Governors

1 Introduction

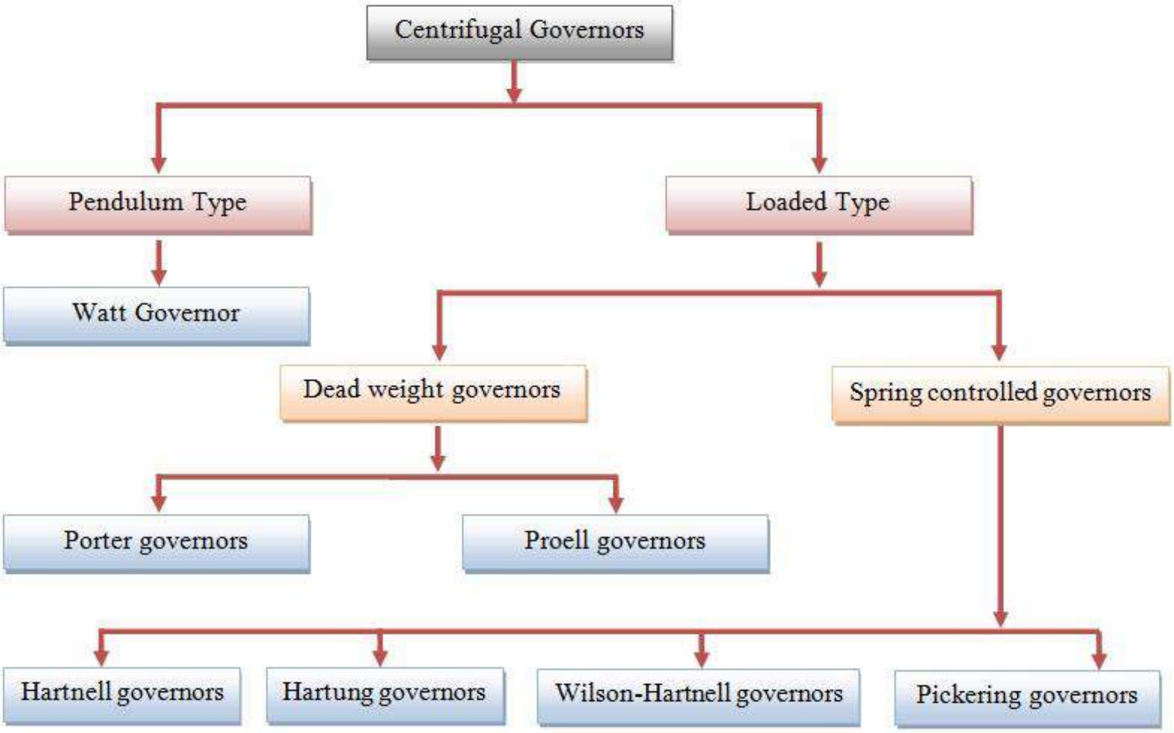
The function of a governor is to regulate the mean speed of an engine, when there are variations in the load *e.g.* (exemplī grātiā) when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

2 Types of Governors

The governors may, broadly, be classified as:

1. Centrifugal governors
2. Inertia governors

The centrifugal governors may further be classified as follows:



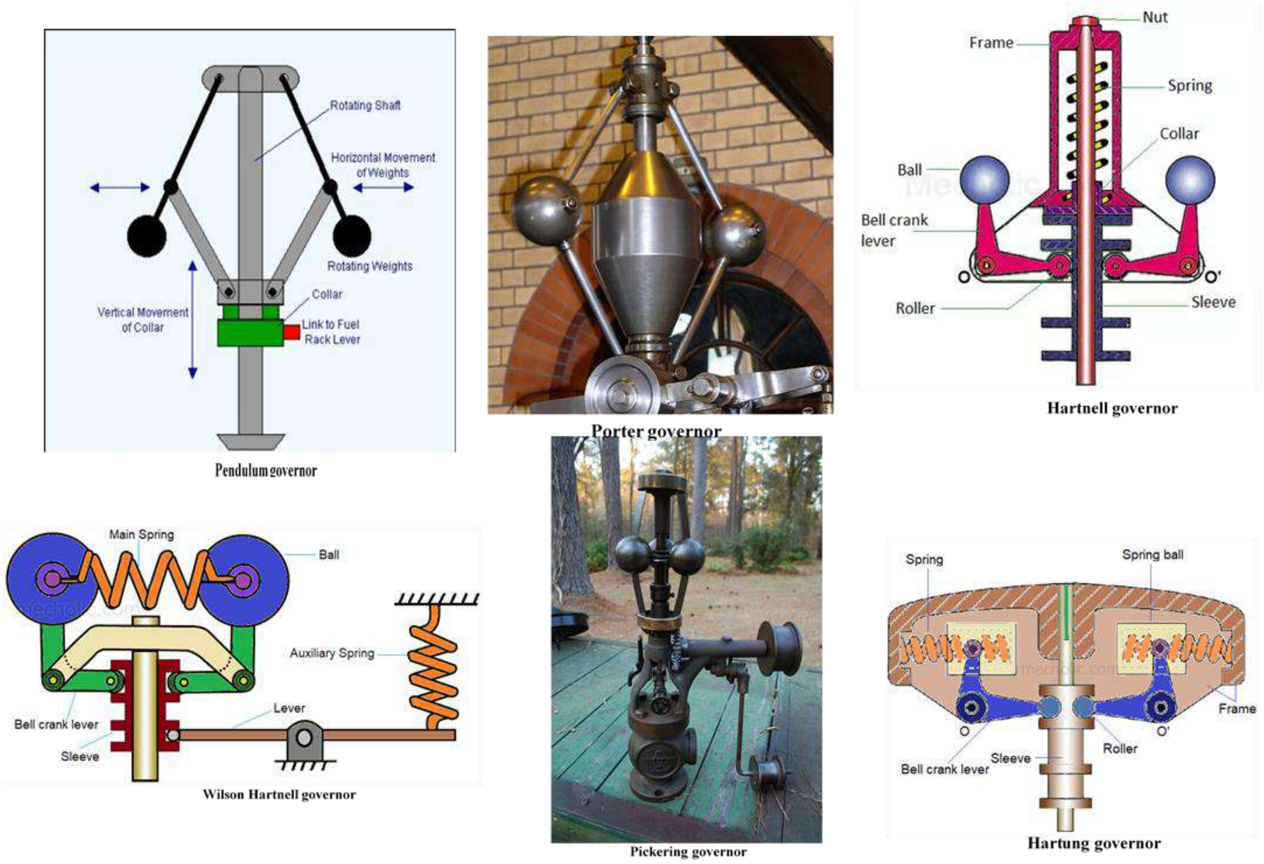
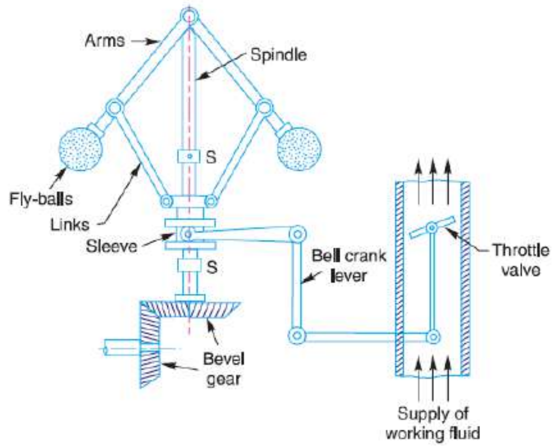


Fig. 1 Types of Governors

3 Centrifugal Governors

the centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the ***controlling force***[*1].it consists of two balls of equal mass, which are attached to the arms. These ***balls are known as governor balls*** or ***fly balls***.



The balls revolve with a spindle, which is driven by the engine through bevel gears. The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis. The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve revolves with the spindle but can slide up and down. The balls and the sleeve rise when the spindle speed increases, and falls when the speed decreases. In order to limit the travel of the sleeve in upward and downward directions, two stops *S*, *S* are provided on the spindle. The sleeve is

[*] The controlling force is provided either by the action of gravity as in Watt governor or by a spring as in case of Hartnell governor.

connected by a bell crank lever to a throttle valve. The supply of the working fluid decreases
when the sleeve rises and increases when it falls.

4. Watt Governor

The simplest form of a centrifugal governor is a Watt governor, as shown in Fig. 2

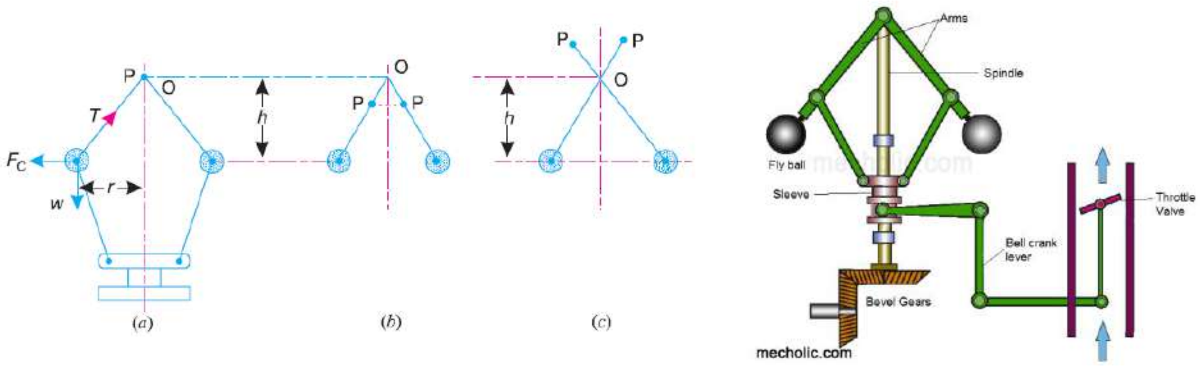


Fig. 2. Watt governor.

It is basically a conical pendulum with links attached to a sleeve of negligible mass. The arms of the governor may be connected to the spindle in the following three ways:

1. The pivot *P* may be on the spindle axis as shown in Fig. 2 (*a*).

2. The pivot *P* may be offset from the spindle axis and the arms when produced intersect at *O*, as shown in Fig.2 (*b*).

3. The pivot *P* may be offset, but the arms cross the axis at *O*, as shown in Fig.2 (*c*).

Let ***m*** is the mass of the ball [kg], ***w*** is the weight of the ball[N]=***mg***, ***T*** is the tension in the arm [N], **ω** is the angular velocity of the arm and ball about the spindle axis in[rad/s], ***r*** is the radius of the path of rotation of the ball *i.e.* horizontal distance from the center of the ball to the spindle axis [m], $\boldsymbol{F_C}$ is the centrifugal force acting on the ball $(\boldsymbol{F_C = m.\omega^2.r})$ [N], and

h is the height of the governor [m].

It is assumed that the weight of the arms, links and the sleeve are negligible as compared to the weight of the balls. Now, the ball is in equilibrium under the action of:

1. the centrifugal force (F_C) acting on the ball,
2. the tension (T) in the arm, and
3. the weight (w) of the ball. Taking moments about point O, we have

$$\sum M_O = 0 \xrightarrow{yields} F_c \cdot h = w \cdot r = m \cdot g \cdot r \xrightarrow{yields} m \cdot \omega^2 \cdot r \cdot h = m \cdot g \cdot r$$

$$\therefore h = \frac{g}{\omega^2} \tag{1}$$

When g is expressed in m/s^2 and ω in rad/s, then h is in meters. If n is the speed in r.p.m., then: $\omega = \frac{2 \cdot \pi \cdot n}{60}$

$$\therefore h = \frac{g}{\omega^2} = \frac{9.81}{(\frac{2 \cdot \pi \cdot n}{60})^2} = \frac{895}{n^2} \quad (2)$$

Note: We see from the above expression that the height of a governor $\boldsymbol{h}$, is inversely proportional to $\boldsymbol{n}^2$. Therefore at high speeds, the value of $\boldsymbol{h}$ is small. At such speeds, the change in the value of $\boldsymbol{h}$ corresponding to a small change in speed is insufficient to enable a governor of this type to operate the mechanism to give the necessary change in the fuel supply. This governor may only work satisfactorily at relatively low speeds *i.e.* from 60 to 80 r.p.m.

Example 1

Calculate the vertical height of a Watt governor when it rotates at 60 r.p.m. Also, find the change in vertical height when its speed increases to 61 r.p.m.

Solution.

Given: n_1 = 60 r.p.m.; n_2 = 61 r.p.m.

Initial height

We know that initial height:

$$h_1 = \frac{895}{n_1^2} = \frac{895}{60^2} = 0.248\, m$$

Change in vertical height

We know that final height:

$$h_2 = \frac{895}{n_2^2} = \frac{895}{61^2} = 0.240\, m$$

$$\therefore \ Change\ in\ vertical\ height = \Delta h = h_2 - h_1 = 0.248 - 0.240 = 0.008\, m = 8\, mm$$

5 Porter Governor

The Porter governor is a modification of a Watt's governor, with a central load attached to the sleeve as shown in Fig.3 (*a*). The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level. Consider the forces acting on one-half of the governor as shown in fig 3b.

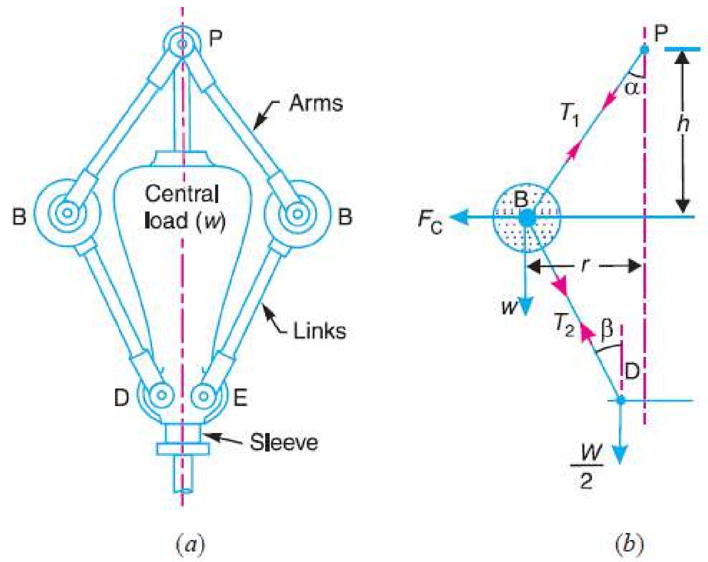


Fig..3. Porter governor

Let: ***m*** is mass of each ball [kg], ***w*** is weight of each ball [N](**w = *m.g)***, ***M*** is mass of the central load [kg], ***W*** is weight of the central load [N] (**W = *M.g)***, ***r*** is radius of rotation [m], ***h*** is height of governor [m] , ***n*** is speed of the balls in r.p.m ., ***ω*** is angular speed of the balls [rad/s] (**ω = 2 π*N*/60** [rad/s]),$\boldsymbol{F_C}$ is centrifugal force acting on the ball [N] ($\boldsymbol{F_C = m.\omega^2.r}$), $\mathbf{T_1}$ is force in the arm in [N], $\mathbf{T_2}$ is force in the link in [N], ***α*** is angle of inclination of the arm (or upper link) to the vertical, and ***β*** is Angle of inclination of the link(or lower link) to the vertical.

Though there are several ways of determining the relation between the height of the governor (***h***) and the angular speed of the balls (**ω**), yet the following two methods are important from the subject point of view:

1. Method of resolution of forces; and

2. Instantaneous center method.

12.5.1. *Method of resolution of forces*

Considering the equilibrium of the forces acting at *D*, we have:

$$T_2 \cdot \cos\beta = \frac{W}{2} = \frac{M\cdot g}{2} \xrightarrow{yields} T_2 = \frac{M\cdot g}{2\cdot cos\,\beta} \qquad (3)$$

Again, considering the equilibrium of the forces acting on *B*. The point *B* is in equilibrium under the action of the following forces, as shown in Fig..3 (*b*).

i. The weight of ball ($w = m.g$),
ii. The centrifugal force (F_C),
iii. The tension in the arm (T_1), and
iv. The tension in the link (T_2).

Resolving the forces vertically,

$$T_1 \cdot \cos\alpha = T_2 \cdot \cos\beta + w = \frac{M\cdot g}{2} + m \cdot g \qquad (4)$$

Resolving the forces horizontally

$$T_1 \cdot sin\,\alpha + T_2 \cdot sin\,\beta = F_c,$$

$substitute\ eq\ (3$

$$T_1 \cdot sin\,\alpha + \frac{M \cdot g}{2 \cdot cos\,\beta} \cdot sin\,\beta = F_c \xrightarrow{yields} T_1 \cdot sin\,\alpha + \frac{M \cdot g}{2} \cdot \tan\beta = F_c$$

$$T_1 \cdot sin\,\alpha = F_c - \frac{M \cdot g}{2} \cdot \tan\beta \qquad (5)$$

Dividing equation **(5)** by equation ***(4)*,**

$$\frac{T_1 \cdot sin\,\alpha}{T_1 \cdot cos\,\alpha} = \frac{F_c - \frac{M \cdot g}{2} \cdot tan\,\beta}{\frac{M \cdot g}{2} + m \cdot g} = \tan\alpha \xrightarrow{yields} \left(\frac{M \cdot g}{2} + m \cdot g\right) \cdot \tan\alpha = F_c - \frac{M \cdot g}{2} \cdot tan\,\beta$$

$$\left(\frac{M \cdot g}{2} + m \cdot g\right) = \frac{F_c}{tan\,\alpha} - \frac{M \cdot g}{2} \cdot \frac{tan\,\beta}{tan\,\alpha}$$

Substituting: $\frac{tan\,\beta}{tan\,\alpha} = q;\ tan\,\alpha = \frac{r}{h};\ F_c = m \cdot \omega^2 \cdot r$

$$\left(\frac{M \cdot g}{2} + m \cdot g\right) = m \cdot \omega^2 \cdot r \cdot \frac{h}{r} - \frac{M \cdot g}{2} \cdot q \xrightarrow{yields} m \cdot \omega^2 \cdot h = m \cdot g + \frac{M \cdot g}{2} \cdot (1 + q)$$

$$h = m \cdot g + \frac{M \cdot g}{2} \cdot (1 + q) \cdot \frac{1}{m \cdot \omega^2} \xrightarrow{yields} h = \frac{m + \frac{M}{2} \cdot (1+q)}{m} \cdot \frac{g}{\omega^2} \qquad (6)$$

$$\omega^2 = \frac{m + \frac{M}{2} \cdot (1 + q)}{m} \cdot \frac{g}{h} \xrightarrow{yields} \left(\frac{2 \cdot \pi \cdot n}{60}\right)^2 = \frac{m + \frac{M}{2} \cdot (1 + q)}{m} \cdot \frac{g}{h}$$

$$n^2 = \frac{m + \frac{M}{2} \cdot (1+q)}{m} \cdot \frac{g}{h} \cdot \left(\frac{60}{2 \cdot \pi}\right)^2 \xrightarrow{yields} N^2 = \frac{m + \frac{M}{2} \cdot (1+q)}{m} \cdot \frac{895}{h} \qquad (7)$$

Notes: 1. When the length of arms is equal to the length of links and the points *P* and *D* lie on the same vertical line, then

$$\tan\alpha = \tan\beta \xrightarrow{yields} q = \frac{tan\,\alpha}{tan\,\beta} = 1$$

Therefore, equation (*7*) becomes:

$$n^2 = \frac{m + M}{m} \cdot \frac{895}{h}\ or\ h = \frac{m + M}{m} \cdot \frac{895}{n^2} \qquad (8)$$

2. When the loaded sleeve moves up and down the spindle, the frictional force acts on it in a direction opposite to that of the motion of the sleeve.

If *F* = Frictional force acting on the sleeve in newtons, then the equations **(*6*)** and **(*7*)** may be written as:

$$n^2 = \frac{m \cdot g + \frac{(M \cdot g \pm F)}{2} \cdot (1+q)}{m \cdot g} \cdot \frac{895}{h} \qquad (9)$$

$$n^2 = \frac{m \cdot g + (M \cdot g \pm F)}{m \cdot g} \cdot \frac{895}{h}; when\ q = 1 \qquad (10)$$

The **+** sign is used when the sleeve moves upwards or the governor speed increases and the negative sign is used when the sleeve moves downwards or the governor speed decreases.

3. On comparing the equation ***(8)*** with equation **(*2*)** of Watt's governor (Art. 12.4), we find that the mass of the central load (*M*) increases the height of governor in the ratio:

$$\frac{m + M}{m}$$

6. *Instantaneous center method*

In this method, the equilibrium of the forces acting on the link *BD* is considered. The instantaneous center *I* lies at the point of intersection of *PB* produced and a line through *D* perpendicular to the spindle axis, as shown in Fig.4. Taking moments about the point *I*,

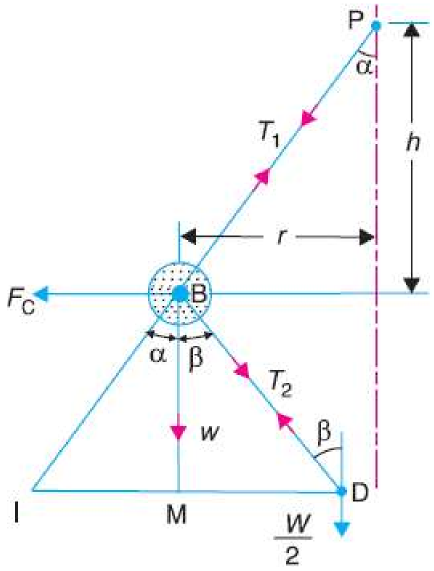


Fig. 4. Instantaneous center method

$$F_c \cdot BM = w \cdot IM + \frac{W}{2} \cdot ID = m \cdot g \cdot IM + \frac{M \cdot g}{2} \cdot ID$$

$$F_c = m \cdot g \cdot \frac{IM}{BM} + \frac{M \cdot g}{2} \cdot \frac{(IM + MD)}{BM} = m \cdot g \cdot \frac{IM}{BM} + \frac{M \cdot g}{2} \cdot (\frac{IM}{BM} + \frac{MD}{BM})$$

$$F_c = m \cdot g \cdot tan\,\alpha + \frac{M \cdot g}{2} \cdot (\tan\alpha + \tan\beta)$$

Dividing throughout by ***tan α***,

$$\frac{F_c}{tan\,\alpha} = m \cdot g \cdot + \frac{M \cdot g}{2} \cdot \left(1 + \frac{\tan\beta}{tan\,\alpha}\right); for\ q = \frac{tan\,\beta}{tan\,\alpha}$$

$$\frac{F_c}{tan\,\alpha} = m \cdot g \cdot + \frac{M \cdot g}{2} \cdot (1 + q)$$

For $Fc = m \cdot \omega^2 \cdot r\ and\ \tan\alpha = \frac{r}{h} \xrightarrow{yields} m \cdot \omega^2 \cdot r\ \cdot \frac{h}{r} = m \cdot g \cdot + \frac{M \cdot g}{2} \cdot (1 + q)$

$$h = \frac{m \cdot g + \frac{M \cdot g}{2} \cdot (1 + q)}{m \cdot \omega^2} = \frac{m + \frac{M}{2} \cdot (1 + q)}{m} \cdot \frac{g}{\omega^2}$$

When ***tan α = tan β or q = 1***, then

$$h = \frac{m + M}{m} \cdot \frac{g}{\omega^2}$$

7. Sensitiveness of Governors

Consider two governors *A* and *B* running at the same speed. When this speed increases or decreases by a certain amount, the lift of the sleeve of governor *A* is greater than the lift of the sleeve of governor *B*. It is then said that the governor *A* is more sensitive than the governor *B*.

the sensitiveness is defined as the ***ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.***

Let

N_1 = Minimum equilibrium speed,

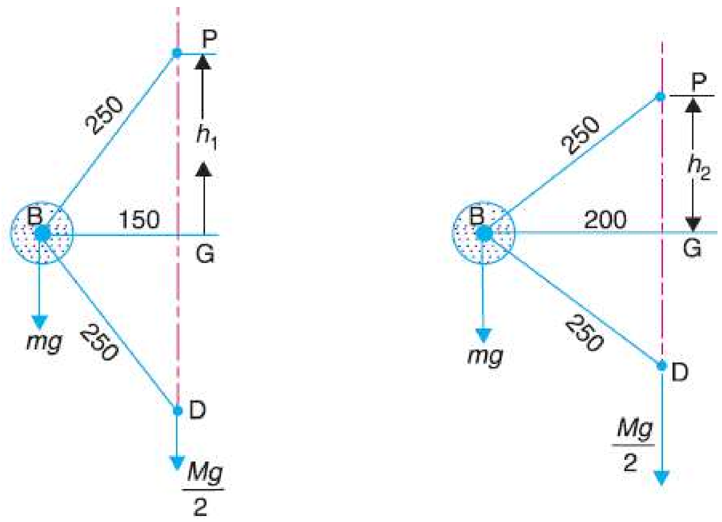
N_2 = Maximum equilibrium speed, and

$$N\ =\ Mean\ equilibrium\ speed = \frac{N_1 + N_2}{2}$$

$$\therefore\ Sensitiveness\ of\ the\ governor = \frac{N_1 - N_2}{N} = \frac{2 \cdot (N_1 - N_2)}{N_1 + N_2}$$

Example 2

A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 15 kg. The radius of rotation of the ball is 150 mm when the government begins to lift and 200 mm when the governor is at maximum speed. Find the minimum and maximum speeds and range of speed of the governor, the sensitiveness of the governor.



(*a*) Minimum position. (*b*) Maximum positio

Fig. 5

Solution. Given: *BP* = *BD* = 250 mm = 0.25 m ; *m* = 5 kg ; *M* = 15 kg ; r_1 = 150 mm = 0.15m; r_2 = 200 mm = 0.2 m.

The minimum and maximum positions of the governor are shown in Fig.5 (*a*) and (*b*) respectively.

Let N_1 = Minimum speed. From Fig.5 (*a*), we find that height of the governor:

$$h_1 = \sqrt{PB^2 - BG^2} = \sqrt{0.25^2 - 0.15^2} = 0.2\,m$$

We know that:

$$N_1^2 = \frac{m+M}{m} \cdot \frac{895}{h} = \frac{5+15}{5} \cdot \frac{895}{0.2} = 17900$$

$$N_1 = 133.79\,rpm\ (minimum\ speed)$$

Maximum speed N_2 when r_2 = BG = 0.2 m see fig. 5b

$$h_2 = \sqrt{PB^2 - BG^2} = \sqrt{0.25^2 - 0.2^2} = 0.15\,m$$

$$\therefore N_2^2 = \frac{m+M}{m} \cdot \frac{895}{h} = \frac{5+15}{5} \cdot \frac{895}{0.15} = 23866.6$$

$$N_2 = 154.5\,rpm\ (maximum\ speed)$$

$$We\ know\ that\ range\ of\ speed = 133.79 < n < 154.5\,rpm$$

$$Rang\ rotating\ speed = N_2 - N_1 = 154.5 - 133.79 = 20.71\,rpm$$

We know that sensitiveness of the governor= $\frac{2\cdot(N_{max}-N_{min})}{N_{max}+N_{min}} = \frac{2\cdot(154.5-133.79)}{154.5+133.79} =$ $0.143 = 14.3\%$

Example 3.

The arms of a Porter governor are each 250 mm long and pivoted on the governor axis. The mass of each ball is 5 kg and the mass of the central sleeve is 30 kg. The radius of rotation of the balls is 150 mm when the sleeve begins to rise and reaches a value of 200 mm for maximum speed. Determine the speed range of the governor. If the friction at the sleeve is equivalent of 20 N of the load at the sleeve, determine how the speed range is modified, the sensitiveness of the governor both state.

Solution.

Given: $BP = BD = 250$ mm; $m = 5$ kg; $M = 30$ kg; $r_1 = 150$ mm; $r_2 = 200$ mm

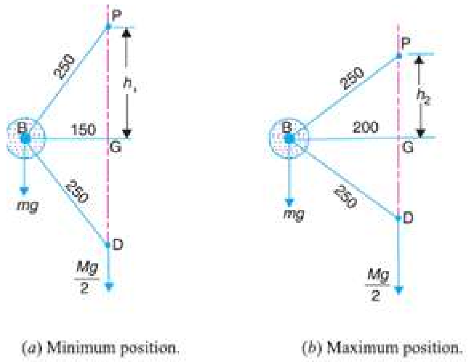


Fig. 6

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig.6 (*a*) and (*b*) respectively.

Let N_1 = Minimum speed when r_1 = ***BG*** = 150 mm, and N_2 = Maximum speed when r_2 = ***BG*** = 200 mm.

Speed range of the governor

From Fig.6 (*a*), we find that height of the governor:

$$h_1 = PG = \sqrt{PB^2 - BG^2} = \sqrt{0.25^2 - 0.15^2} = 0.2\,m$$

$$N_1^2 = \frac{m+M}{m} \cdot \frac{895}{h} = \frac{5+30}{5} \cdot \frac{895}{0.2} = 31325$$

$$N_1 = 117\,rpm\ (minimum\ speed)$$

From Fig.6 (*b*), we find that the height of the governor

$$h_2 = PG = \sqrt{PB^2 - BG^2} = \sqrt{0.25^2 - 0.2^2} = 0.15\,m$$

$$\therefore N_2^2 = \frac{m+M}{m} \cdot \frac{895}{h} = \frac{5+30}{5} \cdot \frac{895}{0.15} = 41767$$

$$N_2 = 204.4\,rpm\ (maximum\ speed)$$

$$Rang\ rotating\ speed = N_2 - N_1 = 204.4\ - 117 = 87.4\,rpm$$

We know that sensitiveness of the governor=$\frac{2\cdot(N_{max}-N_{min})}{N_{max}+N_{min}} = \frac{2\cdot(204.4-117)}{204.4+117} = 0.543 = 54.3\%$

Speed range when friction at the sleeve is equivalent of 20 N of load (i.e. when F = 20 N)

We know that when the sleeve moves downwards, the friction force (*F*) acts upwards and the minimum speed is given by:

$$N_1^2 = \frac{m \cdot g + (M \cdot g - F)}{m \cdot g} \cdot \frac{895}{h_1} = \frac{5 \cdot 9.8 + (30 \cdot 9.8 - 20)}{5 \cdot 9.8} \cdot \frac{895}{0.2} = 29500$$

$$N_1 = 172\, rpm$$

We also know that when the sleeve moves upwards, the frictional force (*F*) acts downwards and the maximum speed is given by

$$N_2^2 = \frac{m \cdot g + (M \cdot g + F)}{m \cdot g} \cdot \frac{895}{h_2} = \frac{5 \cdot 9.8 + (30 \cdot 9.8 + 20)}{5 \cdot 9.8} \cdot \frac{895}{0.15} = 44202.$$

$$N_2 = 210.24\, rpm$$

$$Rang\ rotating\ speed = N_2 - N_1 = 210.24\ - 172 = 38.24\, rpm$$

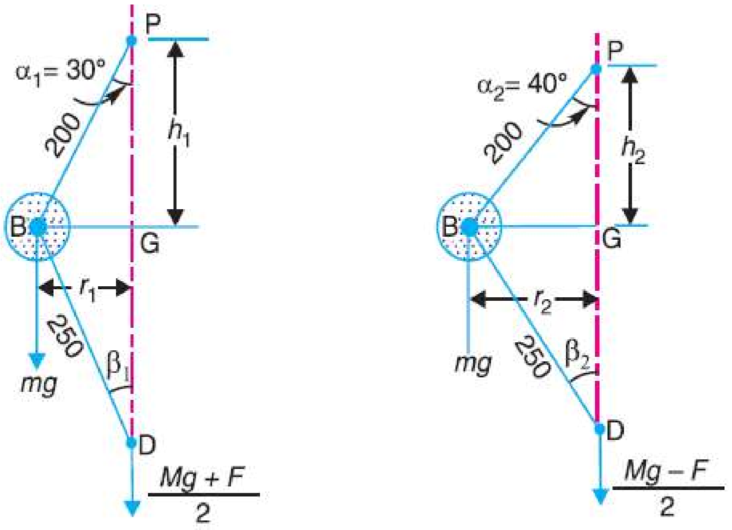
We know that sensitiveness of the governor=$\frac{2\cdot(N_{max}-N_{min})}{N_{max}+N_{min}} = \frac{2\cdot(210.4-172)}{210+172} = 0.201 = 20.1\%$

Example 3

In an engine governor of the Porter type, the upper and lower arms are 200mm and 250 mm respectively and pivoted on the axis of rotation. The mass of the central load is 15 kg, the mass of each ball is 2 kg and friction of the sleeve together with the resistance of the operating gear is equal to a load of 25 N at the sleeve. If the limiting inclinations of the upper arms to the vertical are 30° and 40°, find, taking friction into account, the range of speed of the governor, the sensitiveness of the governor.

Solution

Given: *BP* = 200 mm = 0.2 m ; *BD* = 250 mm = 0.25 m ; *M* = 15 kg ; *m* = 2 kg; *F* = 25 N ; $\alpha_1 = 30°$; $\alpha_2 = 40°$.



All dimensions in mm.

(*a*) Minimum position. (*b*) Maximum positio

Fig. 7

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown Fig.7 (*a*) and (*b*) respectively.

Let n_1 = Minimum speed, and n_2 = Maximum speed. From Fig. 7 (*a*), we find that minimum radius of rotation:

$$r_1 = BG = PB \cdot \sin \alpha_1 = 0.2 \cdot sin\,30 = 0.2 \cdot 0.5 = 1\ m$$

The height of the governor

$$h_1 = PG = PB \cdot cos\,\alpha_1 = 0.2 \cdot cos\,30 = 0.2 \cdot 0.866 = 0.173\,m$$

$$DG = \sqrt{BD^2 - BG^2} = \sqrt{0.25^2 - 0.1^2} = 0.23\,m$$

$$\tan\beta_1 = \frac{BG}{DG} = \frac{0.1}{0.23} = 0.4348 \xrightarrow{yields} \beta = \tan^{-1} 0.4348 = 26°$$

$$\tan\alpha_1 = tan\,30 = 0.5774$$

$$q_1 = \frac{tan\,\beta_1}{tan\,\alpha_1} = \frac{0.4348}{0.5774} = 0.753$$

We know that when the sleeve moves downwards, the frictional force (*F*) acts upwards and the minimum speed is given by:

$$N_1^2 = \frac{m \cdot g + \frac{(M \cdot g - F)}{2} \cdot (1 + q)}{m \cdot g} \cdot \frac{895}{h_1}$$

$$= \frac{2 \cdot 9.81 + \frac{(15 \cdot 9.81 - 25)}{2} \cdot (1 + 0.753)}{2 \cdot 9.81} \cdot \frac{895}{0.173} = 33404.2$$

$$N_1 = 182.77\,rpm$$

Now from Fig.7 (*b*), we find that maximum radius of rotation:

$$r_2 = BG = PB \cdot \sin\alpha_2 = 0.2 \cdot sin\,40 = 0.2 \cdot 0.64 = 0.129\,m$$

The height of the governor,

$$h_2 = PG = PB \cdot cos\,\alpha_2 = 0.2 \cdot cos\,40 = 0.2 \cdot 0.766 = 0.153\,m$$

$$DG = \sqrt{BD^2 - BG^2} = \sqrt{0.25^2 - 0.129^2} = 0.214\,m$$

$$\tan\beta_2 = \frac{BG}{DG} = \frac{0.129}{0.214} = 0.603 \xrightarrow{yields} \beta = \tan^{-1} 0.4348 = 31.08°$$

$$\tan\alpha_2 = tan\,40 = 0.5774$$

$$q_2 = \frac{tan\,\beta_2}{tan\,\alpha_2} = \frac{0.603}{0.5774} = 0.84$$

We know that when the sleeve moves upwards, the frictional force (*F*) acts downwards and the maximum speed is given by:

$$N_2^2 = \frac{m \cdot g + \frac{(M \cdot g + F)}{2} \cdot (1 + q)}{m \cdot g} \cdot \frac{895}{h_2}$$

$$= \frac{2 \cdot 9.81 + \frac{(15 \cdot 9.81 + 25)}{2} \cdot (1 + 0.84)}{2 \cdot 9.81} \cdot \frac{895}{0.153} = 53069.8$$

$$N_2 = 230\, rpm$$

$$Rang\ rotating\ speed = N_2 - N_1 = 230 - 182.77 = 47.23\, rpm$$

We know that sensitiveness of the governor= $\frac{2 \cdot (N_{max} - N_{min})}{N_{max} + N_{min}} = \frac{2 \cdot (230 - 182.77)}{230 + 182.77} = 0.2288 = 22.88\%$