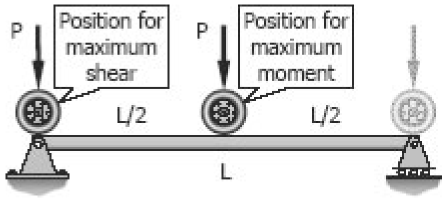
Moving Loads

From the previous section, we see that the maximum moment occurs at a point of zero shears. For beams loaded with concentrated loads, the point of zero shears usually occurs under a concentrated load and so the maximum moment.

Beams and girders such as in a bridge or an overhead crane are subject to moving concentrated loads, which are at fixed distance with each other. The problem here is to determine the moment under each load when each load is in a position to cause a maximum moment. The largest value of these moments governs the design of the beam.

SINGLE MOVING LOAD

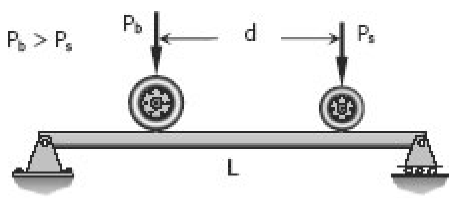
For a single moving load, the maximum moment occurs when the load is at the midspan and the maximum shear occurs when the load is very near the support (usually assumed to lie over the support).



$$M_{max} = \frac{PL}{4} \text{ and } V_{max} = P$$

TWO MOVING LOADS

For two moving loads, the maximum shear occurs at the reaction when the larger load is over that support. The maximum moment is given by



$$M_{max} = \frac{(PL - P_s d)^2}{4PL}$$

where P_s is the smaller load, P_b is the bigger load, and P is the total load ($P = P_s + P_b$).

THREE OR MORE MOVING LOADS

In general, the bending moment under a particular load is a maximum when the center of the beam is midway between that load and the resultant of all the loads then on the span. With this rule, we compute the maximum moment under each load, and use the biggest of the moments for the design. Usually, the biggest of these moments occurs under the biggest load.

The maximum shear occurs at the reaction where the resultant load is nearest. Usually, it happens if the biggest load is over that support and as many a possible of the remaining loads are still on the span.

The maximum shear occurs at the reaction where the resultant load is nearest. Usually, it happens if the biggest load is over that support and as many a possible of the remaining loads are still on the span. In determining the largest moment and shear, it is sometimes necessary to check the condition when the bigger loads are on the span and the rest of the smaller loads are outside.

Solved Problems in Moving Loads

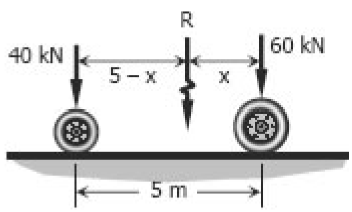
Problem 453

A truck with axle loads of 40 kN and 60 kN on a wheel base of 5 m rolls across a 10-m span. Compute the maximum bending moment and the maximum shearing force.

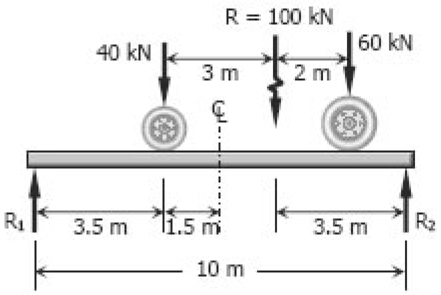
Solution 453

$R = 40 + 60 = 100\text{ kN}$

$xR = 40(5)$
$x = 200/R$
$x = 200/100$
$x = 2\text{ m}$



For maximum moment under 40 kN wheel:



$\Sigma M_{R2} = 0$

$10R_1 = 3.5(100)$

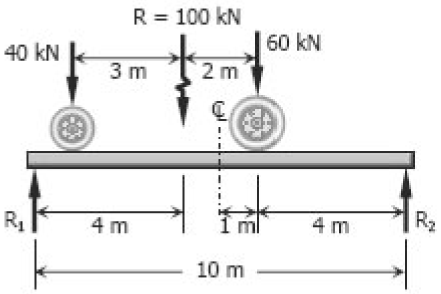
$R_1 = 35\text{ kN}$

$M_{\text{To the left of 40 kN}} = 3.5R_1$

$M_{\text{To the left of 40 kN}} = 3.5(35)$

$M_{\text{To the left of 40 kN}} = 122.5\text{ kN·m}$

For maximum moment under 60 kN wheel:



$\Sigma M_{R1} = 0$

$10R_2 = 4(100)$

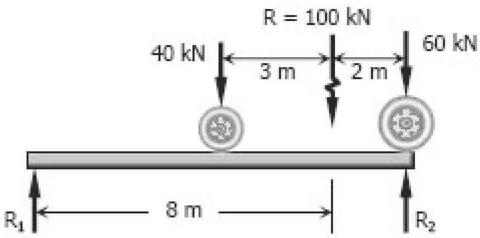
$R_2 = 40\text{ kN}$

$M_{\text{To the right of 60 kN}} = 4R_2$

$M_{\text{To the right of 60 kN}} = 4(40)$

$M_{\text{To the right of 60 kN}} = 160\text{ kN·m}$

Thus, $M_{max} = \mathbf{160\text{ kN·m}}$



The maximum shear will occur when the 60 kN is over a support.

$\Sigma M_{R1} = 0$

$10R_2 = 100(8)$

$R_2 = 80\text{ kN}$

Thus, $V_{max} = 80\text{ kN}$

Problem 454

Repeat Prob. 453 using axle loads of 30 kN and 50 kN on a wheel base of 4 m crossing an 8-m span.

Solution 454

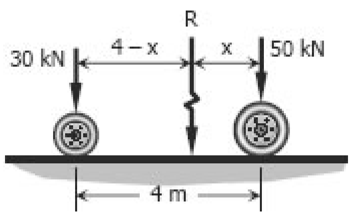
$R = 30 + 50 = 80\text{ kN}$

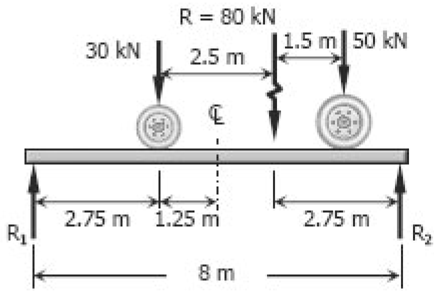
$xR = 4(30)$

$x = 120/R$

$x = 120/80$

$x = 1.5\text{ m}$





Maximum moment under 30 kN wheel:

$\Sigma M_{R2} = 0$

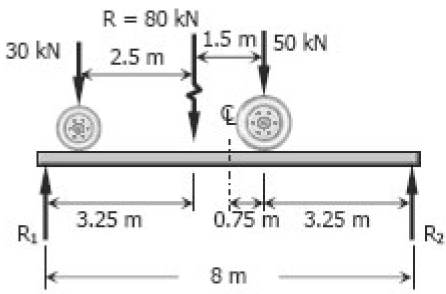
$8R_1 = 2.75(80)$

$R_1 = 27.5$ kN

$M_{\text{To the left of 30 kN}} = 2.75R_1$

$M_{\text{To the left of 30 kN}} = 2.75(27.5)$

$M_{\text{To the left of 30 kN}} = 75.625$ kN·m



Maximum moment under 50 kN wheel:

$\Sigma M_{R1} = 0$

$8R_2 = 3.25(80)$

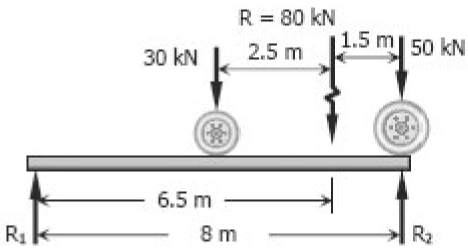
$R_2 = 32.5$ kN

$M_{\text{To the right of 50 kN}} = 3.25R_2$

$M_{\text{To the right of 50 kN}} = 3.25(32.5)$

$M_{\text{To the right of 50 kN}} = 105.625$ kN·m

Thus, $M_{max} =$ **105.625 kN·m**



The maximum shear will occur when the 50 kN is over a support.

$\Sigma M_{R1} = 0$

$8R_2 = 6.5(80)$

$R_2 = 65$ kN

Thus, $V_{max} =$ **65 kN**

Problem 455

A tractor weighing 3000 lb, with a wheel base of 9 ft, carries 1800 lb of its load on the rear wheels. Compute the maximum moment and maximum shear when crossing a 14 ft-span.

Solution 455

$R = W_r + W_f$

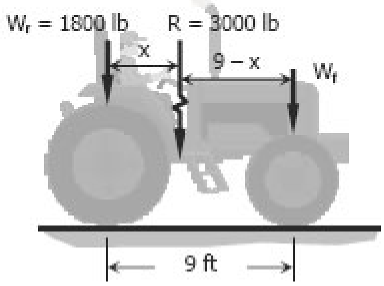
$3000 = 1800 + W_f$

$W_f = 1200$ lb

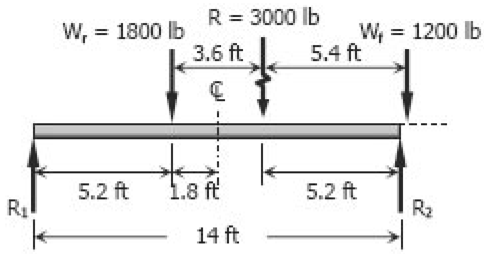
$Rx = 9W_f$

$3000x = 9(1200)$

$x = 3.6$ ft



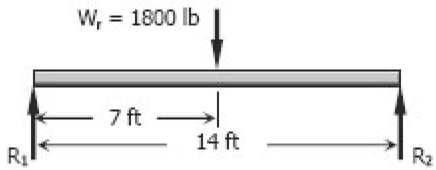
$9 - x = 5.4 \text{ ft}$



When the midspan is midway between W_r and R, the front wheel W_f will be outside the span (see figure). In this case, only the rear wheel W_r = 1800 lb is the load. The maximum moment for this condition is when the load is at the midspan.

$R_1 = R_2 = \frac{1}{2}(1800)$

$R_1 = 900$ lb



Maximum moment under W_r

$M_{\text{To the left of rear wheel}} = 7R_1$

$M_{\text{To the left of rear wheel}} = 7(900)$

$M_{\text{To the left of rear wheel}} = 6300$ lb·ft

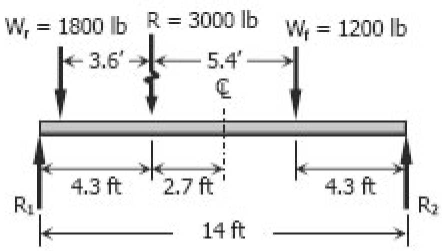
Maximum moment under W_f

$\Sigma M_{R1} = 0$

$14R_2 = 4.3R$

$14R_2 = 4.3(3000)$

$R_2 = 921.43$ lb



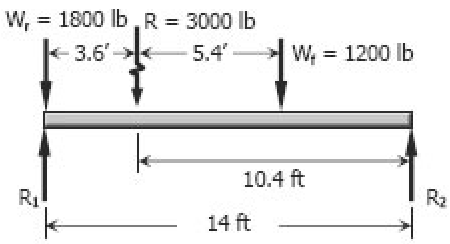
$M_{\text{To the right of front wheel}} = 4.3R_2$

$M_{\text{To the right of front wheel}} = 4.3(921.43)$

$M_{\text{To the right of front wheel}} = 3962.1$ lb·ft

Thus, $M_{max} = M_{\text{To the left of rear wheel}}$

M_{max} = **6300 lb·ft**



The maximum shear will occur when the rear wheel (wheel of greater load) is directly over the support.

$\Sigma M_{R2} = 0$

$14R_1 = 10.4R$

$14R_1 = 10.4(3000)$

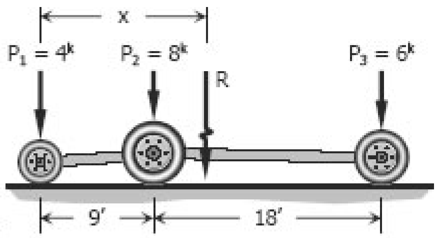
$R_1 = 2228.57$ lb

Thus, V_{max} = **2228.57 lb**

Problem 456

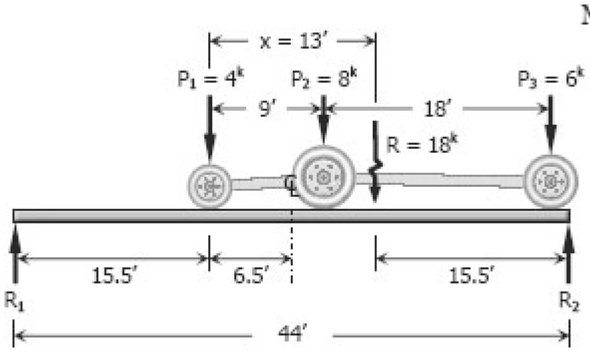
Three wheel loads roll as a unit across a 44-ft span. The loads are P_1 = 4000 lb and P_2 = 8000 lb separated by 9 ft, and P_3 = 6000 lb at 18 ft from P_2. Determine the maximum moment and maximum shear in the simply supported span.

Solution 456



$R = P_1 + P_2 + P_3$
$R = 4^k + 8^k + 6^k$
$R = 18$ kips
$R = 18{,}000$ lbs

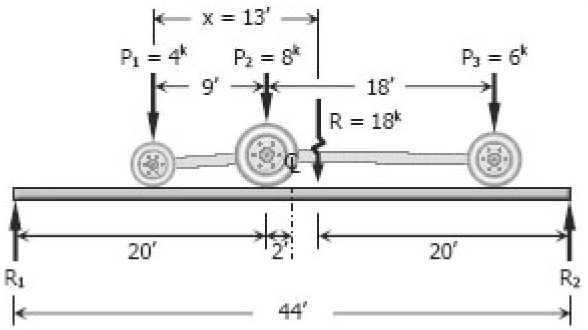
$xR = 9P_2 + (9 + 18)P_3$
$x(18) = 9(8) + (9 + 18)(6)$
$x = 13$ ft → the resultant R is 13 ft from P_1



Maximum moment under P_1

$\Sigma M_{R2} = 0$
$44R_1 = 15.5R$
$44R_1 = 15.5(18)$
$R_1 = 6.34091$ kips
$R_1 = 6{,}340.91$ lbs

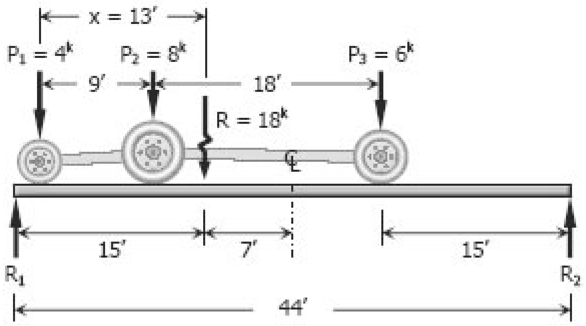
$M_{\text{To the left of } P1} = 15.5R_1$
$M_{\text{To the left of } P1} = 15.5(6340.91)$
$M_{\text{To the left of } P1} = 98{,}284.1$ lb·ft



Maximum moment under P_2

$\Sigma M_{R2} = 0$
$44R_1 = 20R$
$44R_1 = 20(18)$
$R_1 = 8.18182$ kips
$R_1 = 8{,}181.82$ lbs

$M_{\text{To the left of } P2} = 20R_1 - 9P_1$
$M_{\text{To the left of } P2} = 20(8{,}181.82) - 9(4000)$
$M_{\text{To the left of } P2} = 127{,}636.4$ lb·ft



Maximum moment under P_3

$\Sigma R_1 = 0$

$44R_2 = 15R$

$44R_2 = 15(18)$

$R_2 = 6.13636$ kips

$R_2 = 6{,}136.36$ lbs

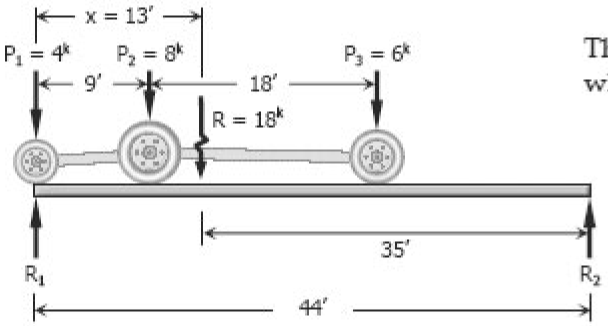
$M_{\text{To the right of P3}} = 15R_2$

$M_{\text{To the right of P3}} = 15(6{,}136.36)$

$M_{\text{To the right of P3}} = 92{,}045.4$ lb·ft

Thus, $M_{max} = M_{\text{To the left of P2}}$

$= \mathbf{127{,}636.4}$ **lb·ft**



The maximum shear will occur when P_1 is over the support.

$\Sigma M_{R2} = 0$

$44R_1 = 35R$

$44R_1 = 35(18)$

$R_1 = 14.3182$ kips

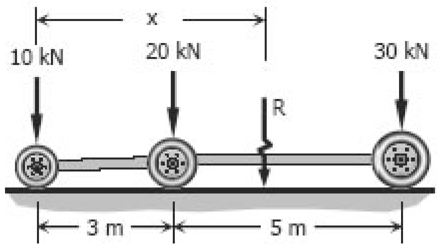
$R_1 = 14{,}318.2$ lbs

Thus, $V_{max} = \mathbf{14{,}318.2}$ **lbs**

Problem 457

A truck and trailer combination crossing a 12-m span has axle loads of 10, 20, and 30 kN separated respectively by distances of 3 and 5 m. Compute the maximum moment and maximum shear developed in the span.

Solution 457



$R = 10 + 20 + 30$

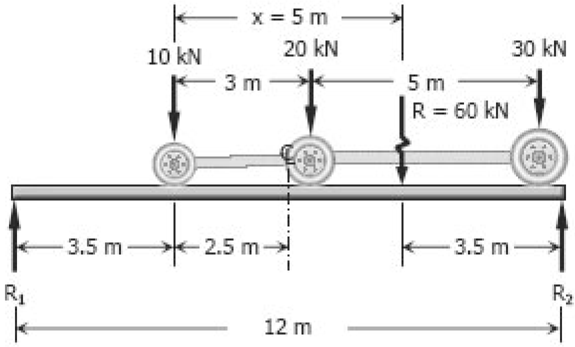
$R = 60$ kN

$xR = 3(20) + 8(30)$

$x(60) = 3(20) + 8(30)$

$x = 5$ m

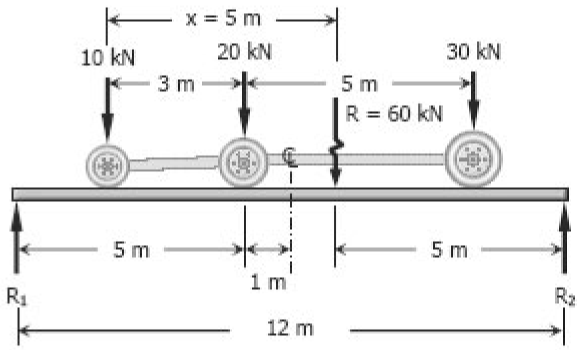
Maximum moment under 10 kN



$\Sigma M_{R2} = 0$
$12R_1 = 3.5R$
$12R_1 = 3.5(60)$
$12R_1 = 210$
$R_1 = 12.7$ kN

$M_{\text{To the left of 10 kN}} = 3.5R_1$
$= 3.5(12.7)$
$= 61.25$ kN·m

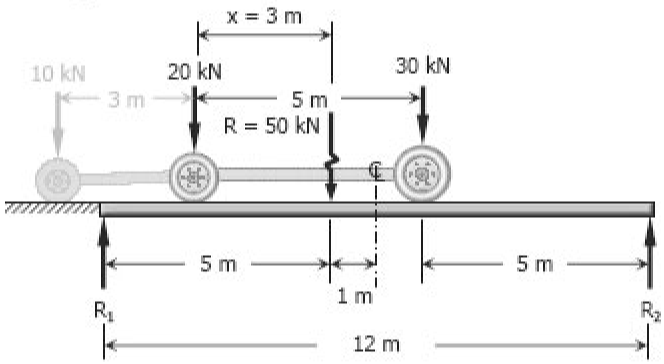
Maximum moment under 20 kN



$\Sigma M_{R2} = 0$
$12R_1 = 5R$
$12R_1 = 5(60)$
$R_1 = 25$ kN

$$M_{\text{To the left of 20 kN}} = 5R_1 - 3(10)$$
$$= 5(25) - 30$$
$$= 95 \text{ kN·m}$$

When the centerline of the beam is midway between reaction R = 60 kN and 30 kN, the 10 kN comes off the span.



$R = 20 + 30$
$R = 50$ kN

$xR = 5(30)$
$x(50) = 150$
$x = 3$ m from 20 kN

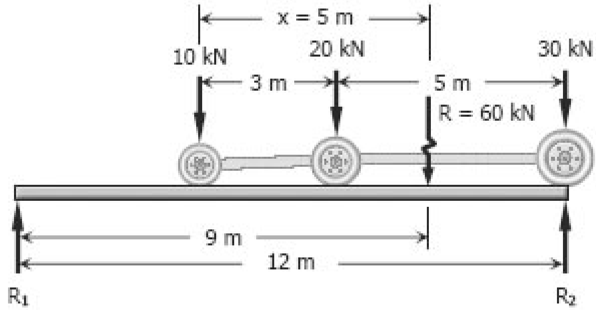
$\Sigma M_{R1} = 0$
$12R_2 = 5R$
$12R_2 = 5(50)$
$R_2 = 20.83$ kN

$$M_{\text{To the right of 30 kN}} = 5R_2$$
$$= 5(20.83)$$
$$= 104.17 \text{ kN·m}$$

Thus, the maximum moment will occur when only the 20 and 30 kN loads are on the span.

$M_{max} = M_{\text{To the right of 30 kN}}$
$M_{max} = \mathbf{104.17 \text{ kN·m}}$

The maximum shear will occur when the three loads are on the span and the 30 kN load is directly over the support.



$\Sigma M_{R1} = 0$

$12R_2 = 9R$

$12R_2 = 9(60)$

$R_2 = 45$ kN

Thus, V_{max} = **45 kN**

Stresses in Beams

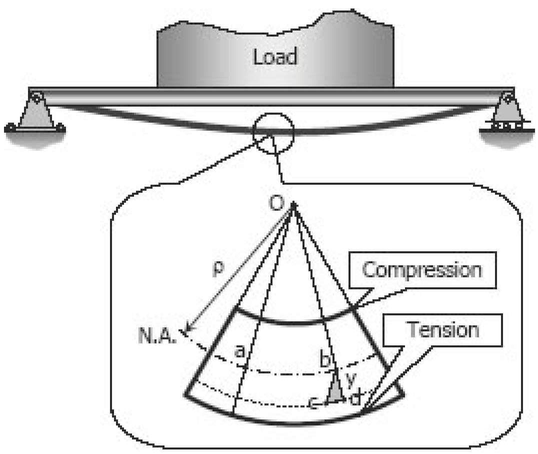
Forces and couples acting on the beam cause bending (flexural stresses) and shearing stresses on any cross section of the beam and deflection perpendicular to the longitudinal axis of the beam. If couples are applied to the ends of the beam and no forces act on it, the bending is said to be pure bending. If forces produce the bending, the bending is called ordinary bending.

ASSUMPTIONS

In using the following formulas for flexural and shearing stresses, it is assumed that a plane section of the beam normal to its longitudinal axis prior to loading remains plane after the forces and couples have been applied, and that the beam is initially straight and of uniform cross section and that the moduli of elasticity in tension and compression are equal.

Flexure Formula

Stresses caused by the bending moment are known as flexural or bending stresses. Consider a beam to be loaded as shown.



Consider a fiber at a distance y from the neutral axis, because of the beam's curvature, as the effect of bending moment, the fiber is stretched by an amount of cd. Since the curvature of the beam is very small, bcd and Oba are considered as similar triangles. The strain on this fiber is

$$\varepsilon = \frac{cd}{ab} = \frac{y}{\rho}$$

By Hooke's law, $\varepsilon = \sigma / E$, then

$$\frac{\sigma}{E} = \frac{y}{\rho};\ \sigma = \frac{y}{\rho}E$$

which means that the stress is proportional to the distance y from the neutral axis.