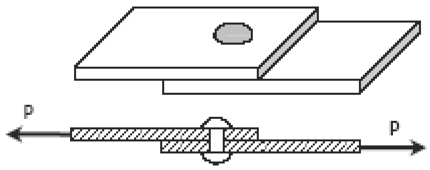
Shearing Stress

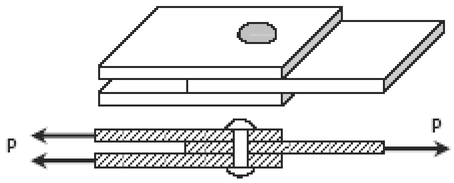
Forces parallel to the area resisting the force cause shearing stress. It differs to tensile and compressive stresses, which are caused by forces perpendicular to the area on which they act. Shearing stress is also known as tangential stress.

$$\tau = \frac{V}{A}$$

where V is the resultant shearing force which passes which passes through the centroid of the area A being sheared.



Single Shear



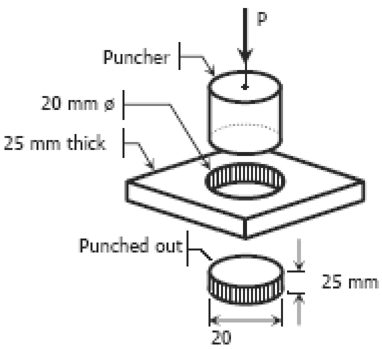
Double Shear

SOLVED PROBLEMS IN SHEARING STRESS

Problem 115

What force is required to punch a 20-mm-diameter hole in a plate that is 25 mm thick? The shear strength is 350 MN/m^2.

Solution 115



The resisting area is the shaded area along the perimeter and the shear force V is equal to the punching force P.

$$V = \tau A$$

$$P = 350[\pi(20)(25)]$$

$$= 549\ 778.7\ \text{N}$$

$$= \mathbf{549.8\ kN}$$

Problem 116

As in Fig. 1-11c, a hole is to be punched out of a plate having a shearing strength of 40 ksi. The compressive stress in the punch is limited to 50 ksi. (**a**) Compute the maximum thickness of plate in which a hole 2.5 inches in diameter can be punched. (**b**) If the plate is 0.25 inch thick, determine the diameter of the smallest hole that can be punched.

Solution 116

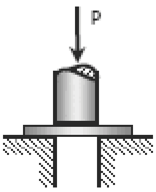


Figure 1-11c

(a) Maximum thickness of plate:

Based on puncher strength:

$P = \sigma A$

$= 50[\frac{1}{4}\pi(2.5^2)]$

$= 78.125\pi$ kips → Equivalent shear force of the plate

Based on shear strength of plate:

$V = \tau A$ → V = P

$78.125\pi = 40[\pi(2.5t)]$

$t = 0.781$ inch

(b) Diameter of smallest hole:

Based on compression of puncher:

$P = \sigma A$

$= 50(\frac{1}{4}\pi d^2)$

$= 12.5\pi d^2$ → Equivalent shear force for plate

Based on shearing of plate:

$V = \tau A$ → V = P

$12.5\pi d^2 = 40[\pi d(0.25)]$

$d = 0.8$ in

Problem 117

Find the smallest diameter bolt that can be used in the clevis shown in Fig. 1-11b if P = 400 kN. The shearing strength of the bolt is 300 MPa.

Solution 117

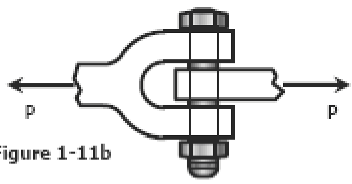


Figure 1-11b

The bolt is subject to double shear.

$V = \tau A$

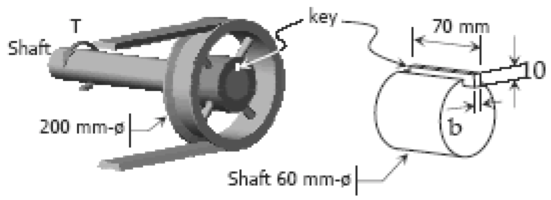
$400(1000) = 300[2(\frac{1}{4}\pi d^2)]$

$d = 29.13$ mm

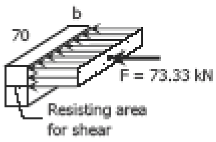
Problem 118

A 200-mm-diameter pulley is prevented from rotating relative to 60-mm-diameter shaft by a 70-mm-long key, as shown in Fig. P-118. If a torque T = 2.2 kN·m is applied to the shaft, determine the width b if the
allowable shearing stress in the key is 60 MPa.

Figure P-118



Solution 118



FBD of Pin

$T = 0.03F$

$2.2 = 0.03F$

$F = 73.33$ kN

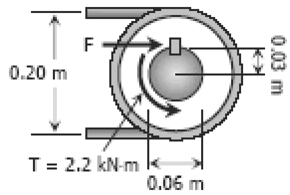
$V = \tau A$

Where: $V = F = 73.33$ kN

$A = 70b$; $\tau = 60$ MPa

$73.33(1000) = 60(70b)$

$b = \mathbf{17.46\ mm}$



Problem 119

Compute the shearing stress in the pin at B for the member supported as shown in Fig. P-119. The pin diameter is 20 mm.

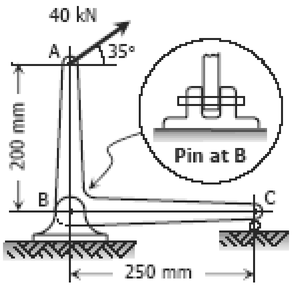
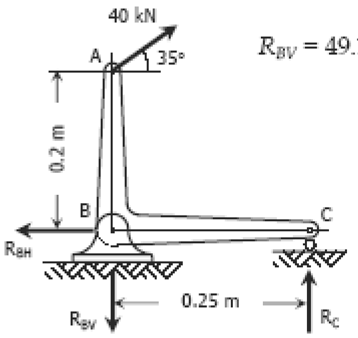


Figure P-119

Solution 119



Free Body Diagram

From the FBD:

$\Sigma M_C = 0$

$0.25R_{BV} = 0.25(40 \sin 35°) + 0.2(40 \cos 35°)$

$R_{BV} = 49.156$ kN

$\Sigma F_H = 0$

$R_{BH} = 40 \cos 35°$

$= 32.766$ kN

$R_B = \sqrt{R_{BH}^2 + R_{BV}^2}$

$= \sqrt{32.766^2 + 49.156^2}$

$= 59.076$ kN → shear force of pin at B

$V_B = \tau_B A$ → double shear

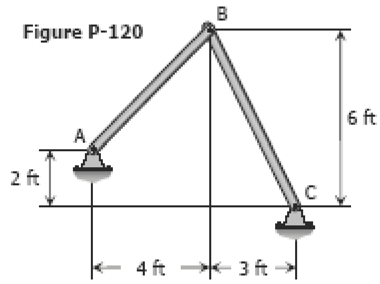
$59.076\,(1000) = \tau_B\,\{2[\tfrac{1}{4}\pi(20^2)]\}$

$\tau_B = \mathbf{94.02\ MPa}$

Problem 120

The members of the structure in Fig. P-120 weigh 200 lb/ft. Determine the smallest diameter pin that can be used at A if the shearing stress is limited to 5000 psi. Assume single shear.

Figure P-120

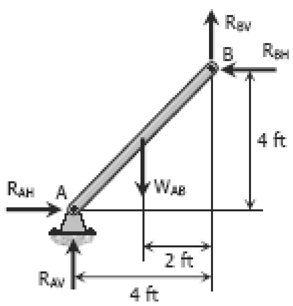


Solution 120

For member AB:

$$\text{Length, } L_{AB} = \sqrt{4^2 + 4^2} = 5.66 \text{ ft}$$

$$\text{Weight, } W_{AB} = 5.66(200) = 1132 \text{ lb}$$



FBD of member

$$\Sigma M_A = 0$$

$$4R_{BH} + 4R_{BV} = 2W_{AB}$$

$$4R_{BH} + 4R_{BV} = 2(1132)$$

$$R_{BH} + R_{BV} = 566 \quad \rightarrow (1)$$

For member BC:

$$\text{Length, } L_{BC} = \sqrt{3^2 + 6^2} = 6.71 \text{ ft}$$

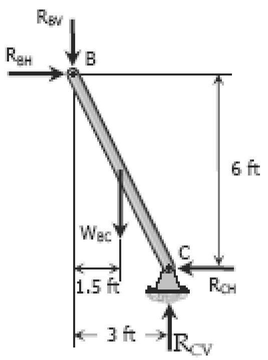
$$\text{Weight, } W_{BC} = 6.71(200) = 1342 \text{ lb}$$

$$\Sigma M_C = 0$$

$$6R_{BH} = 1.5W_{BC} + 3R_{BV}$$

$$6R_{BH} - 3R_{BV} = 1.5(1342)$$

$$2R_{BH} - R_{BV} = 671 \quad \rightarrow (2)$$



FBD of member BC

Add equations (1) and (2)

$$R_{BH} + R_{BV} = 566 \quad \rightarrow (1)$$

$$2R_{BH} - R_{BV} = 671 \quad \rightarrow (2)$$

$$3R_{BH} = 1237$$

$$R_{BH} = 412.33 \text{ lb}$$

From equation (1):

$$412.33 + R_{BV} = 566$$

$$R_{BV} = 153.67 \text{ lb}$$

From the FBD of member AB

$$\Sigma F_H = 0$$

$$R_{AH} = R_{BH} = 412.33 \text{ lb}$$

$$\Sigma F_V = 0$$

$$R_{AV} + R_{BV} = W_{AB}$$

$$R_{AV} + 153.67 = 1132$$

$$R_{AV} = 978.33 \text{ lb}$$

$R_A = \sqrt{R_{AH}^2 + R_{AV}^2}$

$= \sqrt{412.33^2 + 978.33^2}$

$= 1061.67$ lb → shear force of pin at A

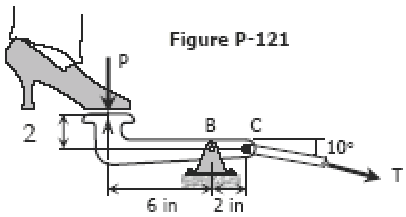
$V = \tau A$

$1061.67 = 5000(\frac{1}{4}\pi d^2)$

$d = \mathbf{0.520\ in}$

Problem 121

Referring to Fig. P-121, compute the maximum force P that can be applied by the machine operator, if the shearing stress in the pin at B and the axial stress in the control rod at C are limited to 4000 psi and 5000 psi, respectively. The diameters are 0.25 inch for the pin, and 0.5 inch for the control rod. Assume single shear for the pin at B.



Solution 121

$[\Sigma M_B = 0]$ $6P = 2T \sin 10°$ → (1)

$[\Sigma F_H = 0]$ $B_H = T \cos 10°$ → from (1), T = 3P/sin 10°

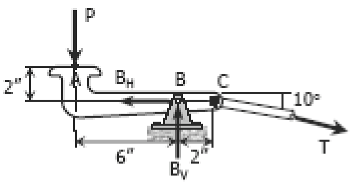
$B_H = (3P/\sin 10°) \cos 10°$

$B_H = 3 \cot 10°\ P$

$[\Sigma F_V = 0]$ $B_V = T \sin 10° + P$ → from (1), T sin 10° = 3P

$B_V = 3P + P$

$B_V = 4P$



$R_B^2 = B_H^2 + B_V^2$

$R_B^2 = (3 \cot 10°\ P)^2 + (4P)^2$

$R_B^2 = 305.47P^2$

$R_B = 17.48P$

$P = R_B/17.48$ → (2)

Based on tension of rod (equation 1):

$P = \frac{1}{3} T \sin 10°$

$P = \frac{1}{3}[5000 \times \frac{1}{4}\pi(0.5)^2] \sin 10°$

$P = 56.83$ lb

Based on shear of rivet (equation 2):

$P = 4000 \times \frac{1}{4}\pi(0.25)^2 / 17.48$

$P = 11.23$ lb

Safe load $P = \mathbf{11.23\ lb}$

Problem 122

Two blocks of wood, width w and thickness t, are glued together along the joint inclined at the angle θ as shown in Fig. P-122. Using the free-body diagram concept in Fig. 1-4a, show that the shearing stress on the glued joint is τ = P sin 2θ/2A, where A is the cross-sectional area.

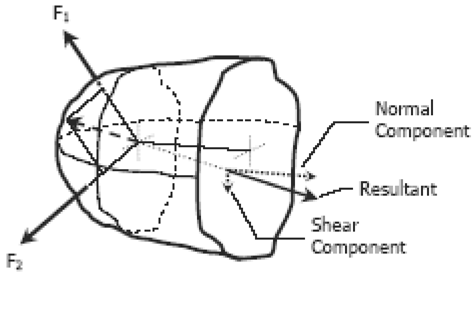


Figure 1-4a Normal and shear components of the resultant on arbitrary section.

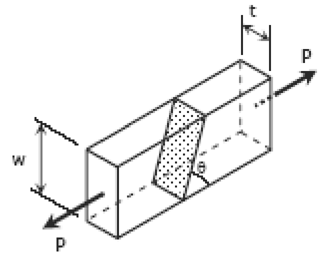
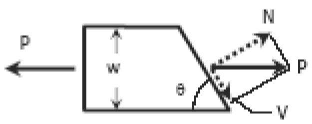


Figure P-122

Solution 122



Shear area, $A_{shear} = t\,(w \csc\theta)$

$= tw \csc\theta$

$= A \csc\theta$

Shear force, $V = P\cos\theta$

$V = \tau A_{shear}$

$P\cos\theta = \tau(A\csc\theta)$

$\tau = P\sin\theta\cos\theta \;/\; A$

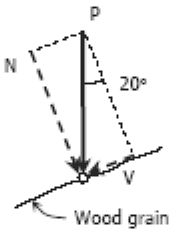
$= P\,(2\sin\theta\cos\theta) \;/\; 2A$

$= P\sin 2\theta \;/\; 2A$ (*ok!*)

Problem 123

A rectangular piece of wood, 50 mm by 100 mm in cross section, is used as a compression block shown in Fig. P-123. Determine the axial force P that can be safely applied to the block if the compressive stress in wood is limited to 20 MN/m^2 and the shearing stress parallel to the grain is limited to 5 MN/m^2. The grain makes an angle of 20° with the horizontal, as shown. (Hint: Use the results in Problem 122.)

Solution 123



Based on maximum compressive stress:

Normal force:

$N = P \cos 20°$

Normal area:

$A_N = 50\ (100 \sec 20°)$

$= 5320.89\ \text{mm}^2$

$N = \sigma A_N$

$P \cos 20° = 20\ (5320.89)$

$P = 113\ 247\ \text{N}$

$= 133.25\ \text{kN}$

Based on maximum shearing stress:

Shear force:

$V = P \sin 20°$

Shear area:

$A_V = A_N$

$= 5320.89\ \text{mm}^2$

$V = \tau A_V$

$P \sin 20° = 5\ (5320.89)$

$P = 77\ 786\ \text{N}$

$= 77.79\ \text{kN}$

For safe compressive force, use P = **77.79 kN**