \cdot T_{q_1}}{60}} = \frac{n_2 \cdot T_{q_2}}{n_1 \cdot T_{q_1}}$$

Because the torque in and out is different, a gear box has to be clamped in order to stop the case or body rotating. A holding torque $\boldsymbol{T_{q3}}$ must be applied to the body through the clamps.



The total torque must add up to zero:

$$T_{q_1} + T_{q_2} + T_{q_3} = 0$$

If we use a convention that anti-clockwise is positive and clockwise is negative, we can determine the holding torque. The direction of rotation of the output shaft depends on the design of the gear box.

EXAMPLE

1. A gear box has an input speed of 1500 rev/min clockwise and an output speed of 300 rev/min anticlockwise. The input power is 20 kW and the efficiency is 70%. Determine the following.

   1. The gear ratio
   2. The input torque,
   3. The output powers.

4. The output torque,
5. The holding torque.

SOLUTION

$$G.R. = \frac{INPUT\ SPEED}{OUTPUT\ SPEED} = \frac{n_1}{n_2} = \frac{1500}{300} = 5$$

$$P_{in} = \frac{2 \cdot \pi \cdot n_1 \cdot T_1}{60} \xrightarrow{yields} T_1 = \frac{60 \cdot P_{in}}{2 \cdot \pi \cdot n_1}$$

$$= \frac{60 \cdot 20000}{2 \cdot \pi \cdot 1500} = 127.3Nm\ (negative\ clockwise)$$

$$\eta = \frac{P_{out}}{P_{in}} \xrightarrow{yields} P_{out} = \eta \cdot P_{in} = 0.7 \cdot 20000 = 14000\ W$$

$$P_{out} = \frac{2 \cdot \pi \cdot n_2 \cdot T_{q_2}}{60} \xrightarrow{yields} T_2 = \frac{60 \cdot P_{out}}{2 \cdot \pi \cdot n_2}$$

$$= \frac{60 \cdot 14000}{2 \cdot \pi \cdot 3000} = 445.6Nm\ (posetive\ anti-clockwise)$$

$$T_1 + T_2 + T_3 = 0 \xrightarrow{yields} -127.3 + 445.6 + T_3 = 0$$

$$T_3 = -318.3\ Nm\ (anticlockwise)$$

8.7 **Gear Trains**

Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called *gear train* or *train of toothed wheels*. The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

8.7.1. Types of Gear Trains

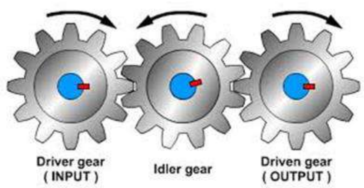
Following are the different types of gear trains, depending upon the arrangement of wheels:

1. Simple gear train,

2. Compound gear train,

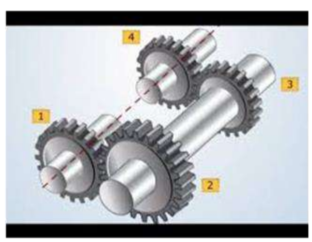
3. Reverted gear train, and

4. Epicyclic gear train.

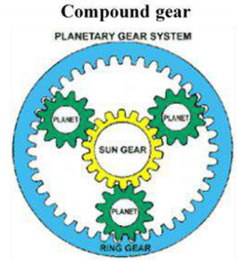


Simple gear train


Compound gear



Reverted gear train

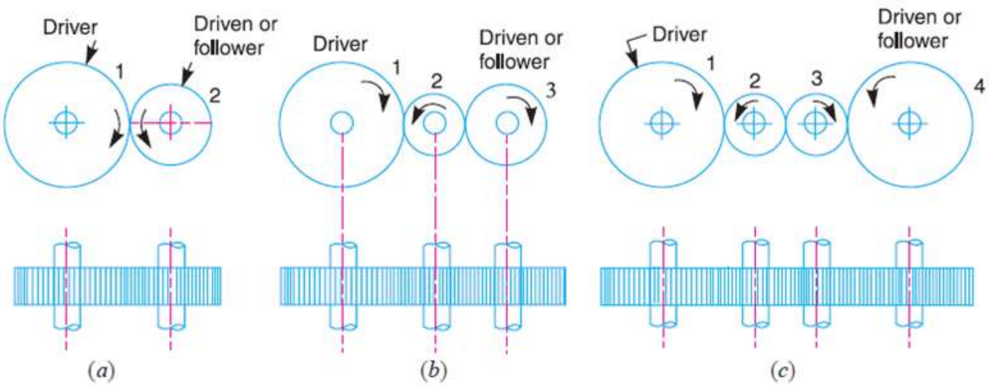


Epicyclic gear train

In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other. But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

8.8. Simple Gear Train

When there is only one gear on each shaft, as shown in Figure below, it is known as ***simple gear train***. The gears are represented by their pitch circles.



simple gear train

When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Figure **a**. Since the gear 1 drives the gear 2, therefore gear 1 is called the ***driver*** and the gear 2 is called the ***driven*** or ***follower*****.** It may be noted that the motion of the driven gear is opposite to the motion of driving gear.

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore:

$$Speed\ ratio = \frac{n_1}{n_2} = \frac{T_2}{T_1}$$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as ***train value*** of the gear train. Mathematically

$$Train\ ratio = \frac{n_2}{n_1} = \frac{T_1}{T_2}$$

$$Speed\ ratio = \frac{1}{Train\ ratio}$$

Note: The gears which mesh must have the same circular pitch or module. Thus gears 1 and 2 must have the same module as they mesh together. Similarly gears 3 and 4, must have the same module.

$$p_c = \frac{\pi \cdot d_1}{T_1} = \pi \cdot m \ (m = \frac{d}{T}\ is\ module\ gear)$$

From above, we see that the train value is the reciprocal of speed ratio.

Sometimes, the distance between the two gears is large. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods:

1. By providing the large sized gear, or **2.** By providing one or more intermediate gears.

A little consideration will show that the former method (i.e. providing large sized gears) is very inconvenient and uneconomical method; whereas the latter method (i.e. providing one or more intermediate gear) is very convenient and economical.

It may be noted that when the number of intermediate gears is odd, the motion of both the gears (i.e. driver and driven or follower) is like as shown in Fig. (b).

But if the number of intermediate gears is even, the motion of the driven or follower will be in the opposite direction of the driver as shown in Fig. (c).

Now consider a simple train of gears with one intermediate gear as shown in Fig. (*b*).

Let:

n_1 = Speed of driver in r.p.m.,

n_2 = Speed of intermediate gear in r.p.m.,

n_3 = Speed of driven or follower in r.p.m.,

T_1 = Number of teeth on driver,

T_2 = Number of teeth on intermediate gear, and

T_3 = Number of teeth on driven or follower.

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is:

$$\frac{n_1}{n_2} = \frac{T_2}{T_1} \tag{1}$$

Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is:

$$\frac{n_2}{n_3} = \frac{T_3}{T_2} \tag{2}$$

The speed ratio of the gear train as shown in Fig. (*b*) is obtained by multiplying the equations **(*1*)** and **(*2*):**

$$\frac{n_1}{n_2} \cdot \frac{n_2}{n_3} = \frac{T_2}{T_1} \cdot \frac{T_3}{T_2} \xrightarrow{yields} \frac{n_1}{n_3} \cdot = \frac{T_3}{T_1}$$

i.e. $Speed\ ratio = \frac{speed\ of\ diver}{speed\ of\ driven} = \frac{number\ of\ teeth\ on\ driven}{number\ of\ teeth\ on\ driver}$

$$train\ ratio = \frac{speed\ of\ driven}{speed\ of\ diver} = \frac{number\ of\ teeth\ on\ driver}{number\ of\ teeth\ on\ driven}$$

From above, we see that the speed ratio and the train value, in a simple train of gears, is independent of the size and number of intermediate gears. These intermediate gears are called ***idle gears,*** as they do not affect the speed ratio or train value of the system. The idle gears are used for the following two purposes:

1. To connect gears where a large centre distance is required, and
2. To obtain the desired direction of motion of the driven gear (*i.e*. clockwise or anticlockwise).

Example

A simple train has 3 gears. Gear A is the input and has 50 teeth. Gear C is the output and has 150 teeth. Gear A rotates at 1500 rev/min anticlockwise. Calculate the gear ratio and the output speed.

The input torques on A is 12 Nm and the efficiency is 75%. Calculate the output power and the holding torque.

Solution

$$gear\ ratio = \frac{n_A}{n_C} = \frac{T_C}{T_A} = \frac{150}{50} = 3$$

$$\frac{n_A}{n_C} = 3 \xrightarrow{yields} n_C = \frac{n_A}{3} = \frac{1500}{3} = 500\, rev/\,min \text{ (anticlockwise)}$$

$$\eta = \frac{P_{output}}{P_{input}} \xrightarrow{yields} P_{output} = \eta \cdot P_{input} = 0.75 \cdot 1885 = 1413.7\, W$$

$$T_C = \frac{P_{output}}{\omega_C} = \frac{60 \cdot P_{output}}{2 \cdot \pi \cdot N_C} = \frac{60 \cdot 1413.7}{2 \cdot \pi \cdot 500} = 27\, Nm)) \text{ (anticlockwise)}$$

$$T_A + T_C + T_{hold} = 0 \xrightarrow{yields} T_{hold} = -(T_A + T_C) = -(12 + 27) = -39\, Nm(clockwise)$$

8.9. Design of Spur Gears

Spur gears or straight-cut gears are the simplest type of gear. They consist of a cylinder or disk with teeth projecting radially as shown in figure bellow

Spur gears

Sometimes, the spur gears (*i.e*. driver and driven) are to be designed for the given velocity ratio and distance between the centers of their shafts.

Let:
x = Distance between the centers of two shafts,
n_1 = Speed of the driver,
T_1 = Number of teeth on the driver,
d_1 = Pitch circle diameter of the driver,
n_2, T_2 and d_2 = Corresponding values for the driven or follower, and
p_c = Circular pitch.
We know that the distance between the centers of two shafts:

$$x = \frac{d_1 + d_2}{2} \tag{3}$$

and speed ratio or velocity ratio:

$$\frac{n_1}{n_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1} \tag{4}$$

From the above equations, we can conveniently find out the values of $\boldsymbol{d_1}$ and $\boldsymbol{d_2}$ (or $\boldsymbol{T_1}$ and $\boldsymbol{T_2}$) and the circular pitch ($\boldsymbol{p_c}$). The values of $\boldsymbol{T_1}$ and $\boldsymbol{T_2}$, as obtained above, may or may not be whole numbers. But in a gear since the number of its teeth is always a whole number, therefore a slight alteration must be made in the values of $\boldsymbol{x}$, $\boldsymbol{d_1}$ and $\boldsymbol{d_2}$, so that the number of teeth in the two gears may be a complete number.

Example

Two parallel shafts, about 600 mm apart are to be connected by spur gears.
One shaft is to run at 360 r.p.m. and the other at 120 r.p.m. Design the gears, if the circular pitch is to be 25 mm.

Solution.

Given: x = 600 mm; n_1 = 360 r.p.m.; n_2 = 120 r.p.m.; pc = 25 mm

Solution.

Given: x = 600 mm; N_1 = 360 r.p.m.; N_2 = 120 r.p.m.; pc = 25 mm

Let d_1 = Pitch circle diameter of the first gear, and d_2 = Pitch circle diameter of the second gear. We know that speed ratio:

$$\frac{n_1}{n_2} = \frac{T_2}{T_1} \xrightarrow{yields} \frac{360}{120} = \frac{T_2}{T_1} \xrightarrow{yields} 3 = \frac{T_2}{T_1}$$

$$T_2 = 3 \cdot T_1 \tag{1}$$

and centre distance between the shafts (x):

$$x = \frac{d_1 + d_2}{2} \xrightarrow{yields} 600 = \frac{1}{2} \cdot (d_1 + d_2)$$

$$d_1 + d_2 = 1200\,mm \tag{2}$$

$$p_{c1} = p_{c2} \xrightarrow{yields} \frac{\pi \cdot d_1}{T_1} = \frac{\pi \cdot d_2}{T_2} = 25$$

From this is given:

$$\frac{d_1}{T_1} = \frac{d_2}{T_2} \xrightarrow{yields} d_2 = \frac{T_2}{T_1} \cdot d_1 = 3 \cdot d_1 \tag{3}$$

$$d_1 = \frac{25}{\pi} \cdot T_1 \tag{4}$$

$$d_2 = \frac{25}{\pi} \cdot T_2 \tag{5}$$

Offset (3) in (2)

$$d_1 + 3 \cdot d_1 = 1200\,mm$$

$$d_1 = 300\,mm;\; d_2 = 900\,mm$$

From (4) and (5)

$$d_1 = \frac{25}{\pi} \cdot T_1 \xrightarrow{yields} 300 = \frac{25}{\pi} \cdot T_1 \xrightarrow{yields} T_1 = 37.69 \approx 38$$

$$d_2 = \frac{25}{\pi} \cdot T_2 \xrightarrow{yields} 900 = \frac{25}{\pi} \cdot T_2 \xrightarrow{yields} \cdot T_2 = 113.09 \approx 114$$

$$\acute{d}_1 = \frac{T_1 \cdot p_c}{\pi} = \frac{38 \cdot 25}{\pi} = 302.39\,mm; \acute{d}_2 = \frac{T_2 \cdot p_c}{\pi} = \frac{114 \cdot 25}{\pi} = 907.18\,mm$$

∴ Exact distance between the two shafts:

$$x = \frac{\acute{d}_1 + \acute{d}_2}{2} = \frac{302.39\ +\ 907.18}{2} = 604.785\ mm$$

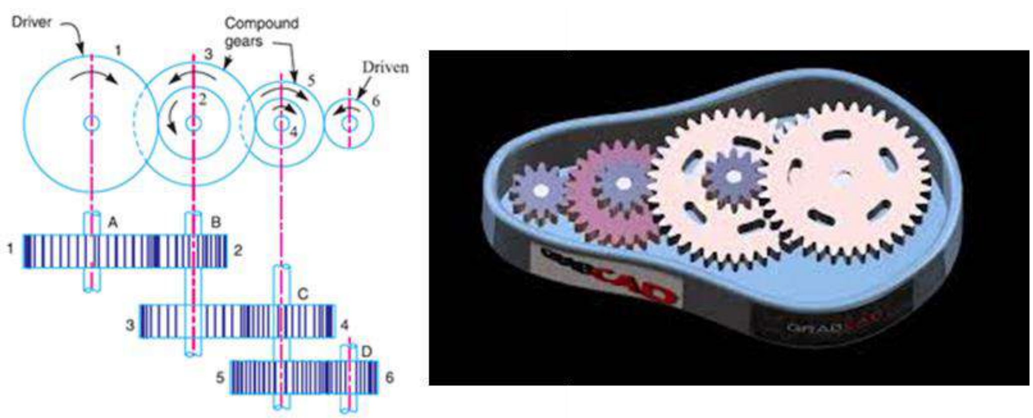
Hence the number of teeth on the first and second gear must be 38 and 114 and their pitch circle diameters must be 302.39 mm and 907.18 mm respectively. The exact distance between the two shafts must be 604.785 mm.

Example 2

Two parallel shafts are to be connected by spur gearing. The approximate distance between the shafts is 600 mm. If one shaft runs at 120 r.p.m. and the other at 360 r.p.m., find the number of teeth on each wheel, if the module is 8 mm. Also determine the exact distance apart of the shafts.

8.10. Compound Gear Train

When there is more than one gear on a shaft, as shown in Figure below, it is called a ***compound train of gear***.



Compound Gear Train

In a compound train of gears, as shown in Figure above the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let:

n_1 = *Speed of driving gear 1 in r.p.m,*

T_1 = *Number of teeth on driving gear 1,*

n_2, n_3 ..., n_6 = Speed of respective gears in r.p.m., and

T_2, T_3..., T_6 = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is:

$$\frac{n_1}{n_2} = \frac{T_2}{T_1} \qquad (3)$$

Similarly, for gears 3 and 4, speed ratio is:

$$\frac{n_3}{n_4} = \frac{T_4}{T_3} \qquad (4)$$

and for gears 5 and 6, speed ratio is:

$$\frac{n_5}{n_6} = \frac{T_6}{T_5} \qquad (5)$$

*The speed ratio of compound gear train is obtained by multiplying the equations **(3), (4)** and **(5),***

$$\therefore \frac{n_1}{n_2} \cdot \frac{n_3}{n_4} \cdot \frac{n_5}{n_6} = \frac{T_2}{T_1} \cdot \frac{T_4}{T_3} \cdot \frac{T_6}{T_5}$$

Since gears 2 and 3 are mounted on one shaft *B*, therefore $n_2 = n_3$. Similarly gears 4 and 5 are mounted on shaft *C*, therefore $n_4 = n_5$.

$$\therefore \frac{n_1}{n_6} = \frac{T_2 \cdot T_4 \cdot T_6}{T_1 \cdot T_3 \cdot T_5}$$

$$Speed\ ratio = \frac{Speed\ of\ the\ first\ driver}{Speed\ of\ the\ last\ driven\ or\ follower}$$
$$= \frac{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivens}{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivers}$$

$$Train\ value = \frac{Speed\ of\ the\ last\ driven\ or\ follower}{Speed\ of\ the\ first\ driver}$$
$$= \frac{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivers}{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivens}$$

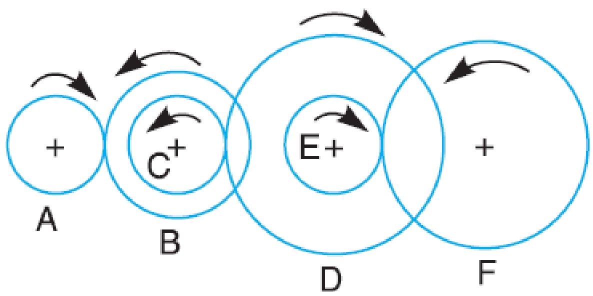
The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears. If a simple gear train is used to give a large speed reduction, the last gear has to be very large.

Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

Example1

The gearing of a machine tool is shown in Figure below. The motor shaft is connected to gear A and rotates at 975 r.p.m. The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft. What is the speed of gear F? The number of teeth on each gear are as given below:

Gear	*A*	*B*	*C*	*D*	*E*	*F*
No. of teeth	*20*	*50*	*25*	*75*	*26*	*65*



Solution

Given: n_A = 975 r.p.m.; T_A = 20; T_B = 50; T_C = 25; T_D = 75; T_E = 26; T_F = 65

From Figure above, we see that gears ***A***, ***C*** and ***E*** are drivers while the gears ***B***, ***D*** and ***F*** are driven or followers. Let the gear ***A*** rotates in clockwise direction. Since the gears ***B*** and ***C*** are mounted on the same shaft, therefore it is a ***compound gear*** and the direction or rotation of both these gears is same (*i.e.* anticlockwise). Similarly, the gears ***D*** and ***E*** are mounted on the same shaft, therefore it is also a ***compound gear*** and the direction of rotation of both these gears is same (*i.e.* clockwise). The gear ***F*** will rotate in anticlockwise direction.

Let N_F = Speed of gear *F*, *i.e.* last driven or follower. We know that

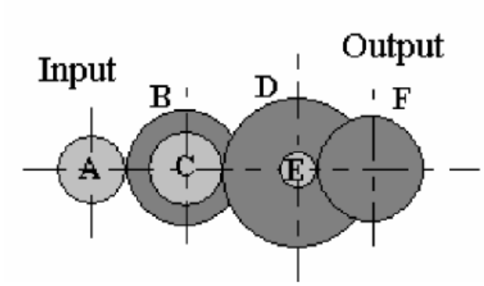
$$\frac{Speed\ of\ the\ first\ driver}{Speed\ of\ the\ last\ driven\ or\ follower} = \frac{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivens}{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivers}$$

$$\therefore \frac{n_A}{n_F} = \frac{T_B \cdot T_D \cdot T_F}{T_A \cdot T_C \cdot T_E} \xrightarrow{yields} \frac{975}{n_F} = \frac{50 \cdot 75 \cdot 65}{20 \cdot 25 \cdot 26} = 18.75$$

$$n_F = \frac{975}{18.75} = 52\, rev/min$$

Example2

Calculate the gear ratio for the compound chain shown below. If the input gear rotates clockwise, in which direction does the output rotate?



Gear A has 20 teeth
Gear B has 100 teeth
Gear C has 40 teeth
Gear D has 100 teeth
Gear E has 10 teeth
Gear F has 100 teeth

SOLUTION

The driving teeth are A, C and E.
The driven teeth are B, D and F
Gear ratio = product of driven teeth/product of driving teeth

Gear ratio = (100 x 100 x 100)/ (20 x 40 x 10) = 125

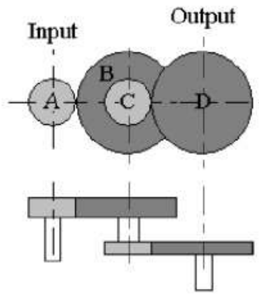
Alternatively we can say there are three simple gear trains and work ot the ratio for each.

First chain GR = 100/20 = 5
Second chain GR = 100/40 = 2.5
Third chain GR = 100/10 = 10
The overall ratio = 5 x 2.5 x 10 = 125

Each chain reverses the direction of rotation so if A is clockwise, B and C rotate anticlockwise so D and E rotate clockwise. The output gear F hence rotates anticlockwise.

Example3

Gear A is the input and revolves at 1200 rev/min clockwise viewed from the left end. The input torque is 30 Nm and the efficiency is 70%.
Gear A has 50 teeth
Gear B has 150 teeth
Gear C has 30 teeth
Gear D has 60 teeth



Calculate the following.

i. The output speed and its direction
ii. The output power.
iii. The fixing torque.

$$\therefore \frac{n_A}{n_D} = \frac{T_B \cdot T_D}{T_A \cdot T_C} \xrightarrow{yields} \frac{1200}{n_D} = \frac{150 \cdot 60}{50 \cdot 30} = 6$$

$$n_D = \frac{1200}{6} = 200rpm\ clockwise\ direction$$

$$P_{input} = \omega_{input} \cdot T_{input} = \frac{2 \cdot \pi \cdot n}{60} \cdot T_{input} = \frac{2 \cdot \pi \cdot 1200}{60} \cdot 30 =$$

$$P_{input} = 3769.9\ W$$

$$\eta = \frac{P_{output}}{P_{input}} \xrightarrow{yields} P_{output} = \eta \cdot P_{input} = 0.7 \cdot 3769.9$$

$$P_{output} = 2638.93W$$

$$P_{output} = \omega_{output} \cdot T_{output} \xrightarrow{yields} T_{output} = \frac{P_{output}}{\omega_{output}} = \frac{60 \cdot 2638.93}{2 \cdot \pi \cdot n_{output}}$$

$$T_{output} = \frac{60 \cdot 2638.93}{2 \cdot \pi \cdot 200} = 125.999Nm \approx 126Nm$$

$$T_{input} + T_{output} + T_{fix} = 0$$

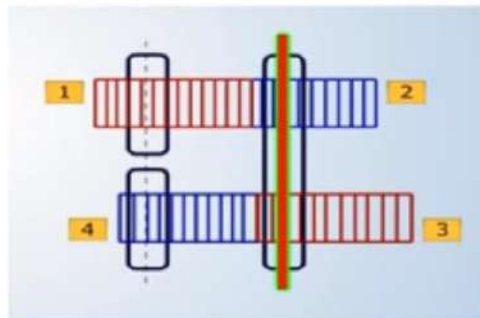
$$\because -T_{input}(clockwise) and -T_{output}(clockwise)$$

$$\therefore -30 - 126 + T_{fix} \xrightarrow{yields} T_{fix} = +156Nm\ anticlockwise$$

8.11. Reverted Gear Train

Reverted Gear Trains are used when the space between the Input and Output Shaft is small. This type of Gear Train requires large changes in Speed.

When the axis of the first gear (*i.e.* first driver) and the last gear (*i.e.* last driven or follower) are co-axial, then the gear train is known as ***reverted gear* train** as shown in figure below.



We see that gear 1 (*i.e.* first driver) drives the gear 2 (*i.e.* first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (*i.e.* the last driven or follower) in the same direction as that of gear 1. Thus, we see that in a reverted gear train, the motion of the first gear and the last gear is *like*.

Let:

T_1 = Number of teeth on gear 1,

r_1 = Pitch circle radius of gear 1, and

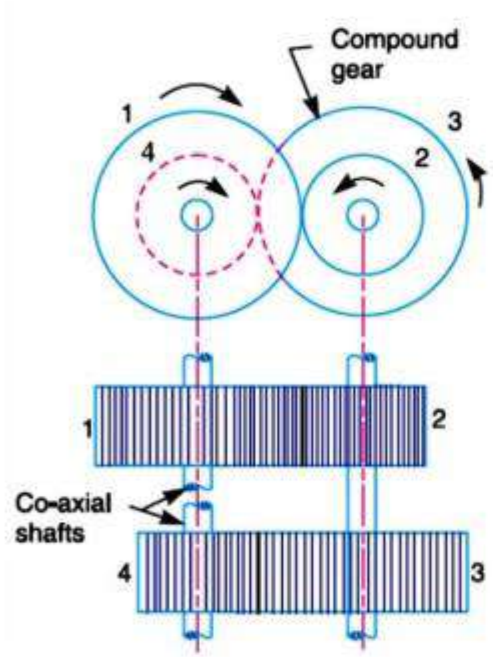
n_1 = Speed of gear 1 in r.p.m.

Similarly,

T_2, T_3, T_4 = Number of teeth on respective gears,

r_2, r_3, r_4 = Pitch circle radii of respective gears, and

n_2, n_3, n_4 = Speed of respective gears in r.p.m.



Since the distance between the centres of the shafts of gears 1 and 2 as well as gears 3 and 4 is same, therefore

$$r_1 + r_2 = r_3 + r_4 \tag{6}$$

Also, the circular pitch or module of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius.

$$p_c = \frac{2 \cdot \pi \cdot r}{T} \xrightarrow{yields} r = \frac{T \cdot p_c}{2 \cdot \pi}\ , or\ r = \frac{T \cdot m}{2} \left(m = \frac{2 \cdot r}{T} = \frac{p_c}{\pi}\ module\ gear \right)$$

Now from equation 6

$$\frac{T_1 \cdot m}{2} + \frac{T_2 \cdot m}{2} = \frac{T_3 \cdot m}{2} + \frac{T_4 \cdot m}{2} \xrightarrow{yields} T_1 + T_2 = T_3 + T_4 \tag{7}$$

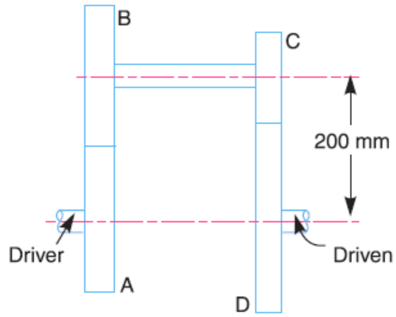
$$Speed\ ratio = \frac{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivens}{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivers} \xrightarrow{yields} \frac{n_1}{n_4} = \frac{T_2 \cdot T_4}{T_1 \cdot T_3} \tag{8}$$

From equations (6), (7) and (8), we can determine the number of teeth on each gear for the given centre distance, speed ratio and module only when the number of teeth on one gear is chosen arbitrarily.

The reverted gear trains are used in automotive transmissions, lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).

Example 1

The speed ratio of the reverted gear train, as shown in figure bellow, is to be 12. The module pitch of gears A and B is 3.125 mm and of gears C and D is 2.5 mm. Calculate the suitable numbers of teeth for the gears. No gear is to have less than 24 teeth.



Solution.

Given : Speed ratio, $n_A/n_D = 12$;

$m_A = m_B = 3.125$ mm ; $m_C = m_D = 2.5$ mm

Let n_A = Speed of gear A,

T_A = Number of teeth on gear A,

r_A = Pitch circle radius of gear A,

n_B, n_C , n_D = Speed of respective gears,

T_B, T_C , T_D = Number of teeth on respective gears, and

r_B, r_C , r_D = Pitch circle radii of respective gears.

Since the speed ratio between the gears ***A*** and ***B*** and between the gears ***C*** and ***D*** are to be same, therefore, We know that

$$speed\ ratio = \frac{Speed\ of\ the\ first\ driver}{Speed\ of\ the\ last\ driven\ or\ follower}$$

$$= \frac{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivens}{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivers}$$

$$speed\ ratio = \frac{n_A}{n_D} = 12$$

Also

$$\frac{n_A}{n_D} = \frac{n_A}{n_B} \cdot \frac{n_C}{n_D} = 12, (n_B\ =\ n_C, being\ on\ the\ same\ shaft)$$

$$For \frac{n_A}{n_B}\ and \frac{n_C}{n_D}\ to\ be\ same, each\ speed\ ratio\ should\ be\ 12\ so\ that$$

$$\frac{n_A}{n_D} = \frac{n_A}{n_B} \cdot \frac{n_C}{n_D} = \sqrt{12} \cdot \sqrt{12} = 12,$$

$$\frac{n_A}{n_B} = \frac{n_C}{n_D} = \sqrt{12} = 3.464$$

Also the speed ratio of any pair of gears in mesh is the inverse of their number of teeth, Therefore

$$speed\ ratio = \frac{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivens}{Product\ of\ the\ number\ of\ teeth\ on\ the\ drivers}$$

$$\frac{T_B}{T_A} = \frac{T_D}{T_C} = 3.464 \qquad (9)$$

We know that the distance between the shafts

$$x = r_A + r_B = r_C + r_D = 200$$

$$\because r = \frac{m \cdot T}{2}$$

$$\therefore \frac{m_A \cdot T_A}{2} + \frac{m_B \cdot T_B}{2} = \frac{m_C \cdot T_C}{2} + \frac{m_D \cdot T_D}{D}$$

$$= 200, (m_A = m_B = 3.125mm\ and\ m_C = m_C = 2.5mm\ )$$

$$3.125 \cdot (T_A + T_B) = 2.5 \cdot (T_C + T_D) = 400$$

$$(T_A + T_B) = \frac{400}{3.125} = 128 \qquad (10)$$

$$(T_C + T_D) = \frac{400}{2.5} = 160 \qquad (11)$$

From equation (9)

$$T_B = 3.464 \cdot T_A$$

Substituting this value of T_B in equation (10),

$$(T_A + 3.464 \cdot T_A) = 128 \xrightarrow{yields} T_A = \frac{128}{4.464} = 28.67\ say\ 28$$

$$(28 + T_B) = 128 \xrightarrow{yields} T_B = 100$$

Again from equation (9)

$$T_D = 3.464 \cdot T_C$$

Substituting this value of T_D in equation (11),

$$(T_C + 3.464 \cdot T_C) = 160 \xrightarrow{yields} T_C = \frac{160}{4.464} = 35.84\ say\ 36$$

$$(36 + T_D) = 160 \xrightarrow{yields} T_D = 160 - 36 = 124$$

The speed ratio of the reverted gear train with the calculated values of number of teeth on each gear is:

$$speed\ ratio = \frac{T_B}{T_A} \cdot \frac{T_D}{T_C} = \frac{100}{28} \cdot \frac{124}{36} = 12.3$$

8.11.1. Applications of reverted gear train

1-Epicyclic Gear

2-Train Machine Tools

3- Clocks

3- Clocks

1-Epicyclic Gear

2-Train Machine