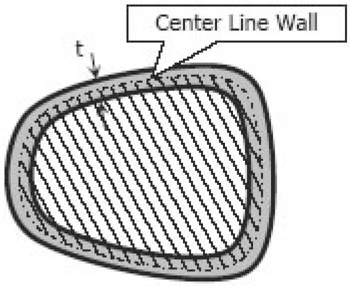
Torsion of Thin-Walled Tubes

The torque applied to thin-walled tubes is expressed as



$$T = 2Aq$$

where T is the torque in N·mm, A is the area enclosed by the centerline of the tube (as shown in the stripefilled portion) in mm^2, and q is the shear flow in N/mm.

The average shearing stress across any thickness t is

$$\tau = \frac{q}{t} = \frac{T}{2At}.$$

Thus, torque T ca also be expressed as

$$T = 2At\tau.$$

Solved Problems in Torsion of Thin-Walled Tubes

Problem 337

A torque of 600 N·m is applied to the rectangular section shown in Fig. P-337. Determine the wall thickness t so as not to exceed a shear stress of 80 MPa. What is the shear stress in the short sides? Neglect stress concentration at the corners.

Solution 337

$$T = 2At\tau$$

Where: $T = 600$ N·m = 600 000 N·mm

$A = 30(80) = 2400\ mm^2$

$\tau = 80$ MPa

$$600\,000 = 2(2400)(t)(80)$$

$$t = \mathbf{1.5625\ mm}$$

At any convenient center O within the section, the farthest side is the shortest side, thus, it is induced with the maximum allowable shear stress of **80 MPa**.

Problem 338

A tube 0.10 in. thick has an elliptical shape shown in Fig. P-338. What torque will cause a shearing stress of 8000 psi?

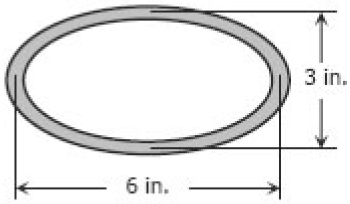
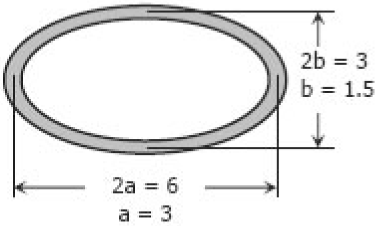


Figure P-338

Solution 338



$T = 2At\tau$

Where: $A = \pi ab = \pi(3)(1.5) = 4.5\pi \text{ in}^2$

$T = 0.10$ in

$\tau = 8000$ psi

$T = 2(4.5\pi)(0.10)(8000)$

$T = 22619.47$ lb·in

$T = \mathbf{22.62 \text{ kip·in}}$

Problem 339

A torque of 450 lb·ft is applied to the square section shown in Fig. P-339. Determine the smallest permissible dimension a if the shearing stress is limited to 6000 psi.

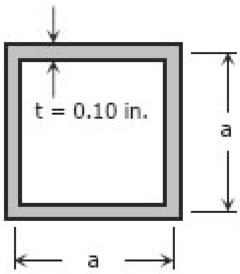


Figure P-339

Solution 339

$T = 2At\tau$

Where: $T = 450$ lb·ft

$T = 450(12)$ lb·in

$A = a^2$

$\tau = 6000$ psi

$450(12) = 2a^2(0.10)(6000)$

$a = \mathbf{2.12 \text{ in}}$

Problem 340

A tube 2 mm thick has the shape shown in Fig. P-340. Find the shearing stress caused by a torque of 600 N·m.

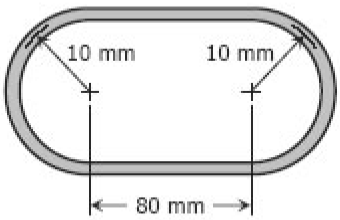


Figure P-340

Solution 340

$T = 2At\tau$

Where: $A = \pi(10^2) + 80(20) = 1914.16 \text{ mm}^2$

$t = 2$ mm

$T = 600$ N·m $= 600\,000$ N·mm

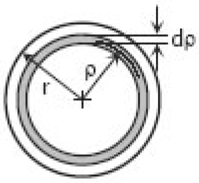
$600\,000 = 2(1914.16)(2)\tau$

$\tau = \mathbf{78.36 \text{ MPa}}$

Problem 341

Derive the torsion formula $\tau = T\rho/J$ for a solid circular section by assuming the section is composed of a series of concentric thin circular tubes. Assume that the shearing stress at any point is proportional to its radial distance.

Solution 341



$$T = 2At\tau$$

Where: $T = dT$; $A = \pi\rho^2$; $t = d\rho$

$$\frac{\tau}{\rho} = \frac{\tau_{max}}{r}; \quad \tau = \frac{\tau_{max}\rho}{r}$$

$$dT = 2\pi(\rho^2)\,d\rho\left(\frac{\tau_{max}\rho}{r}\right)$$

$$T = \frac{2\pi\tau_{max}}{r}\int_0^r \rho^3\,d\rho$$

$$T = \frac{2\pi\tau_{max}}{r}\left[\frac{\rho^4}{4}\right]_0^r$$

$$T = \frac{2\pi\tau_{max}}{r}\left(\frac{r^4}{4}\right)$$

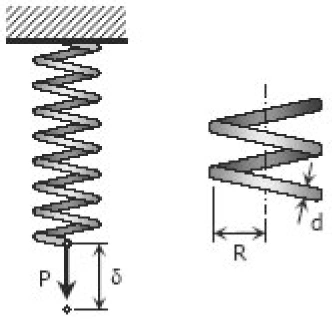
$$T = \frac{\tau_{max}}{r}\left(\frac{\pi r^4}{2}\right)$$

$$T = \frac{\tau_{max}}{r}J$$

$$\tau_{max} = \frac{Tr}{J} \text{ and it follows that } \tau = \frac{T\rho}{J}$$

Helical Springs

When close-coiled helical spring, composed of a wire of round rod of diameter d wound into a helix of mean radius R with n number of turns, is subjected to an axial load P produces the following stresses and elongation:



The maximum shearing stress is the sum of the direct shearing stress τ_1 = P/A and the torsional shearing stress τ_2 = Tr/J, with T = PR.

$$\tau = \tau_1 + \tau_2 = \frac{P}{\pi d^2/4} + \frac{16(PR)}{\pi d^3}$$

$$\tau = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$$

This formula neglects the curvature of the spring. This is used for light spring where the ratio d/4R is small.

For heavy springs and considering the curvature of the spring, a more precise formula is given by: (A.M.Wahl Formula)

$$\tau = \frac{16PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right)$$

where m is called the spring index and (4m – 1) / (4m – 4) is the Wahl Factor.

The elongation of the bar is

$$\delta = \frac{64PR^3n}{Gd^4}$$

Notice that the deformation δ is directly proportional to the applied load P. The ratio of P to δ is called the spring constant k and is equal to

$$k = \frac{P}{\delta} = \frac{Gd^4}{64R^3n} \text{ in N/mm}$$