Statically Indeterminate Members

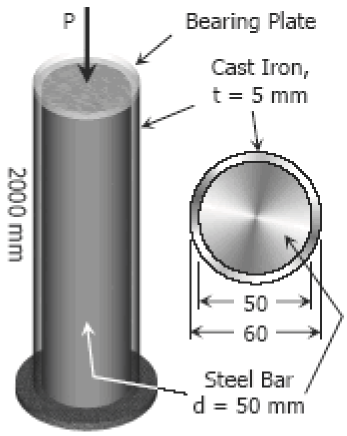
When the reactive forces or the internal resisting forces over a cross section exceed the number of independent equations of equilibrium, the structure is called **statically indeterminate**. These cases require the use of additional relations that depend on the elastic deformations in the members.

Solved Problems in Statically Indeterminate Members

Problem 233

A steel bar 50 mm in diameter and 2 m long is surrounded by a shell of a cast iron 5 mm thick. Compute the load that will compress the combined bar a total of 0.8 mm in the length of 2 m. For steel, E = 200 GPa, and for cast iron, E = 100 GPa.

Solution 233



$$\delta = \frac{PL}{AE}$$

$$\delta = \delta_{cast\ iron} = \delta_{steel} = 0.8\ \text{mm}$$

$$\delta_{cast\ iron} = \frac{P_{cast\ iron}(2000)}{[\frac{1}{4}\pi(60^2 - 50^2)](100\,000)} = 0.8$$

$$P_{cast\ iron} = 11\,000\pi\ \text{N}$$

$$\delta_{steel} = \frac{P_{steel}(2000)}{[\frac{1}{4}\pi(50^2)](200\,000)} = 0.8$$

$$P_{steel} = 50\,000\pi\ \text{N}$$

$$\sum F_V = 0$$

$$P = P_{cast\ iron} + P_{steel}$$

$$P = 11\,000\pi + 50\,000\pi$$

$$P = 61\,000\pi\ \text{N}$$

$$P = \mathbf{191.64\ kN}$$

Problem 234

A reinforced concrete column 200 mm in diameter is designed to carry an axial compressive load of 300 kN. Determine the required area of the reinforcing steel if the allowable stresses are 6 MPa and 120 MPa for the concrete and steel, respectively. Use E_{co} = 14 GPa and E_{st} = 200 GPa.

Solution 234

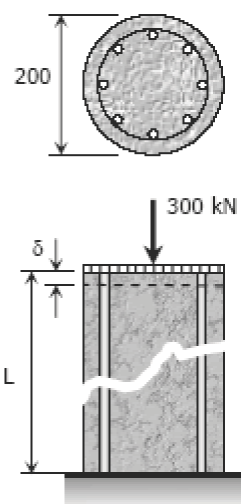
$$\delta_{co} = \delta_{st} = \delta$$

$$\left(\frac{PL}{AE}\right)_{co} = \left(\frac{PL}{AE}\right)_{st}$$

$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{\sigma_{co} L}{14\,000} = \frac{\sigma_{st} L}{200\,000}$$

$$100\sigma_{co} = 7\sigma_s$$



When σ_{st} = 120 MPa

$$100\sigma_{co} = 7(120)$$

$$\sigma_{co} = 8.4 \text{ MPa} > 6 \text{ MPa (not ok!)}$$

When σ_{co} = 6 MPa

$$100(6) = 7\sigma_{st}$$

$$\sigma_{st} = 85.71 \text{ MPa} < 120 \text{ MPa (ok!)}$$

Use σ_{co} = 6 MPa and σ_{st} = 85.71 MPa

$$\sum Fv = 0$$

$$P_{st} + P_{co} = 300$$

$$\sigma_{st} A_{st} + \sigma_{co} A_{co} = 300$$

$$85.71A_{st} + 6[\tfrac{1}{4}\pi(200)^2 - A_{st}] = 300(1000)$$

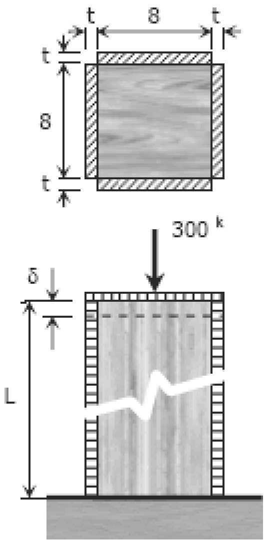
$$79.71A_{st} + 60\,000\pi = 300\,000$$

$$A_{st} = \mathbf{1398.9 \ mm^2}$$

Problem 235

A timber column, 8 in. × 8 in. in cross section, is reinforced on each side by a steel plate 8 in. wide and t in. thick. Determine the thickness t so that the column will support an axial load of 300 kips without exceeding a maximum timber stress of 1200 psi or a maximum steel stress of 20 ksi. The moduli of elasticity are 1.5×10^6 psi for timber, and 29×10^6 psi for steel.

Solution 235



$\delta_{st} = \delta_t$

$$\left(\frac{\sigma L}{E}\right)_{st} = \left(\frac{\sigma L}{E}\right)_{timber}$$

$$\frac{\sigma_{st} L}{29\times 10^6} = \frac{\sigma_{timber} L}{1.5\times 10^6}$$

$1.5\sigma_{st} = 29\sigma_{timber}$

When $\sigma_{timber} = 1200$ psi

$1.5\sigma_{st} = 29(1200)$

$\sigma_{st} = 23\,200$ psi $= 23.2$ ksi > 20 ksi (not ok!)

When $\sigma_{st} = 20$ ksi

$1.5(20 \times 1000) = 29\sigma_{timber}$

$\sigma_{timber} = 1034.48$ psi < 1200 psi (ok!)

Use $\sigma_{st} = 20$ ksi and $\sigma_{timber} = 1.03$ ksi

$\sum F_V = 0$

$F_{steel} + F_{timber} = 300$

$\sigma_{st} A_{st} + \sigma_{timber} A_{timber} = 300$

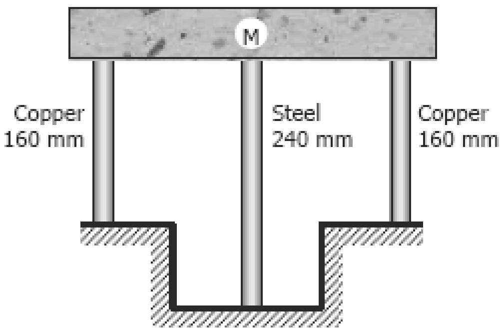
$20[4(8t)] + 1.03(8^2) = 300$

t = **0.365 in**

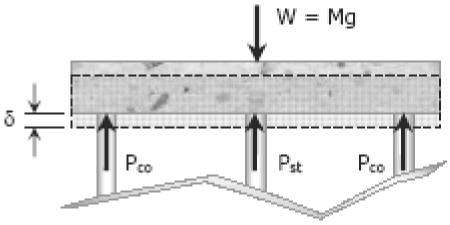
Problem 236

A rigid block of mass M is supported by three symmetrically spaced rods as shown in fig P-236. Each copper rod has an area of 900 mm^2; E = 120 GPa; and the allowable stress is 70 MPa. The steel rod has an area of 1200 mm^2; E = 200 GPa; and the allowable stress is 140 MPa. Determine the largest mass M which can be supported.

Figure P-236 and P-237



Solution 236



$\delta_{co} = \delta_{st}$

$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{\sigma_{co}(160)}{120\,000} = \frac{\sigma_{st}(240)}{200\,000}$$

$10\sigma_{co} = 9\sigma_{st}$

When $\sigma_{st} = 140$ MPa

$\sigma_{co} = \frac{9}{10}(140)$

$\sigma_{co} = 126 \text{ MPa} > 70 \text{ MPa}$ (not ok!)

When $\sigma_{co} = 70$ MPa

$\sigma_{st} = \frac{10}{9}(70)$

$\sigma_{st} = 77.78 \text{ MPa} < 140 \text{ MPa}$ (ok!)

Use $\sigma_{co} = 70$ MPa and $\sigma_{st} = 77.78$ MPa

$\sum F_V = 0$

$2P_{co} + P_{st} = W$

$2(\sigma_{co}A_{co}) + \sigma_{st}A_{st} = Mg$

$2[70(900)] + 77.78(1200) = M(9.81)$

$M = \mathbf{22\,358.4\ kg}$

Problem 237

In Prob. 236, how should the lengths of the two identical copper rods be changed so that each material will be stressed to its allowable limit?

Solution 237

Use $\sigma_{co} = 70$ MPa and $\sigma_{st} = 140$ MPa

$\delta_{co} = \delta_{st}$

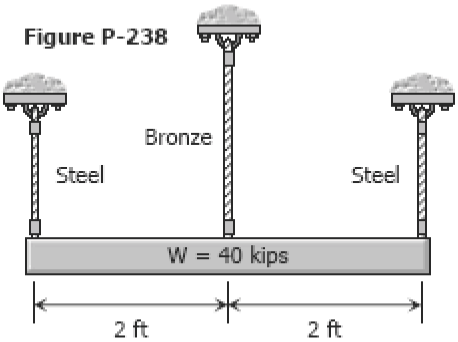
$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{70L_{co}}{120\,000} = \frac{140(240)}{200\,000}$$

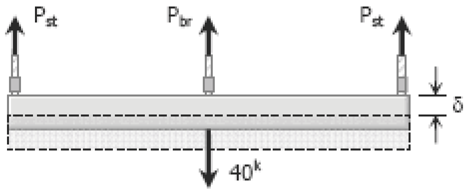
$L_{co} = \mathbf{288\ mm}$

Problem 238

The lower ends of the three bars in Fig. P-238 are at the same level before the uniform rigid block weighing 40 kips is attached. Each steel bar has a length of 3 ft, and area of 1.0 in.^{2}, and E = 29 × 10^6 psi. For the bronze bar, the area is 1.5 in.^{2} and E = 12 × 10^6 psi. Determine (a) the length of the bronze bar so that the load on each steel bar is twice the load on the bronze bar, and (b) the length of the bronze that will make the steel stress twice the bronze stress.



Solution 238



(a) Condition: $P_{st} = 2P_{br}$

$\sum F_V = 0$

$2P_{st} + P_{br} = 40$

$2(2P_{br}) + P_{br} = 40$

$P_{br} = 8$ kips

$P_{st} = 2(8) = 16$ kips

$\delta_{br} = \delta_{st}$

$$\left(\frac{PL}{AE}\right)_{br} = \left(\frac{PL}{AE}\right)_{st}$$

$$\frac{8000L_{br}}{1.5(12\times10^6)} = \frac{16\,000(3\times12)}{1.0(29\times10^6)}$$

$L_{br} = 44.69$ in

$L_{br} = \mathbf{3.72\ ft}$

(b) Condition: $\sigma_{st} = 2\sigma_{br}$

$\sum F_V = 0$

$2P_{st} + P_{br} = 40$

$2(\sigma_{st}A_{st}) + \sigma_{br}A_{br} = 40$

$2[(2\sigma_{br})A_{st}] + \sigma_{br}A_{br} = 40$

$4\sigma_{br}(1.0) + \sigma_{br}(1.5) = 40$

$\sigma_{br} = 7.27$ ksi

$\sigma_{st} = 2(7.27) = 14.54$ ksi

$\delta_{br} = \delta_{st}$

$$\left(\frac{\sigma L}{E}\right)_{br} = \left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{7.27(1000)L_{br}}{12\times10^6} = \frac{14.54(1000)(3\times12)}{29\times10^6}$$

$L_{br} = 29.79$ in

$L_{br} = \mathbf{2.48\ ft}$

Problem 239

The rigid platform in Fig. P-239 has negligible mass and rests on two steel bars, each 250.00 mm long. The center bar is aluminum and 249.90 mm long. Compute the stress in the aluminum bar after the center load P = 400 kN has been applied. For each steel bar, the area is 1200 mm^2 and E = 200 GPa. For the aluminum bar, the area is 2400 mm^2 and E = 70 GPa.

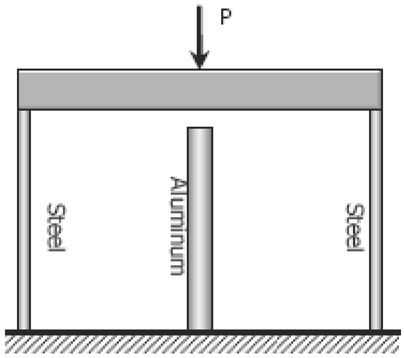
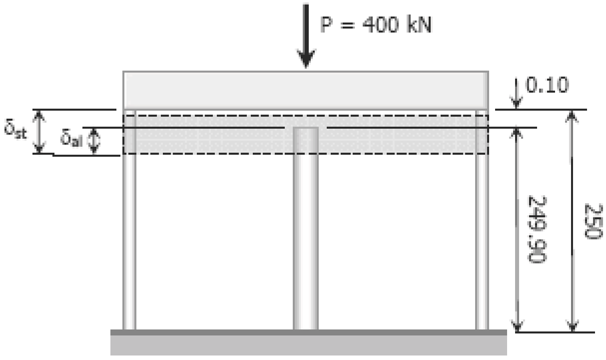


Figure P-239

Solution 239



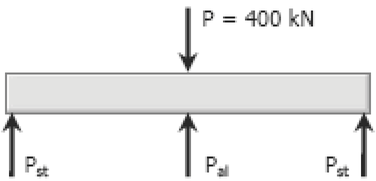
$$\delta_{st} = \delta_{al} + 0.10$$

$$\left(\frac{\sigma L}{E}\right)_{st} = \left(\frac{\sigma L}{E}\right)_{al} + 0.10$$

$$\frac{\sigma_{st}(250)}{200\,000} = \frac{\sigma_{al}(249.90)}{70\,000} + 0.10$$

$$0.00125\sigma_{st} = 0.00357\sigma_{al} + 0.10$$

$$\sigma_{st} = 2.856\sigma_{al} + 80$$



$$\sum F_V = 0$$

$$2P_{st} + P_{al} = 400\,000$$

$$2\sigma_{st}\,A_{st} + \sigma_{al}\,A_{al} = 400\,000$$

$$2(2.856\sigma_{al} + 80)1200 + \sigma_{al}\,(2400) = 400\,000$$

$$9254.4\sigma_{al} + 192\,000 = 400\,000$$

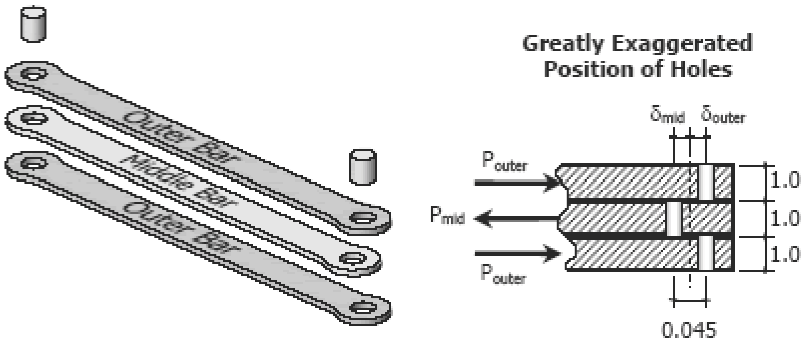
$$\sigma_{al} = \mathbf{22.48\ MPa}$$

Problem 240

Three steel eye-bars, each 4 in. by 1 in. in section, are to be assembled by driving rigid 7/8-in.-diameter drift pins through holes drilled in the ends of the bars. The center-line spacing between the holes is 30 ft in the two outer bars, but 0.045 in. shorter in the middle bar. Find the shearing stress developed in the drip pins. Neglect local deformation at the holes.

Solution 240

Middle bar is 0.045 inch shorter between holes than outer bars.



$\sum F_H = 0$

$P_{mid} = 2P_{outer}$

$\delta_{outer} + \delta_{mid} = 0.045$

$$\left(\frac{PL}{AE}\right)_{outer} + \left(\frac{PL}{AE}\right)_{mid} = 0.045$$

$$\frac{P_{outer}(30\times 12)}{[1.0(4.0)]E} + \frac{P_{mid}(30\times 12-0.045)}{[1.0(4.0)]E} = 0.045$$

$360P_{outer} + 359.955P_{mid} = 0.18E$

$360P_{outer} + 359.955(2P_{outer}) = 0.18E$

(For steel: E = 29 × 106 psi)

$1079.91P_{outer} = 0.18(29 \times 10^6)$

$P_{outer} = 4833.74$ lb

$P_{mid} = 2(4833.74)$

$P_{mid} = 9667.48$ lb

Use shear force $V = P_{mid}$

Shearing stress of drip pins (double shear):

$$\tau = \frac{V}{A} = \frac{9667.48}{2[\frac{1}{4}\pi(\frac{7}{8})^2]}$$

τ = **8038.54 psi**

Problem 241

As shown in Fig. P-241, three steel wires, each 0.05 in.^{2} in area, are used to lift a load W = 1500 lb. Their unstressed lengths are 74.98 ft, 74.99 ft, and 75.00 ft. (a) What stress exists in the longest wire? (b) Determine the stress in the shortest wire if W = 500 lb.

Solution 241

Let L_1 = 74.98 ft; L_2 = 74.99 ft; and L_3 = 75.00 ft

(a) Bring L_1 and L_2 into L_3 = 75 ft length:
(For steel: E = 29 × 106 psi)

$$\delta = \frac{PL}{AE}$$

For L_1:

$$(75 - 74.98)(12) = \frac{P_1(74.98 \times 12)}{0.05(29 \times 10^6)}$$

P_1 = 386.77 lb

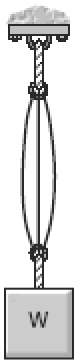


Figure P-241

For L_2

$$(75 - 74.99)(12) = \frac{P_2(74.99 \times 12)}{0.05(29 \times 10^6)}$$

P_2 = 193.36 lb

Let $P = P_3$ (Load carried by L_3)
$P + P_2$ (Total load carried by L_2)
$P + P_1$ (Total load carried by L_1)

$\sum F_V = 0$
$(P + P_1) + (P + P_2) + P = W$
$3P + 386.77 + 193.36 = 1500$
$P = 306.62$ lb $= P_3$

$$\sigma_3 = \frac{P_3}{A} = \frac{306.62}{0.05}$$

σ_3 = **6132.47 psi**

(b) From the above solution:
$P_1 + P_2$ = 580.13 lb > 500 lb (L_3 carries no load)

Bring L_1 into L_2 = 74.99 ft

$$\left[\delta = \frac{PL}{AE}\right] \quad (74.99 - 74.98)(12) = \frac{P_1(74.98 \times 12)}{0.05(29 \times 10^6)}$$

P_1 = 193.38 lb

Let $P = P_2$ (Load carried by L_2)
$P + P_1$ (Total load carried by L_1)

$\sum F_V = 0$
$(P + P_1) + P = 500$
$2P + 193.38 = 500$
$P = 153.31$ lb

$P + P_1 = 153.31 + 193.38$
$P + P_1 = 346.69$ lb

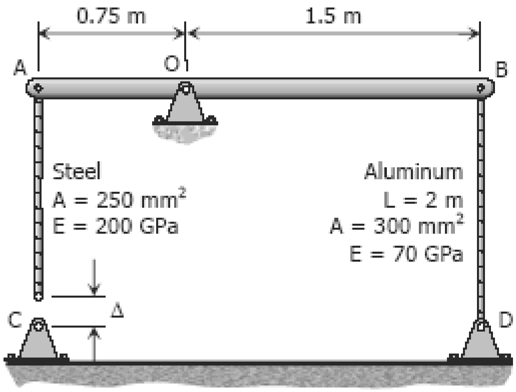
$$\sigma = \frac{P + P_1}{A} = \frac{346.69}{0.05}$$

σ = **6933.8 psi**

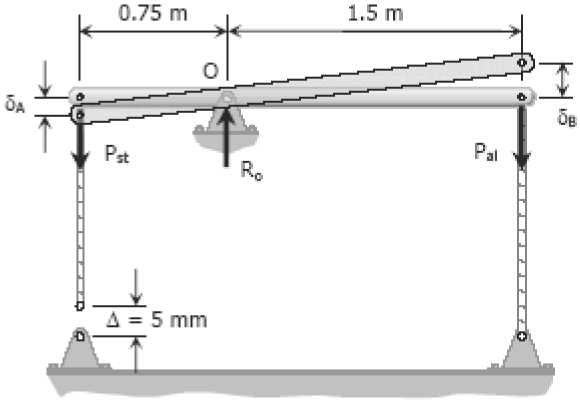
Problem 242

The assembly in Fig. P-242 consists of a light rigid bar AB, pinned at O, that is attached to the steel and aluminum rods. In the position shown, bar AB is horizontal and there is a gap, Δ = 5 mm, between the lower end of the steel rod and its pin support at C. Compute the stress in the aluminum rod when the lower end of the steel rod is attached to its support.

Figure P-242



Solution 242



$\Sigma M_O = 0$

$0.75P_{st} = 1.5P_{al}$

$P_{st} = 2P_{al}$

$\sigma_{st}\,A_{st} = 2(\sigma_{al}\,A_{al})$

$$\sigma_{st} = \frac{2(\sigma_{al}A_{al})}{A_{st}}$$

$$\sigma_{st} = \frac{2[\sigma_{al}(300)]}{250}$$

$\sigma_{st} = 2.4\sigma_{al}$

$\delta_{al} = \delta_B$

By ratio and proportion:

$$\frac{\delta_A}{0.75} = \frac{\delta_B}{1.5}$$

$\delta_A = 0.5\delta_B$

$\delta_A = 0.5\delta_{al}$

$\Delta = \delta_{st} + \delta_A$

$5 = \delta_{st} + 0.5\delta_{al}$

$$5 = \frac{\sigma_{st}(2000-5)}{250(200\,000)} + 0.5\left[\frac{\sigma_{al}(2000)}{300(70\,000)}\right]$$

$5 = (3.99 \times 10^{-5})\,\sigma_{st} + (4.76 \times 10^{-5})\,\sigma_{al}$

$\sigma_{al} = 105\,000 - 0.8379\sigma_{st}$

$\sigma_{al} = 105\,000 - 0.8379(2.4\sigma_{al})$

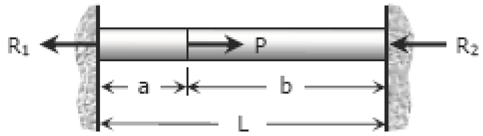
$3.01096\sigma_{al} = 105\,000$

$\sigma_{al} = \mathbf{34\,872.6\ MPa}$

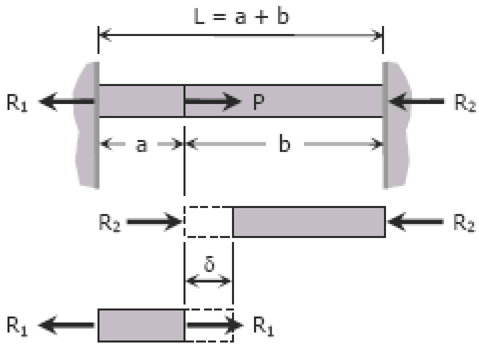
Problem 243

A homogeneous rod of constant cross section is attached to unyielding supports. It carries an axial load P applied as shown in Fig. P-243. Prove that the reactions are given by $R_1 = Pb/L$ and $R_2 = Pa/L$.

Figure P-243



Solution 243



$$\sum F_H = 0$$
$$R_1 + R_2 = P$$
$$R_2 = P - R_1$$

$$\delta_1 = \delta_2 = \delta$$
$$\left(\frac{PL}{AE}\right)_1 = \left(\frac{PL}{AE}\right)_2$$
$$\frac{R_1 a}{AE} = \frac{R_2 b}{AE}$$
$$R_1 a = R_2 b$$

$$R_1 a = (P - R_1) b$$
$$R_1 a = Pb - R_1 b$$
$$R_1 (a + b) = Pb$$
$$R_1 L = Pb$$
$$R_1 = Pb / L \qquad \text{ok!}$$

$$R_2 = P - Pb/L$$
$$R_2 = \frac{P(L-b)}{L}$$
$$R_2 = Pa / L \qquad \textit{ok!}$$

Problem 244

A homogeneous bar with a cross sectional area of 500 mm^2 is attached to rigid supports. It carries the axial loads P1 = 25 kN and P2 = 50 kN, applied as shown in Fig. P-244. Determine the stress in segment BC. (Hint: Use the results of Prob. 243, and compute the reactions caused by P_1 and P_2 acting separately. Then use the principle of superposition to compute the reactions when both loads are applied.)

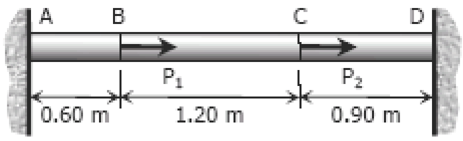
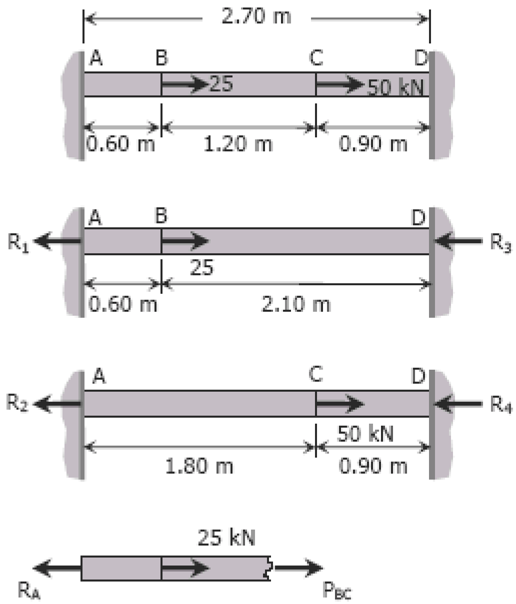


Figure P-244

Solution 244



From the result of Prob. 243:

$R_1 = 25(2.10)/2.70$

$R_1 = 19.44$ kN

$R_2 = 50(0.90)/2.70$

$R_2 = 16.67$ kN

$R_A = R_1 + R_2$

$R_A = 19.44 + 16.67$

$R_A = 36.11$ kN

For segment BC

$P_{BC} + 25 = R_A$

$P_{BC} + 25 = 36.11$

$P_{BC} = 11.11$ kN

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{11.11(1000)}{500}$$

$\sigma_{BC} = 22.22$ MPa

Problem 245

The composite bar in Fig. P-245 is firmly attached to unyielding supports. Compute the stress in each material caused by the application of the axial load P = 50 kips.

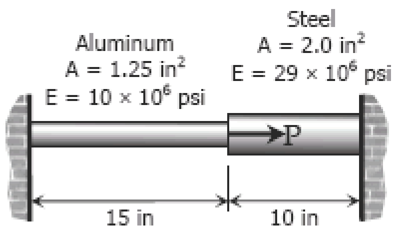
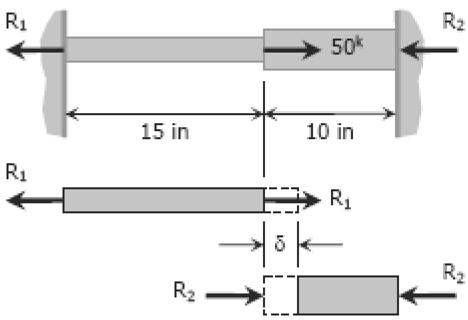


Figure P-245 and P-246

Solution 245

$$\sum F_H = 0$$
$$R_1 + R_2 = 50\,000$$
$$R_1 = 50\,000 - R_2$$



$$\delta_{al} = \delta_{st} = \delta$$
$$\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st}$$
$$\frac{R_1(15)}{1.25(10\times10^6)} = \frac{R_2(10)}{2.0(29\times10^6)}$$
$$R_2 = 6.96R_1$$
$$R_2 = 6.96(50\,000 - R_2)$$
$$7.96R_2 = 348\,000$$
$$R_2 = 43\,718.59 \text{ lb}$$

$$\sigma_{st} = \frac{R_2}{A_{st}} = \frac{43718.59}{2.0}$$
$$\sigma_{st} = \mathbf{21\,859.30 \text{ psi}}$$

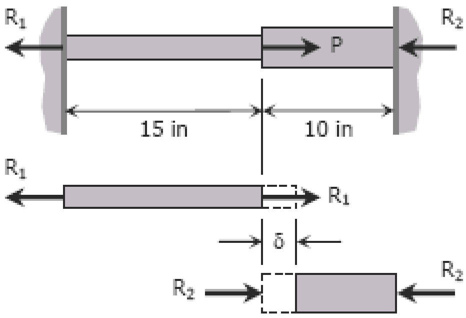
$$R_1 = 50\,000 - 43\,718.59$$
$$R_1 = 6281.41 \text{ lb}$$

$$\sigma_{al} = \frac{R_1}{A_{al}} = \frac{6281.41}{1.25}$$
$$\sigma_{al} = \mathbf{5025.12 \text{ psi}}$$

Problem 246

Referring to the composite bar in Prob. 245, what maximum axial load P can be applied if the allowable stresses are 10 ksi for aluminum and 18 ksi for steel.

Solution 246



$$\delta_{st} = \delta_{al} = \delta$$

$$\left(\frac{\sigma L}{E}\right)_{st} = \left(\frac{\sigma L}{E}\right)_{al}$$

$$\frac{\sigma_{st}(10)}{29\times 10^6} = \frac{\sigma_{al}(15)}{10\times 10^6}$$

$$\sigma_{st} = 4.35\sigma_{al}$$

When $\sigma_{al} = 10$ ksi

$$\sigma_{st} = 4.35(10)$$

$\sigma_{st} = 43.5$ ksi > 18 ksi (not ok!)

When $\sigma_{st} = 18$ ksi

$$18 = 4.35\sigma_{al}$$

$\sigma_{al} = 4.14$ ksi < 10 ksi (ok!)

Use $\sigma_{al} = 4.14$ ksi and $\sigma_{st} = 18$ ksi

$$\Sigma F_H = 0$$

$$P = R_1 + R_2$$

$$P = \sigma_{al} A_{al} + \sigma_{st} A_{st}$$

$$P = 4.14(1.25) + 18(2.0)$$

$$\mathbf{P = 41.17\ kips}$$

Problem 247

The composite bar in Fig. P-247 is stress-free before the axial loads P1 and P2 are applied. Assuming that the walls are rigid, calculate the stress in each material if P_1 = 150 kN and P_2 = 90 kN.

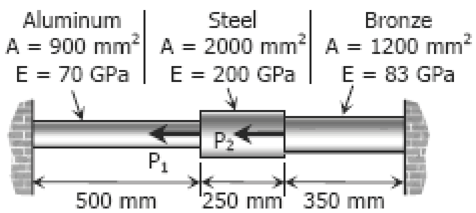
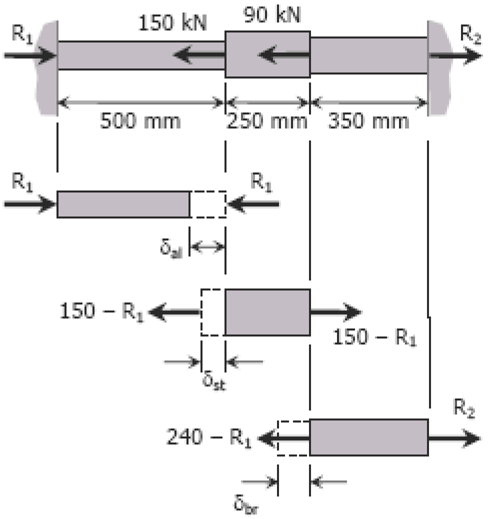


Figure P-247 and P-248

Solution 247



From the *FBD* of each material shown:

δ_{al} is shortening

δ_{st} and δ_{br} are lengthening

$R_2 = 240 - R_1$

$P_{al} = R_1$

$P_{st} = 150 - R_1$

$P_{br} = R_2 = 240 - R_1$

$$\delta_{al} = \delta_{st} + \delta_{br}$$

$$\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{br}$$

$$\frac{R_1(500)}{900(70000)} = \frac{(150 - R_1)(250)}{2000(200000)} + \frac{(240 - R_1)(350)}{1200(83000)}$$

$$\frac{R_1}{126000} = \frac{150 - R_1}{1600000} + \frac{(240 - R_1)7}{1992000}$$

$$\tfrac{1}{63} R_1 = \tfrac{1}{800}(150 - R_1) + \tfrac{7}{996}(240 - R_1)$$

$$\left(\tfrac{1}{63} + \tfrac{1}{800} + \tfrac{7}{996}\right)R_1 = \tfrac{1}{800}(150) + \tfrac{7}{996}(240)$$

$R_1 = 77.60$ kN

$P_{al} = R_1 = 77.60$ kN

$P_{st} = 150 - 77.60 = 72.40$ kN

$P_{br} = 240 - 77.60 = 162.40$ kN

$\sigma = P/A$

$\sigma_{al} = 77.60(1000)/900$

$= \mathbf{86.22\ MPa}$

$\sigma_{st} = 72.40(1000)/2000$

$= \mathbf{36.20\ MPa}$

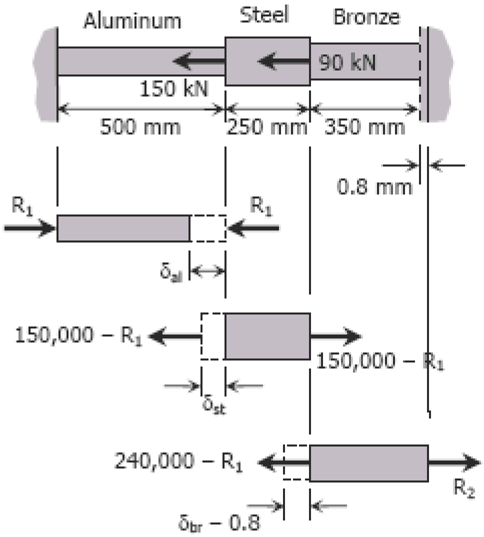
$\sigma_{br} = 162.40(1000)/1200$

$= \mathbf{135.33\ MPa}$

Problem 248

Solve Prob. 247 if the right wall yields 0.80 mm.

Solution 248



$\delta_{al} = \delta_{st} + (\delta_{br} + 0.8)$

$\left(\dfrac{PL}{AE}\right)_{al} = \left(\dfrac{PL}{AE}\right)_{st} + \left(\dfrac{PL}{AE}\right)_{br} + 0.8$

$\dfrac{R_1(500)}{900(70000)} = \dfrac{(150000 - R_1)(250)}{2000(200000)}$

$\qquad + \dfrac{(240000 - R_1)(350)}{1200(83000)} + 0.8$

$\dfrac{R_1}{126000} = \dfrac{150000 - R_1}{1600000}$

$\qquad + \dfrac{7(240000 - R_1)}{1992000} + 0.8$

$\frac{1}{63} R_1 = \frac{1}{800}(150000 - R_1)$

$\qquad + \frac{7}{996}(240000 - R_1) + 1600$

$(\frac{1}{63} + \frac{1}{800} + \frac{7}{996})R_1 = \frac{1}{800}(150000)$

$\qquad + \frac{7}{996}(240000) + 1600$

$R_1 = 143\ 854 \text{ N} = 143.854 \text{ kN}$

$P_{al} = R_1 = 143.854 \text{ kN}$

$P_{st} = 150 - R_1 = 150 - 143.854 = 6.146 \text{ kN}$

$P_{br} = R_2 = 240 - R_1 = 240 - 143.854 = 96.146 \text{ kN}$

$\sigma = P/A$

$\sigma_{al} = 143.854(1000)/900$

$\quad = \mathbf{159.84 \text{ MPa}}$

$\sigma_{st} = 6.146(1000)/2000$

$\quad = \mathbf{3.073 \text{ MPa}}$

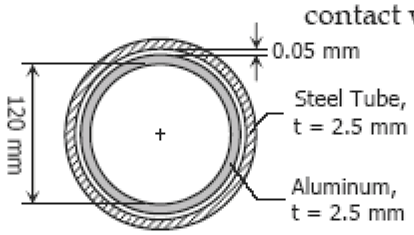
$\sigma_{br} = 96.146(1000)/1200$

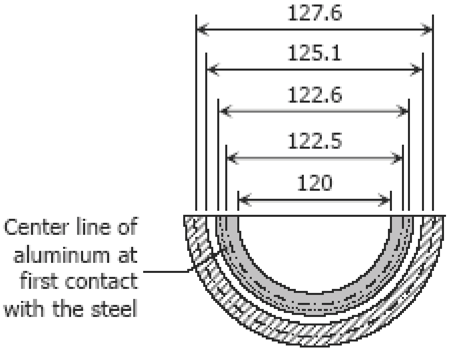
$\quad = \mathbf{80.122 \text{ MPa}}$

Problem 249

There is a radial clearance of 0.05 mm when a steel tube is placed over an aluminum tube. The inside diameter of the aluminum tube is 120 mm, and the wall thickness of each tube is 2.5 mm. Compute the contact pressure and tangential stress in each tube when the aluminum tube is subjected to an internal pressure of 5.0 MPa.

Solution 249





Internal pressure of aluminum tube to cause contact with the steel:

$$\delta_{al} = \left(\frac{\sigma L}{E}\right)_{al}$$

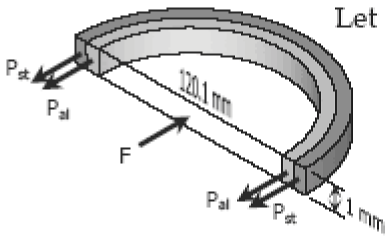
$$\pi(122.6 - 122.5) = \frac{\sigma_1(122.5\pi)}{70\,000}$$

$\sigma_1 = 57.143$ MPa

$$\frac{p_1 D}{2t} = 57.143$$

$$\frac{p_1(120)}{2(2.5)} = 57.143$$

$p_1 = 2.381$ MPa → pressure that causes aluminum to contact with the steel, further increase of pressure will expand both aluminum and steel tubes.



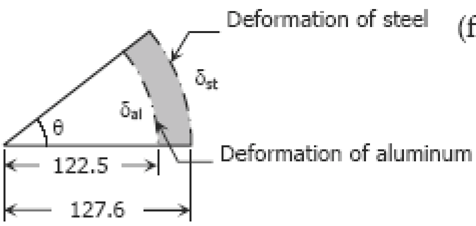
FBD for p ≥ 2.381 MPa

Let p_c = contact pressure between steel and aluminum tubes

$2P_{st} + 2P_{al} = F$

$2P_{st} + 2P_{al} = 5.0(120.1)(1)$

$P_{st} + P_{al} = 300.25$ → Equation (1)



Geometric relation of deformations

The relationship of deformations is (from the figure):

$\delta_{st} = 127.6\theta$

$\theta = \delta_{st} / 127.6$

$\delta_{al} = 122.5\theta$

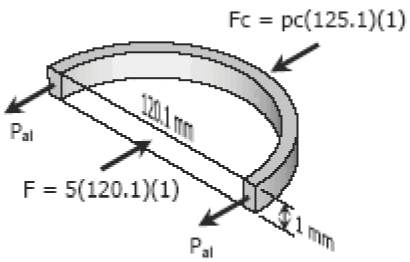
$\delta_{al} = 122.5(\delta_{st} / 127.6)$

$\delta_{al} = 0.96\delta_{st}$

$$\left(\frac{PL}{AE}\right)_{al} = 0.96\left(\frac{PL}{AE}\right)_{st}$$

$$\frac{P_{al}(122.5\pi)}{2.5(70\,000)} = 0.96\left[\frac{P_{st}(127.6\pi)}{2.5(200\,000)}\right]$$

$P_{al} = 0.35P_{st}$ → Equation (2)



From Equation (1)

$P_{st} + 0.35P_{st} = 300.25$

$P_{st} = 222.41$ N

$P_{al} = 0.35(222.41)$

$P_{al} = 77.84$ N

Contact Force

$F_c + 2P_{st} = F$

$p_c\,(125.1)(1) + 2(77.84) = 5(120.1)(1)$

p_c = **3.56 MPa**

Problem 250

In the assembly of the bronze tube and steel bolt shown in Fig. P-250, the pitch of the bolt thread is p = 1/32 in.; the cross-sectional area of the bronze tube is 1.5 in.^{2} and of steel bolt is ¾ in.^{2} The nut is turned until there is a compressive stress of 4000 psi in the bronze tube. Find the stresses if the nut is given one additional turn. How many turns of the nut will reduce these stresses to zero? Use Ebr = 12 × 10^6 psi and Est = 29 × 10^6 psi.

Figure P-250

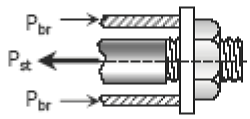
Solution 250

$P_{st} = P_{br}$

$A_{st}\,\sigma_{st} = P_{br}\,\sigma_{br}$

$\frac{3}{4}\,\sigma_{st} = 1.5\,\sigma_{br}$

$\sigma_{st} = 2\,\sigma_{br}$



For one turn of the nut:

$$\delta_{st} + \delta_{br} = \frac{1}{32}$$

$$\left(\frac{\sigma L}{E}\right)_{st} + \left(\frac{\sigma L}{E}\right)_{br} = \frac{1}{32}$$

$$\frac{\sigma_{st}(40)}{29\times10^6} + \frac{\sigma_{br}(40)}{12\times10^6} = \frac{1}{32}$$

$$\sigma_{st} + \frac{29}{12}\sigma_{br} = 22\,656.25$$

$$2\sigma_{br} + \frac{29}{12}\sigma_{br} = 22\,656.25$$

$$\sigma_{br} = 5129.72 \text{ psi}$$

$$\sigma_{st} = 2(5129.72) = 10\,259.43 \text{ psi}$$

Initial stresses:

$$\sigma_{br} = 4000 \text{ psi}$$

$$\sigma_{st} = 2(4000) = 8000 \text{ psi}$$

Final stresses:

$$\sigma_{br} = 4000 + 5129.72 = \mathbf{9129.72\ psi}$$

$$\sigma_{st} = 2(9129.72) = \mathbf{18\,259.4\ psi}$$

Required number of turns to reduce σ_{br} to zero:

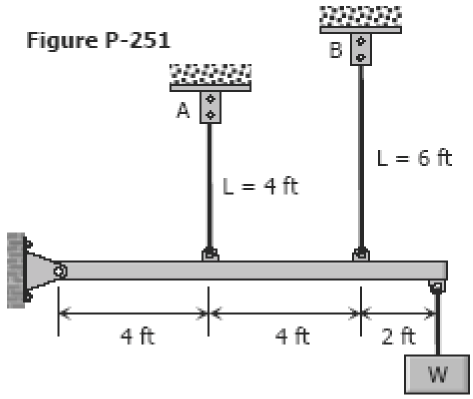
$$n = \frac{9129.72}{5129.72} = \mathbf{1.78\ turns}$$

The nut must be turned back by 1.78 turns

Problem 251

The two vertical rods attached to the light rigid bar in Fig. P-251 are identical except for length. Before the load W was attached, the bar was horizontal and the rods were stress-free. Determine the load in each rod if W = 6600 lb.

Figure P-251

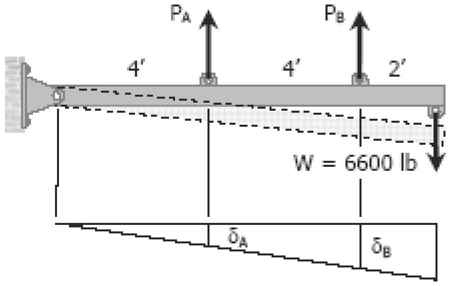


Solution 251

$$\sum M_{pin\ support} = 0$$

$$4P_A + 8P_B = 10(6600)$$

$$P_A + 2P_B = 16500 \quad \rightarrow (1)$$



$$\frac{\delta_A}{4} = \frac{\delta_B}{8}$$

$$\delta_A = 0.5\delta_B$$

$$\left(\frac{PL}{AE}\right)_A = 0.5\left(\frac{PL}{AE}\right)_B$$

$$\frac{P_A(4)}{AE} = \frac{0.5P_B(6)}{AE}$$

$$P_A = 0.75P_B$$

From equation (1)

$$0.75P_B + 2P_B = 16500$$

$$P_B = \mathbf{6000\ lb}$$

$$P_A = 0.75(6000)$$

$$P_A = \mathbf{4500\ lb}$$

Problem 252

The light rigid bar ABCD shown in Fig. P-252 is pinned at B and connected to two vertical rods. Assuming that the bar was initially horizontal and the rods stress-free, determine the stress in each rod after the load after the load P = 20 kips is applied.

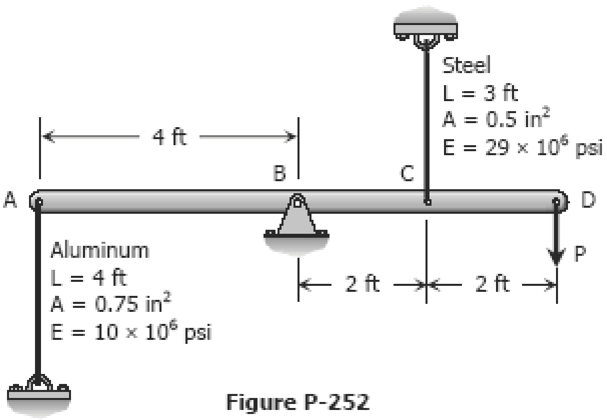
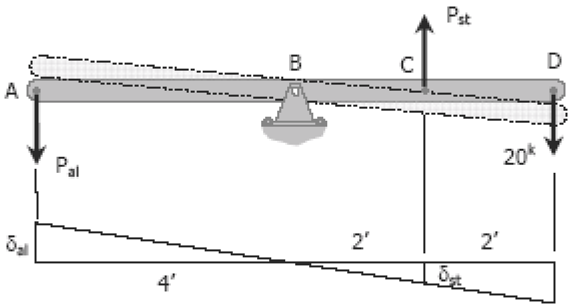


Figure P-252

Solution 252

$$\sum M_B = 0$$
$$4P_{al} + 2P_{st} = 4(20\ 000)$$
$$4(\sigma_{al}\ A_{al}) + 2\sigma_{st}\ A_{st} = 80\ 000$$
$$4[\sigma_{al}\ (0.75)] + 2[\sigma_{st}\ (0.5)] = 80\ 000$$
$$3\sigma_{al} + \sigma_{st} = 80\ 000 \quad \rightarrow (1)$$
$$\frac{\delta_{st}}{2} = \frac{\delta_{al}}{4}$$



$$\delta_{st} = 0.5\delta_{al}$$
$$\left(\frac{\sigma L}{E}\right)_{st} = 0.5\left(\frac{\sigma L}{E}\right)_{al}$$
$$\frac{\sigma_{st}(3)}{29\times 10^6} = 0.5\left[\frac{P_{al}(4)}{10\times 10^6}\right]$$
$$\sigma_{st} = \tfrac{29}{15}\ \sigma_{al}$$

From equation (1)
$$3\sigma_{al} + \tfrac{29}{15}\ \sigma_{al} = 80\ 000$$
$$\sigma_{al} = 16\ 216.22\ \text{psi}$$
$$\sigma_{al} = \mathbf{16.22\ ksi}$$

$$\sigma_{st} = \tfrac{29}{15}\ (16.22)$$
$$\sigma_{st} = \mathbf{31.35\ ksi}$$

Problem 253

As shown in Fig. P-253, a rigid beam with negligible weight is pinned at one end and attached to two vertical rods. The beam was initially horizontal before the load W = 50 kips was applied. Find the vertical movement of W.

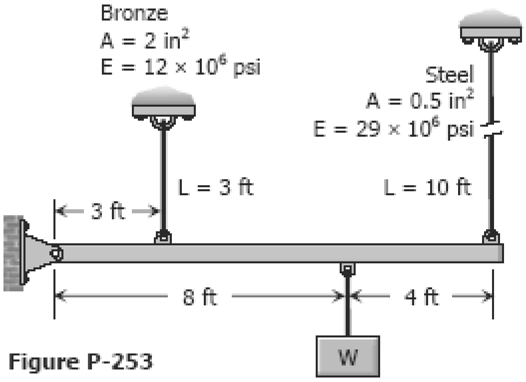
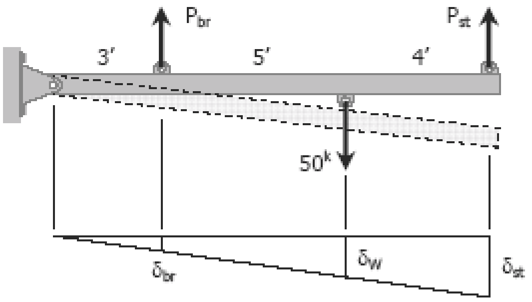


Figure P-253



Solution 253

$\sum M_{pin\ support} = 0$

$3P_{br} + 12P_{st} = 8(50\ 000)$

$3P_{br} + 12P_{st} = 400\ 000 \quad \rightarrow (1)$

$$\frac{\delta_{st}}{12} = \frac{\delta_{br}}{3};\ \delta_{st} = 4\delta_{br}$$

$$\left(\frac{PL}{AE}\right)_{st} = 4\left(\frac{PL}{AE}\right)_{br}$$

$$\frac{P_{st}(10)}{0.5(29\times10^6)} = 4\left[\frac{P_{br}(3)}{2(12\times10^6)}\right]$$

$P_{st} = 0.725P_{br}$

From equation (1)

$3P_{br} + 12(0.725P_{br}) = 400\ 000$

$P_{br} = 34\ 188.03$ lb

$$\delta_{br} = \left(\frac{PL}{AE}\right)_{br} = \frac{34188.03(3\times12)}{2(12\times10^6)}$$

$\delta_{br} = 0.0513$ in

$$\frac{\delta_W}{8} = \frac{\delta_{br}}{3}$$

$$\delta_W = \tfrac{8}{3}\,\delta_{br}$$

$$\delta_W = \tfrac{8}{3}(0.0513)$$

$$\delta_W = \mathbf{0.1368\ in}$$

Check by δ_{st}:

$$P_{st} = 0.725P_{br} = 0.725(34\,188.03)$$

$$P_{st} = 24\,786.32\ \text{lb}$$

$$\delta_{st} = \left(\frac{PL}{AE}\right)_{st}$$

$$\delta_{st} = \frac{24\,786.32(10\times12)}{0.5(29\times10^6)}$$

$$\delta_{st} = 0.2051\ \text{in}$$

$$\frac{\delta_W}{8} = \frac{\delta_{st}}{12}$$

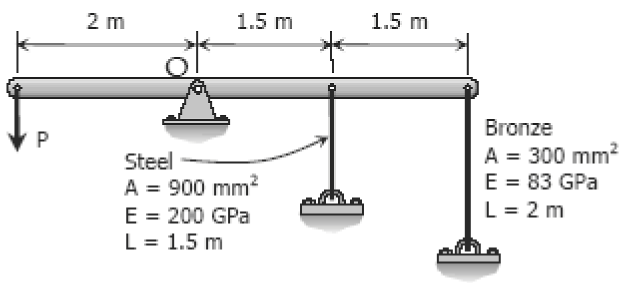
$$\delta_W = \tfrac{2}{3}\,\delta_{st}$$

$$\delta_W = \tfrac{2}{3}(0.2051) = \mathbf{0.1368\ in} \qquad \textit{ok!}$$

Problem 254

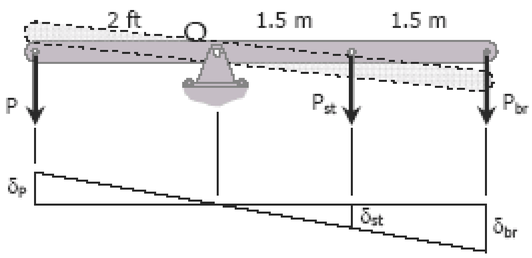
As shown in Fig. P-254, a rigid bar with negligible mass is pinned at O and attached to two vertical rods. Assuming that the rods were initially tress-free, what maximum load P can be applied without exceeding stresses of 150 MPa in the steel rod and 70 MPa in the bronze rod.

Figure P-254



Solution 254

$\sum M_O = 0$
$2P = 1.5P_{st} + 3P_{br}$
$2P = 1.5(\sigma_{st} A_{st}) + 3(\sigma_{br} A_{br})$
$2P = 1.5[\sigma_{st}(900)] + 3[\sigma_{br}(300)]$
$2P = 1350\sigma_{st} + 900\sigma_{br}$
$P = 675\sigma_{st} + 450\sigma_{br}$



$\dfrac{\delta_{br}}{3} = \dfrac{\delta_{st}}{1.5}$
$\delta_{br} = 2\delta_{st}$
$\left(\dfrac{\sigma L}{E}\right)_{br} = 2\left(\dfrac{\sigma L}{E}\right)_{st}$
$\dfrac{\sigma_{br}(2)}{83} = 2\left[\dfrac{\sigma_{st}(1.5)}{200}\right]$
$\sigma_{br} = 0.6225\sigma_{st}$

When $\sigma_{st} = 150$ MPa
$\sigma_{br} = 0.6225(150)$
$\sigma_{br} = 93.375$ MPa > 70 MPa (*not ok!*)

When $\sigma_{br} = 70$ MPa
$70 = 0.6225\sigma_{st}$
$\sigma_{st} = 112.45$ MPa < 150 MPa (*ok!*)

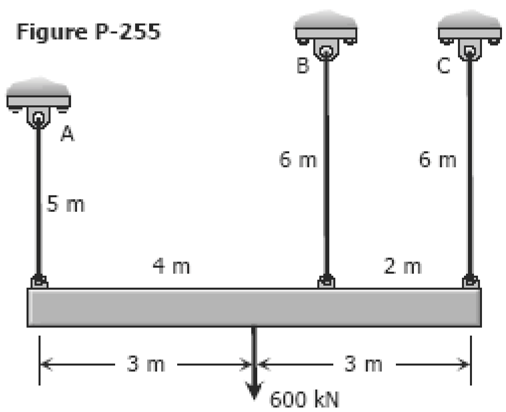
Use $\sigma_{st} = 112.45$ MPa and $\sigma_{br} = 70$ MPa

$P = 675\sigma_{st} + 450\sigma_{br}$
$P = 675(112.45) + 450(70)$
$P = 107\,403.75$ N
$P =$ **107.4 kN**

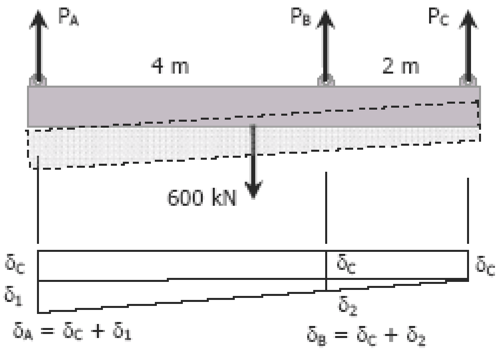
Problem 255

Shown in Fig. P-255 is a section through a balcony. The total uniform load of 600 kN is supported by three rods of the same area and material. Compute the load in each rod. Assume the floor to be rigid, but note that it does not necessarily remain horizontal.

Figure P-255



Solution 255



$\delta_B = \delta_C + \delta_2$

$\delta_2 = \delta_B - \delta_C$

$\dfrac{\delta_1}{6} = \dfrac{\delta_2}{2}; \;\; \delta_1 = 3\delta_2$

$\delta_A = \delta_C + \delta_1 = \delta_C + 3\delta_2$

$\delta_A = \delta_C + 3(\delta_B - \delta_C)$

$\delta_A = 3\delta_B - 2\delta_C$

$$\left(\frac{PL}{AE}\right)_A = 3\left(\frac{PL}{AE}\right)_B - 2\left(\frac{PL}{AE}\right)_C$$

$$\frac{P_A(5)}{AE} = \frac{3P_B(6)}{AE} - \frac{2P_C(6)}{AE}$$

$P_A = 3.6P_B - 2.4P_C$ → (1)

$[\sum F_V = 0]$ $\quad P_A + P_B + P_C = 600$

$(3.6P_B - 2.4P_C) + P_B + P_C = 600$

$4.6P_B - 1.4P_C = 600$ → (2)

$[\sum M_A = 0]$ $\quad 4P_B + 6P_C = 3(600)$

$P_B = 450 - 1.5P_C$ → (3)

Substitute $P_B = 450 - 1.5P_C$ to (2)

$4.6(450 - 1.5P_C) - 1.4P_C = 600$

$8.3P_C = 1470$

$P_C = \mathbf{177.11\ kN}$

From (3)

$P_B = 450 - 1.5(177.11)$

$P_B = \mathbf{184.34\ kN}$

From (1)

$P_A = 3.6(184.34) - 2.4(177.11)$

$P_A = \mathbf{238.56\ kN}$

Problem 256

Three rods, each of area 250 mm2, jointly support a 7.5 kN load, as shown in Fig. P-256. Assuming that there was no slack or stress in the rods before the load was applied, find the stress in each rod. Use E_{st} = 200 GPa and E_{br} = 83 GPa.

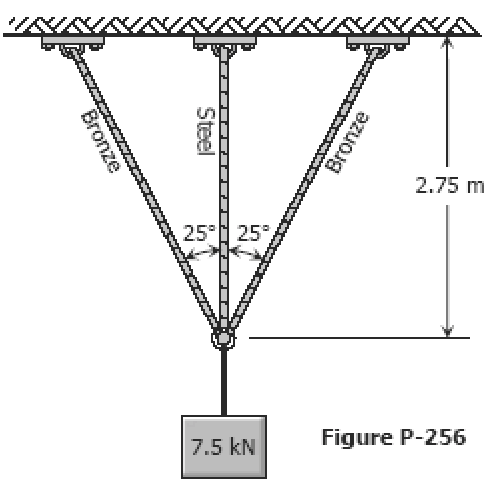
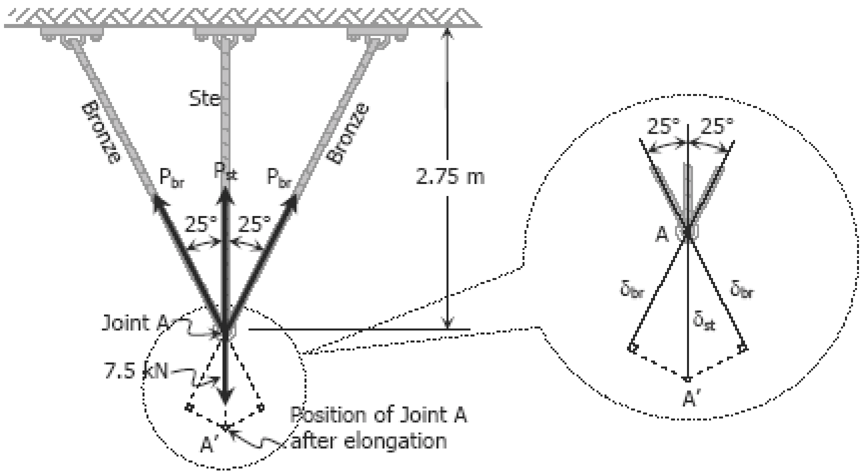


Figure P-256

Solution 256

$$\cos 25° = \frac{2.75}{L_{br}}\ ;\ L_{br} = 3.03 \text{ m}$$



$\sum F_V = 0$

$2P_{br} \cos 25° + P_{st} = 7.5(1000)$

$P_{st} = 7500 - 1.8126 P_{br}$

$\sigma_{st} A_{st} = 7500 - 1.8126 \sigma_{br} A_{br}$

$\sigma_{st} (250) = 7500 - 1.8126[\sigma_{br} (250)]$

$\sigma_{st} = 30 - 1.8126\ \sigma_{br}$ → (1)

$$\cos 25° = \frac{\delta_{br}}{\delta_{st}}$$

$\delta_{br} = 0.9063\ \delta_{st}$

$$\left(\frac{\sigma L}{E}\right)_{br} = 0.9063\left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{\sigma_{br}(3.03)}{83} = 0.9063\left[\frac{\sigma_{st}(2.75)}{200}\right]$$

$\sigma br = 0.3414\sigma_{st}$ → (2)

From equation (1)

$\sigma_{st} = 30 - 1.8126(0.3414\sigma_{st})$

$\sigma_{st} = \mathbf{18.53\ MPa}$

From equation (2)

$\sigma br = 0.3414(18.53)$

$\sigma_{br} = \mathbf{6.33\ MPa}$

Problem 257

Three bars AB, AC, and AD are pinned together as shown in Fig. P-257. Initially, the assembly is stressfree. Horizontal movement of the joint at A is prevented by a short horizontal strut AE. Calculate the stress in each bar and the force in the strut AE when the assembly is used to support the load W = 10 kips. For each steel bar, A = 0.3 in.^{2} and E = 29 × 10^6 psi. For the aluminum bar, A = 0.6 in.^{2} and E = 10 × 10^6 psi.

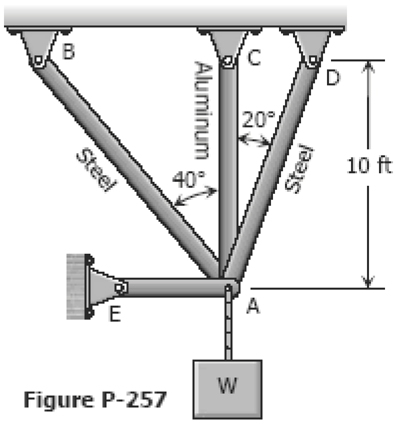
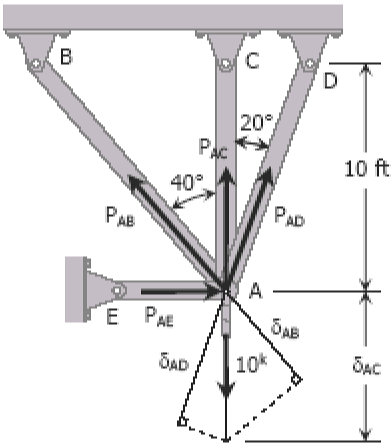


Figure P-257

Solution 257

$\cos 40° = 10 / L_{AB};\ L_{AB} = 13.05$ ft
$\cos 20° = 10 / L_{AD};\ L_{AD} = 10.64$ ft



$\sum F_V = 0$
$P_{AB} \cos 40° + P_{AC} + P_{AD} \cos 20° = 10(1000)$
$0.7660P_{AB} + P_{AC} + 0.9397P_{AD} = 10\ 000$ → (1)

$\delta_{AB} = \cos 40°\ \delta_{AC} = 0.7660\ \delta_{AC}$

$$\left(\frac{PL}{AE}\right)_{AB} = 0.7660\left(\frac{PL}{AE}\right)_{AC}$$

$$\frac{P_{AB}(13.05)}{0.3(29\times10^6)} = 0.7660\left[\frac{P_{AC}(10)}{0.6(10\times10^6)}\right]$$

$P_{AB} = 0.8511P_{AC}$ → (2)

$\delta_{AD} = \cos 20°\ \delta_{AC} = 0.9397\ \delta_{AC}$

$$\left(\frac{PL}{AE}\right)_{AD} = 0.9397\left(\frac{PL}{AE}\right)_{AC}$$

$$\frac{P_{AD}(10.64)}{0.3(29\times10^6)} = 0.9397\left[\frac{P_{AC}(10)}{0.6(10\times10^6)}\right]$$

$P_{AD} = 1.2806P_{AC}$ → (3)

Substitute P_{AB} of (2) and P_{AD} of (3) to (1)
$0.7660(0.8511P_{AC}) + P_{AC} + 0.9397(1.2806P_{AC}) = 10\ 000$
$2.8553P_{AC} = 10\ 000$
$P_{AC} = 3\ 502.23$ lb

$P_{AB} = 0.8511(3\ 502.23)$ → from (2)
$P_{AB} = 2\ 980.75$ lb

$P_{AD} = 1.2806(3\ 502.23)$ → from (3)
$P_{AD} = 4\ 484.96$ lb

Stresses:
$\sigma = P/A$
$\sigma_{AB} = 2980.75/0.3 =$ **9 935.83 psi**
$\sigma_{AC} = 3502.23/0.6 =$ **5 837.05 psi**
$\sigma_{AD} = 4484.96/0.3 =$ **14 949.87 psi**

$\sum F_H = 0$
$P_{AE} + P_{AD} \sin 20° = P_{AB} \sin 40°$
$P_{AE} = 2\ 980.75 \sin 40° - 4\ 484.96 \sin 20°$
$P_{AE} =$ **382.04 lb**