

## Thermal Stress

Temperature changes cause the body to expand or contract. The amount  $\delta_T$ , is given by

$$\delta_T = \alpha L(T_f - T_i) = \alpha L \Delta T$$

where  $\alpha$  is the coefficient of thermal expansion in  $m/m^\circ C$ ,  $L$  is the length in meter, and  $T_i$  and  $T_f$  are the initial and final temperatures, respectively in  $^\circ C$ .

For steel,  $\alpha = 11.25 \times 10^{-6} / ^\circ C$ .

If temperature deformation is permitted to occur freely, no load or stress will be induced in the structure. In some cases where temperature deformation is not permitted, an internal stress is created. The internal stress created is termed as thermal stress.

For a homogeneous rod mounted between unyielding supports as shown, the thermal stress is computed as:



deformation due to temperature changes;

$$\delta_T = \alpha L \Delta T$$

deformation due to equivalent axial stress;

$$\delta_P = \frac{PL}{AE} = \frac{\sigma L}{E}$$

$$\delta_T = \delta_P$$

$$\alpha L \Delta T = \frac{\sigma L}{E}$$

$$\sigma = E \alpha \Delta T$$

where  $\sigma$  is the thermal stress in MPa and  $E$  is the modulus of elasticity of the rod in MPa.

If the wall yields a distance of  $x$  as shown, the following calculations will be made:



$$\delta_T = x + \delta_P$$

$$\alpha L \Delta T = x \frac{\sigma L}{E}$$

where  $\sigma$  represents the thermal stress.

Take note that as the temperature rises above the normal, the rod will be in compression, and if the temperature drops below the normal, the rod is in tension.

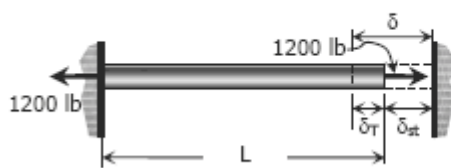
## Solved Problems in Thermal Stress

### Problem 261

A steel rod with a cross-sectional area of  $0.25 \text{ in}^2$  is stretched between two fixed points. The tensile load at  $70^\circ\text{F}$  is 1200 lb. What will be the stress at  $0^\circ\text{F}$ ? At what temperature will the stress be zero? Assume  $\alpha = 6.5 \times 10^{-6} \text{ in} / (\text{in}\cdot^\circ\text{F})$  and  $E = 29 \times 10^6 \text{ psi}$ .

### Solution 261

For the stress at  $0^\circ\text{C}$ :



$$\delta = \delta_T + \delta_{st}$$

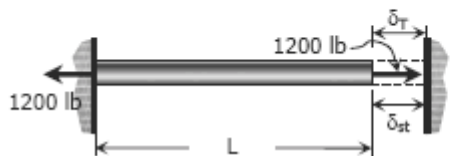
$$\frac{\sigma L}{E} = \alpha L (\Delta T) + \frac{P L}{AE}$$

$$\sigma = \alpha E (\Delta T) + \frac{P}{A}$$

$$\sigma = (6.5 \times 10^{-6})(29 \times 10^6)(70) + \frac{1200}{0.25}$$

$$\sigma = 17\,995 \text{ psi} = 18 \text{ ksi}$$

For the temperature that causes zero stress:



$$\delta_T = \delta_{st}$$

$$\alpha L (\Delta T) = \frac{P L}{AE}$$

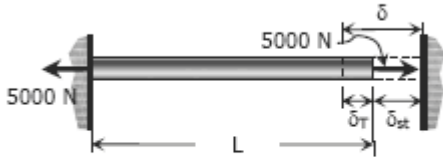
$$(6.5 \times 10^{-6})(T - 70) = \frac{1200}{0.25(29 \times 10^6)}$$

$$T = 95.46^\circ\text{C}$$

### Problem 262

A steel rod is stretched between two rigid walls and carries a tensile load of 5000 N at 20°C. If the allowable stress is not to exceed 130 MPa at -20°C, what is the minimum diameter of the rod? Assume  $\alpha = 11.7 \mu\text{m}/(\text{m}\cdot^\circ\text{C})$  and  $E = 200 \text{ GPa}$ .

### Solution 262



$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma L}{E} = \alpha L(\Delta T) + \frac{PL}{AE}$$

$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

$$130 = (11.7 \times 10^{-6})(200\,000)(40) + \frac{5000}{A}$$

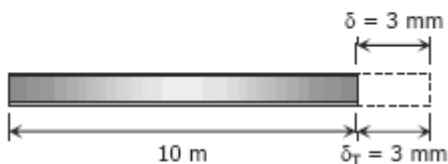
$$A = \frac{5000}{36.4} = 137.36 \text{ mm}^2$$

$$\frac{1}{4}\pi d^2 = 137.36; \quad d = 13.22 \text{ mm}$$

### Problem 263

Steel railroad rails 10 m long are laid with a clearance of 3 mm at a temperature of 15°C. At what temperature will the rails just touch? What stress would be induced in the rails at that temperature if there were no initial clearance? Assume  $\alpha = 11.7 \mu\text{m}/(\text{m}\cdot^\circ\text{C})$  and  $E = 200 \text{ GPa}$ .

### Solution 263



Temperature at which  $\delta_T = 3 \text{ mm}$ :

$$\delta_T = \alpha L(\Delta T)$$

$$\delta_T = \alpha L(T_f - T_i)$$

$$3 = (11.7 \times 10^{-6})(10\,000)(T_f - 15)$$

$$T_f = 40.64^\circ\text{C}$$

Required stress:

$$\delta = \delta_T$$

$$\frac{\sigma L}{E} = \alpha L(\Delta T)$$

$$\sigma = \alpha E(T_f - T_i)$$

$$\sigma = (11.7 \times 10^{-6})(200\,000)(40.64 - 15)$$

$$\sigma = 60 \text{ MPa}$$

### Problem 264

A steel rod 3 feet long with a cross-sectional area of  $0.25 \text{ in.}^2$  is stretched between two fixed points. The tensile force is 1200 lb at 40°F. Using  $E = 29 \times 10^6 \text{ psi}$  and  $\alpha = 6.5 \times 10^{-6} \text{ in.}/(\text{in.}\cdot^\circ\text{F})$ , calculate (a) the temperature at which the stress in the bar will be 10 ksi; and (b) the temperature at which the stress will be zero.

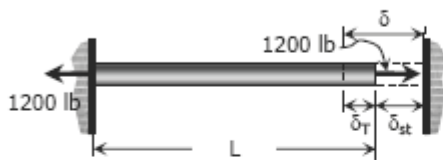
**Solution 264**

(a) Without temperature change:

$$\sigma = P/A = 1200/0.25 = 4800 \text{ psi}$$

$$\sigma = 4.8 \text{ ksi} < 10 \text{ ksi}$$

A drop of temperature is needed to increase the stress to 10 ksi. See accompanying figure.



$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma L}{E} = \alpha L (\Delta T) + \frac{PL}{AE}$$

$$\sigma = \alpha E (\Delta T) + \frac{P}{A}$$

$$10\,000 = (6.5 \times 10^{-6})(29 \times 10^6)(\Delta T) + \frac{1200}{0.25}$$

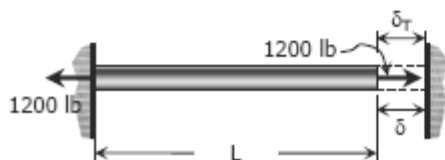
$$\Delta T = 27.59^\circ\text{F}$$

Required temperature:

(temperature must drop from 40°F)

$$T = 40 - 27.59 = 12.41^\circ\text{F}$$

(b) From the figure below:



$$\delta = \delta_T$$

$$\frac{PL}{AE} = \alpha L (\Delta T)$$

$$P = \alpha AE (T_f - T_i)$$

$$1200 = (6.5 \times 10^{-6})(0.25)(29 \times 10^6)(T_f - 40)$$

$$T_f = 65.46^\circ\text{F}$$

**Problem 265**

A bronze bar 3 m long with a cross sectional area of 320 mm<sup>2</sup> is placed between two rigid walls as shown in Fig. P-265. At a temperature of -20°C, the gap  $\Delta = 25$  mm. Find the temperature at which the compressive stress in the bar will be 35 MPa. Use  $\alpha = 18.0 \times 10^{-6} \text{ m}/(\text{m}\cdot^\circ\text{C})$  and  $E = 80 \text{ GPa}$ .



**Solution 265**

$$\delta_T = \delta + \Delta$$

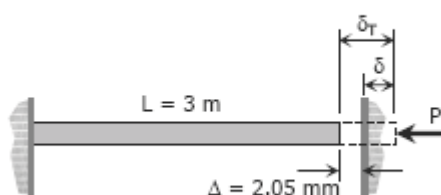
$$\alpha L (\Delta T) = \frac{\sigma L}{E} + 2.5$$

$$(18 \times 10^{-6})(3000)(\Delta T) = \frac{35(3000)}{80000} + 2.5$$

$$\Delta T = 70.6^\circ\text{C}$$

$$T = 70.6 - 20$$

$$T = 50.6^\circ\text{C}$$



### Problem 266

Calculate the increase in stress for each segment of the compound bar shown in Fig. P-266 if the temperature increases by  $100^\circ\text{F}$ . Assume that the supports are unyielding and that the bar is suitably braced against buckling.

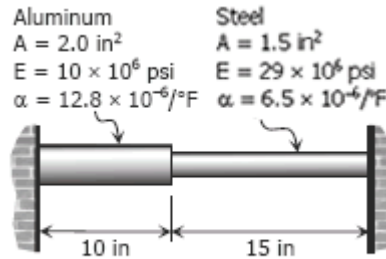
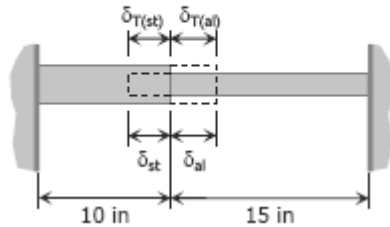


Figure P-266

### Solution 266



$$\delta_T = \alpha L \Delta T$$

$$\delta_{T(st)} = (6.5 \times 10^{-6})(15)(100)$$

$$\delta_{T(st)} = 0.00975$$

$$\delta_{T(al)} = (12.8 \times 10^{-6})(10)(100)$$

$$\delta_{T(al)} = 0.0128 \text{ in}$$

$$\delta_{st} + \delta_{al} = \delta_{T(st)} + \delta_{T(al)}$$

$$\left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{al} = 0.00975 + 0.0128$$

where  $P = P_{st} = P_{al}$

$$\frac{P(15)}{1.5(29 \times 10^6)} + \frac{P(10)}{2(10 \times 10^6)} = 0.02255$$

$$P = 26\,691.84 \text{ psi}$$

$$\sigma = \frac{P}{A}$$

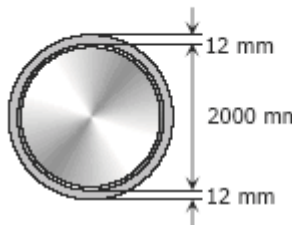
$$\sigma_{st} = \frac{26\,691.84}{1.5} = 17\,794.56 \text{ psi}$$

$$\sigma_{al} = \frac{26\,691.84}{2.0} = 13\,345.92 \text{ psi}$$

### Problem 267

At a temperature of 80°C, a steel tire 12 mm thick and 90 mm wide that is to be shrunk onto a locomotive driving wheel 2 m in diameter just fits over the wheel, which is at a temperature of 25°C. Determine the contact pressure between the tire and wheel after the assembly cools to 25°C. Neglect the deformation of the wheel caused by the pressure of the tire. Assume  $\alpha = 11.7 \mu\text{m}/(\text{m}\cdot^\circ\text{C})$  and  $E = 200 \text{ GPa}$ .

### Solution 267



$$\delta = \delta_T$$

$$\frac{PL}{AE} = \alpha L \Delta T$$

$$P = \alpha \Delta T AE$$

$$P = (11.7 \times 10^{-6})(80 - 25)(90 \times 12)(200\,000)$$

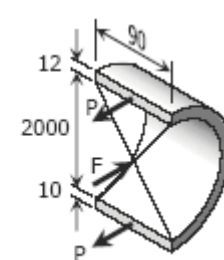
$$P = 138\,996 \text{ N}$$
  

$$F = 2P$$

$$pDL = 2P$$

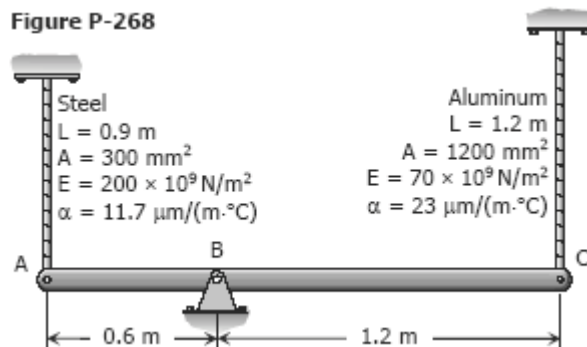
$$p(2000)(90) = 2(138\,996)$$

$$p = 1.5444 \text{ MPa}$$



### Problem 268

The rigid bar ABC in Fig. P-268 is pinned at B and attached to the two vertical rods. Initially, the bar is horizontal and the vertical rods are stress-free. Determine the stress in the aluminum rod if the temperature of the steel rod is decreased by 40°C. Neglect the weight of bar ABC.



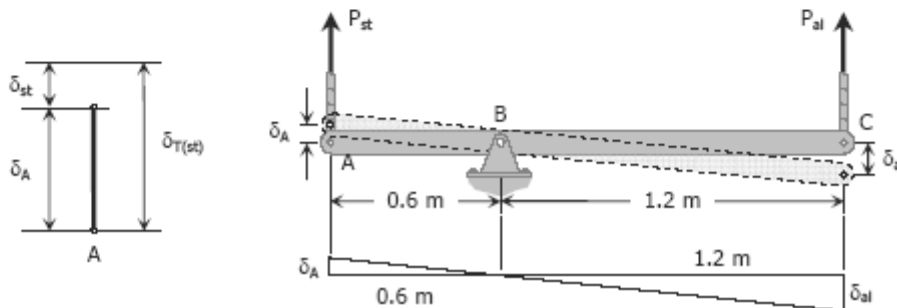
### Solution 268

Contraction of steel rod, assuming complete freedom:

$$\begin{aligned}\delta_{T(st)} &= \alpha L \Delta T \\ &= (11.7 \times 10^{-6})(900)(40) \\ &= 0.4212 \text{ mm}\end{aligned}$$

The steel rod cannot freely contract because of the resistance of aluminum rod. The movement of A (referred to as  $\delta_A$ ), therefore, is less than 0.4212 mm. In terms of aluminum, this movement is (by ratio and proportion):

$$\begin{aligned}\frac{\delta_A}{0.6} &= \frac{\delta_{al}}{1.2} \\ \delta_A &= 0.5\delta_{al}\end{aligned}$$



$$\delta_{T(st)} - \delta_{st} = 0.5 \delta_{al}$$

$$0.4212 - \left(\frac{PL}{AE}\right)_{st} = 0.5 \left(\frac{PL}{AE}\right)_{al}$$

$$0.4212 - \frac{P_{st}(900)}{300(200\,000)} = 0.5 \left[ \frac{P_{al}(1200)}{1200(70\,000)} \right]$$

$$28080 - P_{st} = 0.4762P_{al} \quad \rightarrow \text{Equation (1)}$$

$$\sum M_B = 0$$

$$0.6P_{st} = 1.2P_{al}$$

$$P_{st} = 2P_{al} \quad \rightarrow \text{Equation (2)}$$

Equations (1) and (2)

$$28080 - 2P_{al} = 0.4762P_{al}$$

$$P_{al} = 11340 \text{ N}$$

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{11340}{1200}$$

$$\sigma_{al} = 9.45 \text{ MPa}$$

### Problem 269

As shown in Fig. P-269, there is a gap between the aluminum bar and the rigid slab that is supported by two copper bars. At 10°C,  $\Delta = 0.18$  mm. Neglecting the mass of the slab, calculate the stress in each rod when the temperature in the assembly is increased to 95°C. For each copper bar,  $A = 500$  mm<sup>2</sup>,  $E = 120$  GPa, and  $\alpha = 16.8$   $\mu\text{m}/(\text{m}\cdot^\circ\text{C})$ . For the aluminum bar,  $A = 400$  mm<sup>2</sup>,  $E = 70$  GPa, and  $\alpha = 23.1$   $\mu\text{m}/(\text{m}\cdot^\circ\text{C})$ .

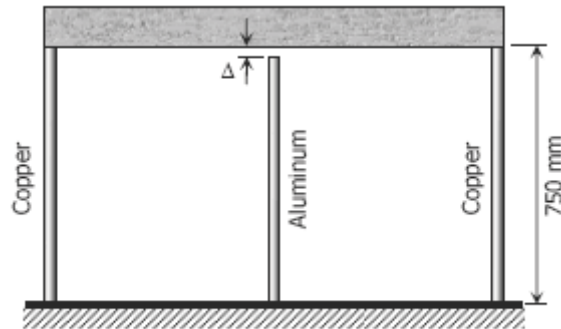


Figure P-269

### Solution 269

Assuming complete freedom:

$$\delta_T = \alpha L \Delta T$$

$$\delta_{T(\text{co})} = (16.8 \times 10^{-6})(750)(95 - 10) = 1.071 \text{ mm}$$

$$\delta_{T(\text{al})} = (23.1 \times 10^{-6})(750 - 0.18)(95 - 10) = 1.472 \text{ mm}$$

From the figure:

$$\delta_{T(\text{al})} - \delta_{\text{al}} = \delta_{T(\text{co})} + \delta_{\text{co}}$$

$$1.472 - \left(\frac{PL}{AE}\right)_{\text{al}} = 1.071 + \left(\frac{PL}{AE}\right)_{\text{co}}$$

$$1.472 - \frac{2F(750 - 0.18)}{400(70000)} = 1.071 + \frac{F(750)}{500(120000)}$$

$$0.401 = (6.606 \times 10^{-5}) F$$

$$F = 6070.37 \text{ N}$$

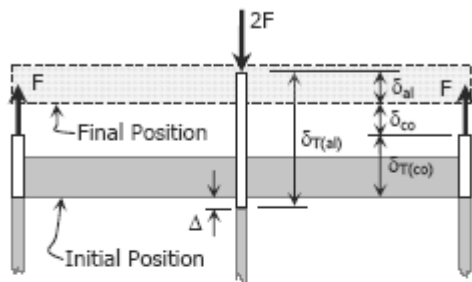
$$P_{\text{co}} = F = 6070.37 \text{ N}$$

$$P_{\text{al}} = 2F = 12140.74 \text{ N}$$

$$\sigma = P/A$$

$$\sigma_{\text{co}} = \frac{6070.37}{500} = 12.14 \text{ MPa}$$

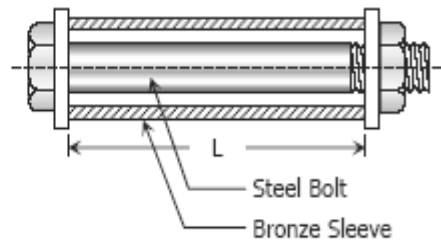
$$\sigma_{\text{al}} = \frac{12140.74}{400} = 30.35 \text{ MPa}$$



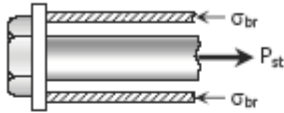


**Problem 270**

A bronze sleeve is slipped over a steel bolt and held in place by a nut that is turned to produce an initial stress of 2000 psi in the bronze. For the steel bolt,  $A = 0.75 \text{ in}^2$ ,  $E = 29 \times 10^6 \text{ psi}$ , and  $\alpha = 6.5 \times 10^{-6} \text{ in}/(\text{in}\cdot^\circ\text{F})$ . For the bronze sleeve,  $A = 1.5 \text{ in}^2$ ,  $E = 12 \times 10^6 \text{ psi}$  and  $\alpha = 10.5 \times 10^{-6} \text{ in}/(\text{in}\cdot^\circ\text{F})$ . After a temperature rise of  $100^\circ\text{F}$ , find the final stress in each material.

**Solution 270**

Before temperature change:



$$\begin{aligned}
 P_{br} &= \sigma_{br} A_{br} \\
 &= 2000(1.5) \\
 &= 3000 \text{ lb compression}
 \end{aligned}$$

$$\begin{aligned}
 \Sigma F_H &= 0 \\
 P_{st} &= P_{br} = 3000 \text{ lb tension} \\
 \sigma_{st} &= P_{st}/A_{st} = 3000/0.75 \\
 &= 4000 \text{ psi tensile stress}
 \end{aligned}$$

$$\delta = \frac{\sigma L}{E}$$

$$a = \delta_{br} = \frac{2000L}{12 \times 10^6} = 1.67 \times 10^{-4}L \text{ shortening}$$

$$b = \delta_{st} = \frac{4000L}{29 \times 10^6} = 1.38 \times 10^{-4}L \text{ lengthening}$$

With temperature rise of 100°F:

(Assuming complete freedom)

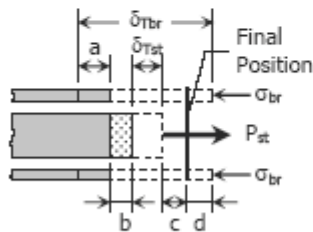
$$\begin{aligned}
 \delta_T &= \alpha L \Delta T \\
 \delta_{Tbr} &= (10.5 \times 10^{-6})L (100) \\
 &= 1.05 \times 10^{-3}L > a
 \end{aligned}$$

$$\begin{aligned}
 \delta_{Tst} &= (6.5 \times 10^{-6})L (100) \\
 &= 6.5 \times 10^{-4}L
 \end{aligned}$$

$$\begin{aligned}
 \delta_{Tbr} - a &= 1.05 \times 10^{-3}L - 1.67 \times 10^{-4}L \\
 &= 8.83 \times 10^{-4}L
 \end{aligned}$$

$$\begin{aligned}
 \delta_{Tst} + b &= 6.5 \times 10^{-4}L + 1.38 \times 10^{-4}L \\
 &= 7.88 \times 10^{-4}L
 \end{aligned}$$

$$\delta_{Tbr} - a > \delta_{Tst} + b \text{ (see figure below)}$$



$$\delta_{Tbr} - a - d = b + \delta_{Tst} + c$$

$$\begin{aligned}
 1.05 \times 10^{-3}L - 1.67 \times 10^{-4}L - \left(\frac{\sigma L}{E}\right)_{br} \\
 = 1.38 \times 10^{-4}L + 6.5 \times 10^{-4}L + \left(\frac{PL}{AE}\right)_{st}
 \end{aligned}$$

$$\begin{aligned}
 8.83 \times 10^{-4}L - \frac{\sigma_{br}L}{12 \times 10^6} \\
 = 7.88 \times 10^{-4}L + \frac{P_{st}L}{0.75(29 \times 10^6)}
 \end{aligned}$$

$$9.5 \times 10^{-4} - \frac{P_{br}}{1.5(12 \times 10^6)} = \frac{P_{st}}{0.75(29 \times 10^6)}$$

$$P_{st} = 20\,662.5 - 1.2083P_{br} \rightarrow \text{Equation (1)}$$

$$\Sigma F_H = 0$$

$$P_{br} = P_{st} \rightarrow \text{Equation (2)}$$

Equations (1) and (2)

$$P_{st} = 20\,662.5 - 1.2083P_{st}$$

$$P_{st} = 9356.74 \text{ lb}$$

$$P_{br} = 9356.74 \text{ lb}$$

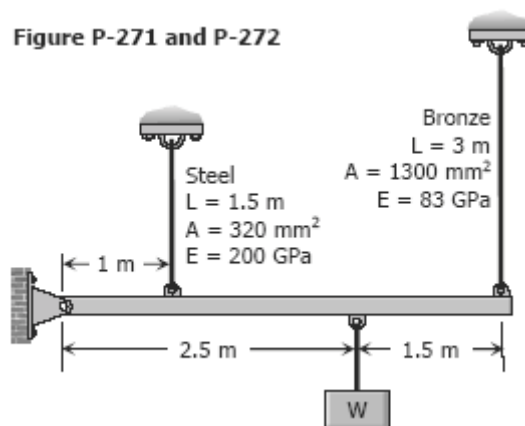
$$\sigma = P/A$$

$$\sigma_{br} = \frac{9356.74}{1.5} = 6237.83 \text{ psi compressive stress}$$

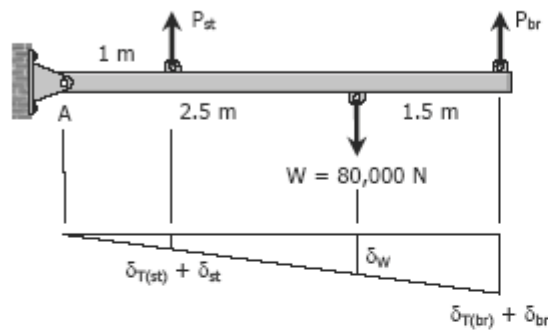
$$\sigma_{st} = \frac{9356.74}{0.74} = 12\,475.66 \text{ psi tensile stress}$$

### Problem 271

A rigid bar of negligible weight is supported as shown in Fig. P-271. If  $W = 80 \text{ kN}$ , compute the temperature change that will cause the stress in the steel rod to be  $55 \text{ MPa}$ . Assume the coefficients of linear expansion are  $11.7 \mu\text{m}/(\text{m}\cdot^\circ\text{C})$  for steel and  $18.9 \mu\text{m}/(\text{m}\cdot^\circ\text{C})$  for bronze.



### Solution 271



Stress in bronze when  $\sigma_{st} = 55$  MPa

$$\Sigma M_A = 0$$

$$4P_{br} + P_{st} = 2.5(80000)$$

$$4\sigma_{br}(1300) + 55(320) = 2.5(80000)$$

$$\sigma_{br} = 35.08 \text{ MPa}$$

By ratio and proportion:

$$\frac{\delta_{T(st)} + \delta_{st}}{1} = \frac{\delta_{T(br)} + \delta_{br}}{4}$$

$$\delta_{T(st)} + \delta_{st} = 0.25[\delta_{T(br)} + \delta_{br}]$$

$$\begin{aligned} (\alpha L \Delta T)_{st} + \left(\frac{\sigma L}{E}\right)_{st} \\ = 0.25 \left[ (\alpha L \Delta T)_{br} + \left(\frac{\sigma L}{E}\right)_{br} \right] \end{aligned}$$

$$(11.7 \times 10^{-6})(1500) \Delta T + \frac{55(1500)}{2000}$$

$$= 0.25 \left[ (18.9 \times 10^{-6})(3000) \Delta T + \frac{35.08(3000)}{83000} \right]$$

$$0.01755 \Delta T + 0.4125 = 0.014175 \Delta T + 0.317$$

$$\Delta T = -28.3 \text{ }^\circ\text{C}$$

A temperature drop of 28.3 °C is needed to stress the steel to 55 MPa.

### Problem 272

For the assembly in Fig. 271, find the stress in each rod if the temperature rises 30°C after a load  $W = 120$  kN is applied.

### Solution 272

$$\Sigma M_A = 0$$

$$4P_{br} + P_{st} = 2.5(80000)$$

$$4\sigma_{br}(1300) + \sigma_{st}(320) = 2.5(80000)$$

$$16.25\sigma_{br} + \sigma_{st} = 625$$

$$\sigma_{st} = 625 - 16.25\sigma_{br}$$

→ Equation (1)

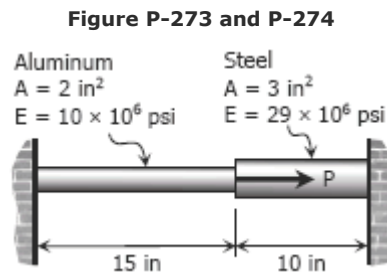
$$\frac{\delta_{T(st)} + \delta_{st}}{1} = \frac{\delta_{T(br)} + \delta_{br}}{4}$$

$$\delta_{T(st)} + \delta_{st} = 0.25[\delta_{T(br)} + \delta_{br}]$$

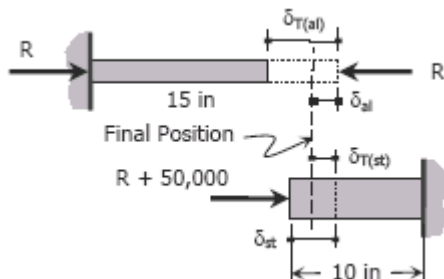
$$\begin{aligned}
(\alpha L \Delta T)_{st} + \left(\frac{\sigma L}{E}\right)_{st} &= 0.25 \left[ (\alpha L \Delta T)_{br} + \left(\frac{\sigma L}{E}\right)_{br} \right] \\
(11.7 \times 10^{-6})(1500)(30) + \frac{\sigma_{st}(1500)}{200000} &= 0.25 \left[ (18.9 \times 10^{-6})(3000)(30) + \frac{\sigma_{br}(3000)}{83000} \right] \\
0.5265 + 0.0075\sigma_{st} &= 0.42525 + 0.00904\sigma_{br} \\
0.0075\sigma_{st} - 0.00904\sigma_{br} &= -0.10125 \\
0.0075(625 - 16.25\sigma_{br}) - 0.00904\sigma_{br} &= -0.10125 \\
4.6875 - 0.121875\sigma_{br} - 0.00904\sigma_{br} &= -0.10125 \\
4.78875 &= 0.130915\sigma_{br} \\
\sigma_{br} &= 36.58 \text{ }^\circ\text{C} \\
\sigma_{st} &= 625 - 16.25(36.58) \\
\sigma_{st} &= 30.58 \text{ }^\circ\text{C}
\end{aligned}$$

### Problem 273

The composite bar shown in Fig. P-273 is firmly attached to unyielding supports. An axial force  $P = 50$  kips is applied at  $60^\circ\text{F}$ . Compute the stress in each material at  $120^\circ\text{F}$ . Assume  $\alpha = 6.5 \times 10^{-6}$  in/(in $\cdot^\circ\text{F}$ ) for steel and  $12.8 \times 10^{-6}$  in/(in $\cdot^\circ\text{F}$ ) for aluminum.



### Solution 273



$$\begin{aligned}
\delta_{T(al)} &= (\alpha L \Delta T)_{al} \\
\delta_{T(al)} &= (12.8 \times 10^{-6})(15)(120 - 60) \\
\delta_{T(al)} &= 0.01152 \text{ inch} \\
\delta_{T(st)} &= (\alpha L \Delta T)_{st} \\
\delta_{T(st)} &= (6.5 \times 10^{-6})(10)(120 - 60) \\
\delta_{T(st)} &= 0.0039 \text{ inch}
\end{aligned}$$

$$\delta_{T(al)} - \delta_{al} = \delta_{st} - \delta_{T(st)}$$

$$0.011\ 52 - \left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st} - 0.0039$$

$$0.011\ 52 - \frac{R(15)}{2(10 \times 10^6)} = \frac{(R + 50\ 000)(10)}{3(29 \times 10^6)} - 0.0039$$

$$100\ 224 - 6.525R = R + 50\ 000 - 33\ 930$$

$$84\ 154 = 7.525R$$

$$R = 11\ 183.25\ \text{lbs}$$

$$P_{al} = R = 11\ 183.25\ \text{lbs}$$

$$P_{st} = R + 50\ 000 = 61\ 183.25\ \text{lbs}$$

$$\sigma = \frac{P}{A}$$

$$\sigma_{al} = \frac{11\ 183.25}{2}$$

$$= 5\ 591.62\ \text{psi}$$

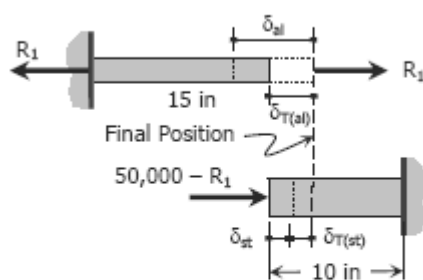
$$\sigma_{st} = \frac{61\ 183.25}{3}$$

$$= 20\ 394.42\ \text{psi}$$

### Problem 274

At what temperature will the aluminum and steel segments in Prob. 273 have numerically equal stress?

### Solution 274



$$\sigma_{al} = \sigma_{st}$$

$$\frac{R_1}{2} = \frac{(50\ 000 - R_1)}{3}$$

$$3R_1 = 100\ 000 - 2R_1$$

$$R_1 = 20\ 000\ \text{lbs}$$

$$\delta = \frac{PL}{AE}$$

$$\delta_{al} = \frac{20\ 000(15)}{2(10 \times 10^6)} = 0.015\ \text{inch}$$

$$\delta_{st} = \frac{(50\ 000 - 20\ 000)(10)}{3(29 \times 10^6)} = 0.003\ 45\ \text{inch}$$

$$\delta_{al} - \delta_{T(al)} = \delta_{st} + \delta_{T(st)}$$

$$0.015 - (12.8 \times 10^{-6})(15) \Delta T = 0.003\ 45 + (6.5 \times 10^{-6})(10) \Delta T$$

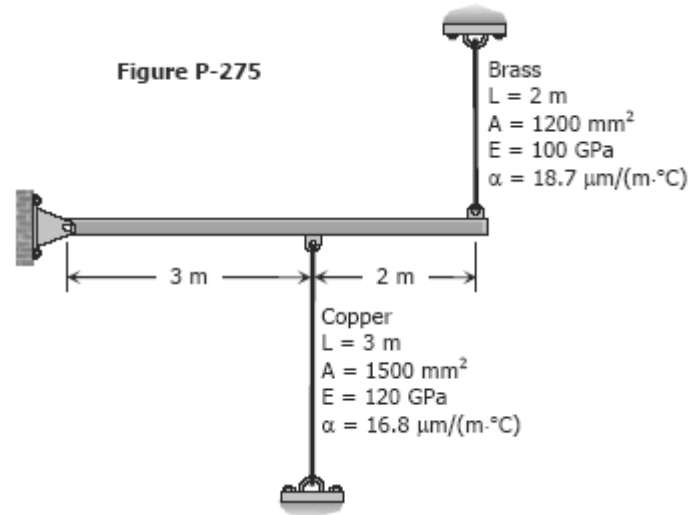
$$0.011\ 55 = 0.000\ 257 \Delta T$$

$$\Delta T = 44.94^\circ\text{F}$$

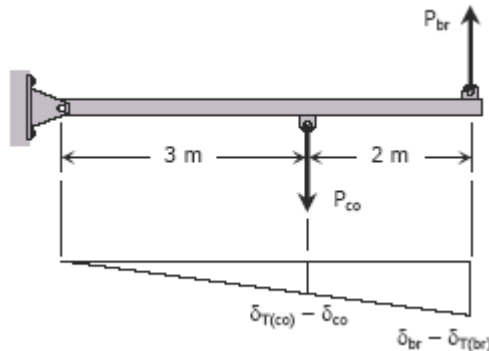
A drop of  $44.94^\circ\text{F}$  from the standard temperature will make the aluminum and steel segments equal in stress.

### Problem 275

A rigid horizontal bar of negligible mass is connected to two rods as shown in Fig. P-275. If the system is initially stress-free. Calculate the temperature change that will cause a tensile stress of 90 MPa in the brass rod. Assume that both rods are subjected to the change in temperature.



### Solution 275



$$\begin{aligned}\Sigma M_{\text{hinge support}} &= 0 \\ 5P_{br} - 3P_{co} &= 0 \\ 5\sigma_{br} A_{br} - 3\sigma_{co} A_{co} &= 0 \\ 5(90)(1200) - 3\sigma_{co} (1500) &= 0 \\ \sigma_{co} &= 120 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\delta &= \sigma L / E \\ \delta_{br} &= 90(2000) / 100\,000 = 1.8 \text{ mm} \\ \delta_{co} &= 120(3000) / 120\,000 = 3 \text{ mm}\end{aligned}$$

$$\frac{\delta_{T(co)} - \delta_{co}}{3} = \frac{\delta_{br} - \delta_{T(br)}}{5}$$

$$\begin{aligned}5\delta_{T(co)} - 5\delta_{co} &= 3\delta_{br} - 3\delta_{T(br)} \\ 5(16.8 \times 10^{-6})(3000) \Delta T - 5(3) &= 3(1.8) - 3(18.7 \times 10^{-6})(2000) \Delta T \\ 0.3642 \Delta T &= 20.4 \\ \Delta T &= 56.01^\circ\text{C drop in temperature}\end{aligned}$$

### Problem 276

Four steel bars jointly support a mass of 15 Mg as shown in Fig. P-276. Each bar has a cross-sectional area of 600 mm<sup>2</sup>. Find the load carried by each bar after a temperature rise of 50°C. Assume  $\alpha = 11.7 \mu\text{m}/(\text{m}\cdot^\circ\text{C})$  and  $E = 200 \text{ GPa}$ .

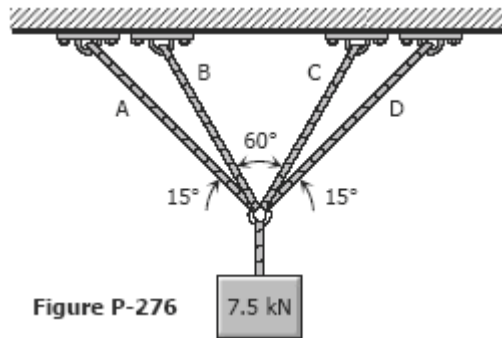
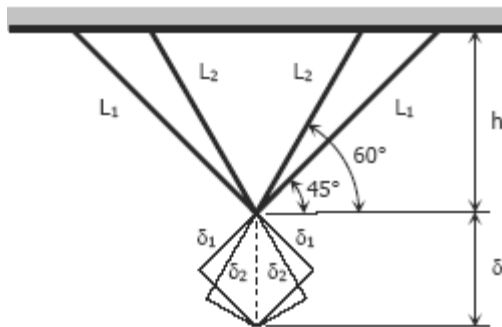


Figure P-276

**Solution 276**



$$h = L_1 \sin 45^\circ$$

$$h = L_2 \sin 60^\circ$$

$$h = h$$

$$L_1 \sin 45^\circ = L_2 \sin 60^\circ$$

$$L_1 = 1.2247L_2$$

$$\delta_1 = \delta \sin 45^\circ$$

$$\delta_2 = \delta \sin 60^\circ$$

$$\frac{\delta_1}{\delta_2} = \frac{\delta \sin 45^\circ}{\delta \sin 60^\circ}$$

$$\delta_1 = 0.8165\delta_2$$

$$\alpha L_1 \Delta T + \frac{P_1 L_1}{AE} = 0.8165 \left[ \alpha L_2 \Delta T + \frac{P_2 L_2}{AE} \right]$$

$$(11.7 \times 10^{-6})L_1 (50) + \frac{P_1 L_1}{600(200000)}$$

$$= 0.8165 \left[ (11.7 \times 10^{-6})L_2 (50) + \frac{P_2 L_2}{600(200000)} \right]$$

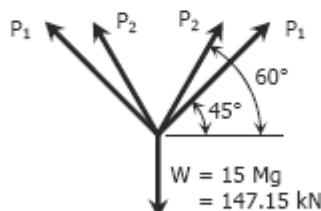
$$70,200L_1 + P_1 L_1 = 0.8165(70,200L_2 + P_2 L_2)$$

$$(70,200 + P_1)L_1 = 0.8165(70,200 + P_2)L_2$$

$$(70,200 + P_1)1.2247L_1 = 0.8165(70,200 + P_2)L_2$$

$$1.5(70,200 + P_1) = 70,200 + P_2$$

$$P_2 = 1.5P_1 + 35,100 \quad \rightarrow \text{Equation (1)}$$



$$\Sigma F_V = 0$$

$$2(P_1 \sin 45^\circ) + 2(P_2 \sin 60^\circ) = 147.15(1000)$$

$$P_1 \sin 45^\circ + P_2 \sin 60^\circ = 72,575$$

$$P_1 \sin 45^\circ + (1.5P_1 + 35,100) \sin 60^\circ = 72,575$$

$$0.7071P_1 + 1.299P_1 + 30,397.49 = 72,575$$

$$2.0061P_1 = 42,177.51$$

$$P_1 = 21,024.63 \text{ N}$$

$$P_2 = 1.5(21,024.63) + 35,100$$

$$P_2 = 66,636.94 \text{ N}$$

$$P_A = P_D = P_1 = 21.02 \text{ kN}$$

$$P_B = P_C = P_2 = 66.64 \text{ kN}$$