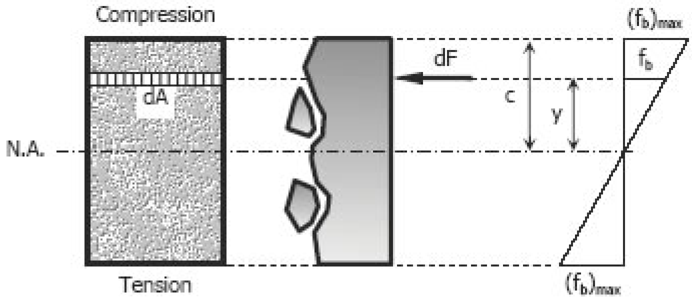
For this chapter, the notation f_b will be used instead of σ, to denote flexural stresses.



Considering a differential area dA at a distance y from N.A., the force acting over the area is

$$dF = f_b\,dA = \frac{y}{\rho} E\,dA = \frac{E}{\rho} y\,dA$$

The resultant of all the elemental moment about N.A. must be equal to the bending moment on the section.

$$M = \int y\,dF = \int y \frac{E}{\rho} y\,dA$$

$$M = \frac{E}{\rho} \int y^2\,dA$$

but $\int y^2\,dA = I,$ then

$$M = \frac{EI}{\rho} \text{ or } \rho = \frac{EI}{M}$$

substituting $\rho = Ey / f_b$

$$\frac{Ey}{f_b} = \frac{EI}{M}$$

then

$$f_b = \frac{My}{I}$$

and

$$(f_b)_{max} = \frac{Mc}{I}$$

The bending stress due to beams curvature is

$$f_b = \frac{Mc}{I} = \frac{\frac{EI}{\rho}c}{I}$$

$$f_b = \frac{Ec}{\rho}$$

The beam curvature is:

$$k = 1 / \rho$$

where ρ is the radius of curvature of the beam in mm (in), M is the bending moment in N·mm (lb·in), f_b is the flexural stress in MPa (psi), I is the centroidal moment of inertia in mm^4 (in^4), and c is the distance from the neutral axis to the outermost fiber in mm (in).

SECTION MODULUS

In the formula

$$(f_b)_{max} = \frac{Mc}{I} = \frac{M}{I/c},$$

the ratio I/c is called the section modulus and is usually denoted by S with units of mm^3 (in^3). The maximum bending stress may then be written as

$$(f_b)_{max} = \frac{M}{S}$$

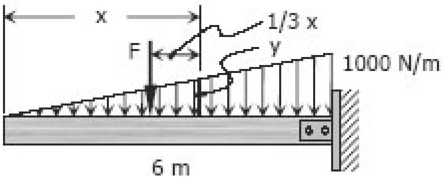
This form is convenient because the values of S are available in handbooks for a wide range of standard structural shapes.

Solved Problems in Flexure Formula

Problem 503

A cantilever beam, 50 mm wide by 150 mm high and 6 m long, carries a load that varies uniformly from zero at the free end to 1000 N/m at the wall. (a) Compute the magnitude and location of the maximum flexural stress. (b) Determine the type and magnitude of the stress in a fiber 20 mm from the top of the beam at a section 2 m from the free end.

Solution 503



$$M = F\left(\tfrac{1}{3}x\right)$$

$$\frac{y}{x} = \frac{1000}{6}$$

$$y = \tfrac{500}{3}x$$

$$F = \tfrac{1}{2}xy$$

$$F = \tfrac{1}{2}x\left(\tfrac{500}{3}x\right)$$

$$F = \tfrac{250}{3}x^2$$

thus

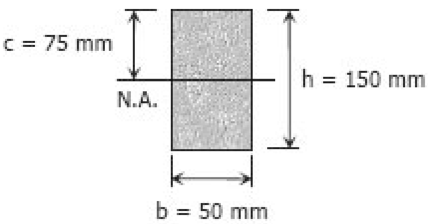
$$M = \tfrac{250}{3}x^2\left(\tfrac{1}{3}x\right)$$

$$M = \tfrac{250}{9}x^3$$

(a) The maximum moment occurs at the support (the wall) or at $x = 6$ m.

$$M = \tfrac{250}{9}x^3 = \tfrac{250}{9}(6^3)$$

$$= 6000 \text{ N·m}$$

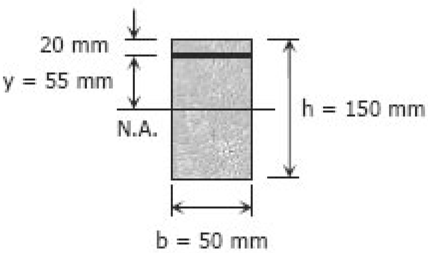


$$(f_b)_{max} = \frac{Mc}{I} = \frac{Mc}{\frac{bh^3}{12}}$$

$$(f_b)_{max} = \frac{6000(1000)(75)}{\frac{50(150)^3}{12}}$$

$$(f_b)_{max} = \textbf{32 MPa}$$

(b) At a section 2 m from the free end or at $x = 2$ m at fiber 20 mm from the top of the beam:



$$M = \tfrac{250}{9}x^3 = \tfrac{250}{9}(2)^3$$

$$M = \tfrac{2000}{9} \text{ N·m}$$

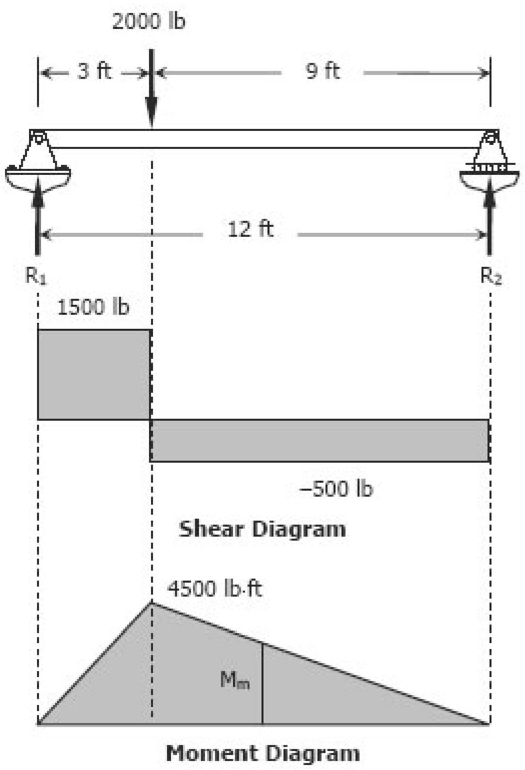
$$f_b = \frac{My}{I} = \frac{\left(\frac{2000}{9}\right)(1000)(55)}{\frac{50(150)^3}{12}}$$

$$f_b = 0.8691 \text{ MPa} = \textbf{869.1 kPa}$$

Problem 504

A simply supported beam, 2 in wide by 4 in high and 12 ft long is subjected to a concentrated load of 2000 lb at a point 3 ft from one of the supports. Determine the maximum fiber stress and the stress in a fiber located 0.5 in from the top of the beam at midspan.

Solution 504



$\Sigma M_{R2} = 0$

$12R_1 = 9(2000)$

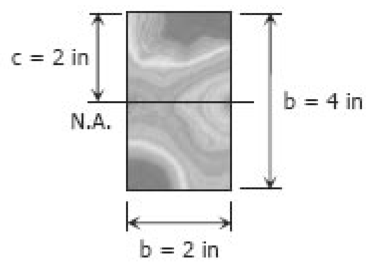
$R_1 = 1500 \text{ lb}$

$\Sigma M_{R1} = 0$

$12R_2 = 3(2000)$

$R_2 = 500 \text{ lb}$

Maximum fiber stress:



$$(f_b)_{max} = \frac{Mc}{I} = \frac{4500(12)(2)}{\frac{2(4)^3}{12}}$$

$$(f_b)_{max} = \mathbf{10{,}125\ psi}$$

Stress in a fiber located 0.5 in from the top of the beam at midspan:

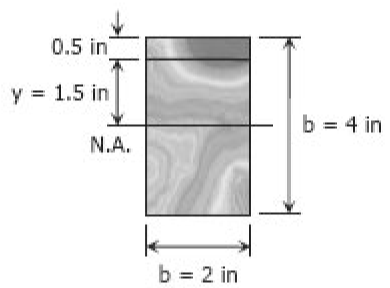
$$\frac{M_m}{6} = \frac{4500}{9}$$

$$M_m = 3000 \text{ lb·ft}$$

$$f_b = \frac{My}{I}$$

$$f_b = \frac{3000(12)(1.5)}{\frac{2(4^3)}{12}}$$

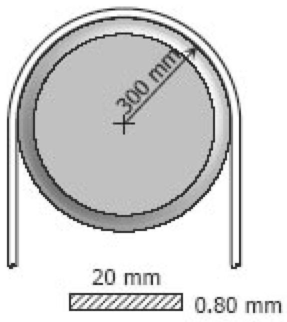
$$f_b = \mathbf{5{,}062.5\ psi}$$



Problem 505

A high strength steel band saw, 20 mm wide by 0.80 mm thick, runs over pulleys 600 mm in diameter. What maximum flexural stress is developed? What minimum diameter pulleys can be used without exceeding a flexural stress of 400 MPa? Assume E = 200 GPa.

Solution 505



Flexural stress developed:

$$M = \frac{EI}{\rho}$$

$$f_b = \frac{Mc}{I} = \frac{(EI/\rho)c}{I}$$

$$f_b = \frac{Ec}{\rho} = \frac{200000(0.80/2)}{300}$$

$$f_b = \mathbf{266.67\ MPa}$$

Minimum diameter of pulley:

$$f_b = \frac{Ec}{\rho}$$

$$400 = \frac{200000(0.80/2)}{\rho}$$

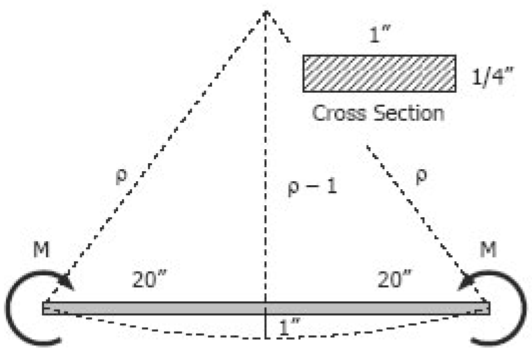
$$\rho = 200 \text{ mm}$$

$$\text{diameter, } d = \mathbf{400\ mm}$$

Problem 506

A flat steel bar, 1 inch wide by ¼ inch thick and 40 inches long, is bent by couples applied at the ends so that the midpoint deflection is 1.0 inch. Compute the stress in the bar and the magnitude of the couples. Use E = 29 × 10^6 psi.

Solution 506



$$(\rho - 1)^2 + 20^2 = \rho^2$$

$$\rho^2 - 2\rho + 1 + 400 = \rho^2$$

$$2\rho = 401$$

$$\rho = 200.5 \text{ in}$$

$$M = \frac{EI}{\rho}$$

$$f_b = \frac{Mc}{I} = \frac{(EI/\rho)c}{I}$$

$$f_b = \frac{Ec}{\rho} = \frac{(29\times10^6)(1/8)}{200.5}$$

$f_b = 18\ 079.8$ psi

$f_b = $ **18.1 ksi**

$$M = \frac{EI}{\rho} = \frac{(29\times10^6)\frac{1(1/4)^3}{12}}{200.5}$$

$M = $ **188.3 lb·in**

Problem 507

In a laboratory test of a beam loaded by end couples, the fibers at layer AB in Fig. P-507 are found to increase 60×10^{-3} mm whereas those at CD decrease 100×10^{-3} mm in the 200-mm-gage length. Using E = 70 GPa, determine the flexural stress in the top and bottom fibers.

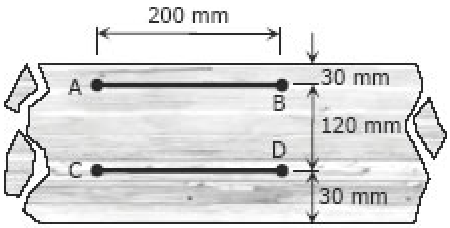
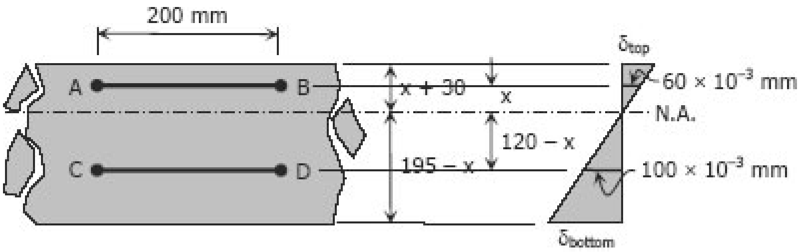


Figure P-507

Solution 507



$$\frac{x}{60\times10^{-3}} = \frac{120-x}{100\times10^{-3}}$$

$x = 0.6(120 - x)$

$x + 0.6x = 0.6(120)$

$1.6x = 72$

$x = 45$ mm

$$\frac{\delta_{top}}{x+30} = \frac{60\times10^{-3}}{x}$$

$$\delta_{top} = \frac{60\times10^{-3}}{45}(45+30)$$

δ_{top} = 0.1 mm lengthening

$$\frac{\delta_{bottom}}{195-x} = \frac{100\times10^{-3}}{120-x}$$

$$\delta_{bottom} = \frac{100\times10^{-3}}{120-45}(195-45)$$

δ_{bottom} = 0.2 mm shortening

From Hooke's Law

$$f_b = E\varepsilon$$

$$f_b = \frac{E\delta}{L}$$

$$(f_b)_{top} = \frac{70000(0.1)}{200}$$

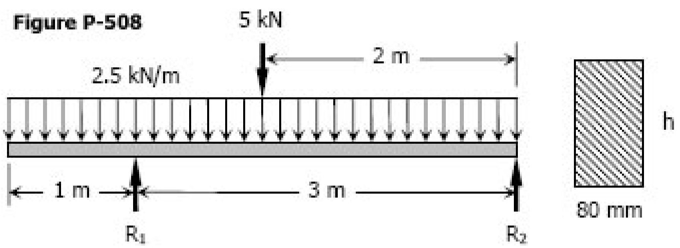
$$= \mathbf{35\ MPa\ tension}$$

$$(f_b)_{bottom} = \frac{70000(0.2)}{200}$$

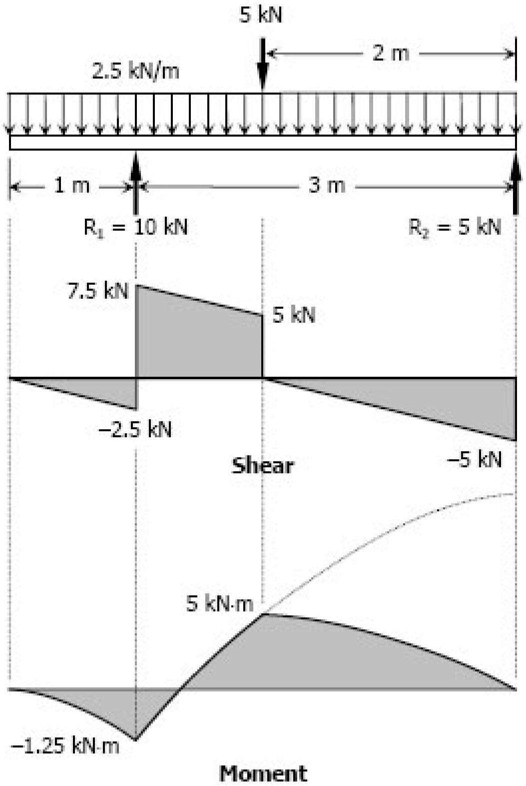
$$= \mathbf{70\ MPa\ compression}$$

Problem 508

Determine the minimum height h of the beam shown in Fig. P-508 if the flexural stress is not to exceed 20 MPa.



Solution 508



$\Sigma M_{R2} = 0$

$3R_1 = 2(5) + 2(2.5)(4)$

$R_1 = 10 \text{ kN}$

$\Sigma M_{R1} = 0$

$3R_2 = 1(5) + 1(2.5)(4)$

$R_2 = 5 \text{ kN}$

$$f_b = \frac{Mc}{I}$$

Where:

$f_b = 20 \text{ MPa}$

$M = 5 \text{ kN}\cdot\text{m}$

$= 5(1000)^2 \text{ N}\cdot\text{mm}$

$c = \frac{1}{2}h$

$$I = \frac{bh^3}{12} = \frac{80h^3}{12}$$

$$= \frac{20}{3}h^3$$

Thus,

$$20 = \frac{5(1000)^2(\frac{1}{2}h)}{\frac{20}{3}h^3}$$

$h^2 = 18\,750$

$h = 137 \text{ mm}$

Problem 509

A section used in aircraft is constructed of tubes connected by thin webs as shown in Fig. P-509. Each tube has a cross-sectional area of 0.20 in2. If the average stress in the tubes is no to exceed 10 ksi, determine the total uniformly distributed load that can be supported in a simple span 12 ft long. Neglect the effect of the webs.

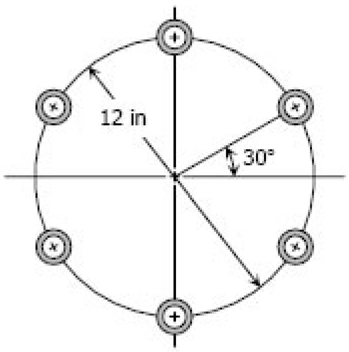
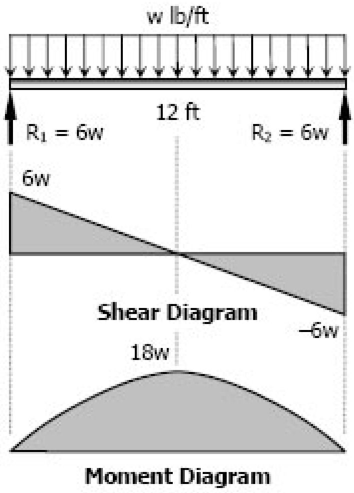


Figure P-509

Solution 509



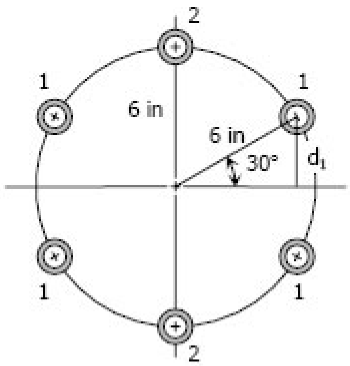
$R_1 = R_2 = \frac{1}{2}(12)(w)$
$R_1 = R_2 = 6w$

$f_b = 10$ ksi $= 10{,}000$ psi
$M = 18w$ lb·ft
$c = 6$

Centroidal moment of inertia of one tube:
$A = \pi r^2 = 0.20$
$r = 0.2523$ in → hollow portion of the tube was neglected

$$\bar{I}_x = \frac{\pi r^4}{4} = \frac{\pi(0.2523)^4}{4}$$
$$\bar{I}_x = 0.0032 \text{ in}^4$$

Moment of inertia at the center of the section:
$d_1 = 6 \sin 30° = 3$ in



$I_1 = \bar{I}_x + Ad_1^2$
$I_1 = 0.0032 + 0.2(3^2)$
$I_1 = 1.8 \text{ in}^4$

$I_2 = \bar{I}_x + Ad_2^2$
$I_2 = 0.0032 + 0.2(6^2)$
$I_2 = 7.2 \text{ in}^4$

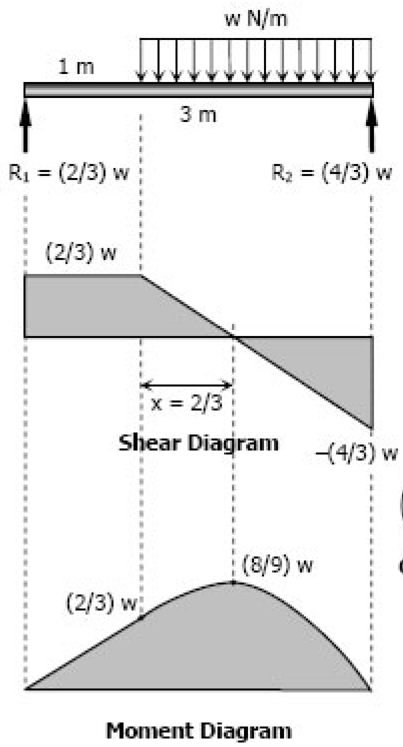
$I = 4I_1 + 2I_2 = 4(1.8) + 2(7.2)$
$I = 21.6 \text{ in}^4$

$$f_b = \frac{Mc}{I}$$
$$10{,}000 = \frac{18w(12)(6)}{21.6}$$
$w = \mathbf{166.7}$ **lb/ft**

Problem 510

A 50-mm diameter bar is used as a simply supported beam 3 m long. Determine the largest uniformly distributed load that can be applied over the right two-thirds of the beam if the flexural stress is limited to 50 MPa.

Solution 510



$\Sigma M_{R1} = 0$

$3R_2 = 2w(2)$

$R_2 = \frac{4}{3}w$

$\Sigma M_{R2} = 0$

$3R_1 = 2w(1)$

$R_1 = \frac{2}{3}w$

$$(f_b)_{max} = \frac{Mc}{I}$$

where $(f_b)_{max} = 50$ MPa

$M = \frac{8}{9}w$ N·m

$c = 25$ mm

$$I = \frac{\pi r^4}{4} = \frac{\pi(25)^4}{4}$$

$I = 97656.25\pi$ mm^4

$$50 = \frac{\frac{8}{9}w(1000)(25)}{97656.25\,\pi}$$

w = **690.29 N/m**

Problem 511

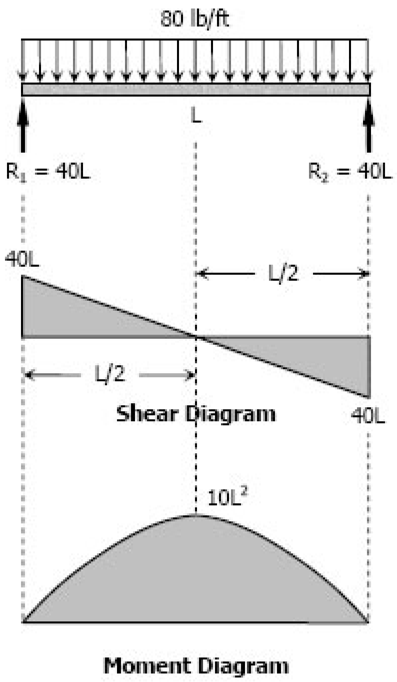
A simply supported rectangular beam, 2 in wide by 4 in deep, carries a uniformly distributed load of 80 lb/ft over its entire length. What is the maximum length of the beam if the flexural stress is limited to 3000 psi?

Solution 511

By symmetry:

$R_1 = R_2 = \frac{1}{2}(80L)$

$R_1 = R_2 = 40L$



$$(f_b)_{max} = \frac{Mc}{I}$$

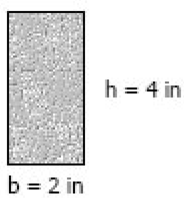
where $(f_b)_{max} = 3000$ psi

$M = 10L^2$ lb·ft

$c = h / 2 = 2$ in

$$I = \frac{bh^3}{12} = \frac{2(4)^3}{12}$$

$= \frac{32}{3}$ in^4



$$3000 = \frac{10L^2(12)(2)}{32/3}$$

L = **11.55 ft**

Problem 512

The circular bar 1 inch in diameter shown in Fig. P-512 is bent into a semicircle with a mean radius of 2 ft. If P = 400 lb and F = 200 lb, compute the maximum flexural stress developed in section a-a. Neglect the deformation of the bar.

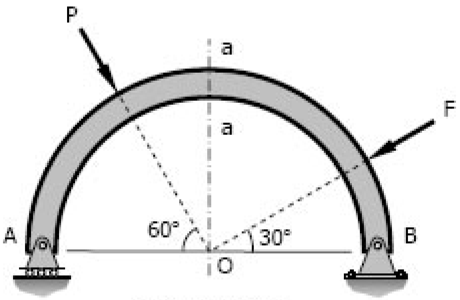
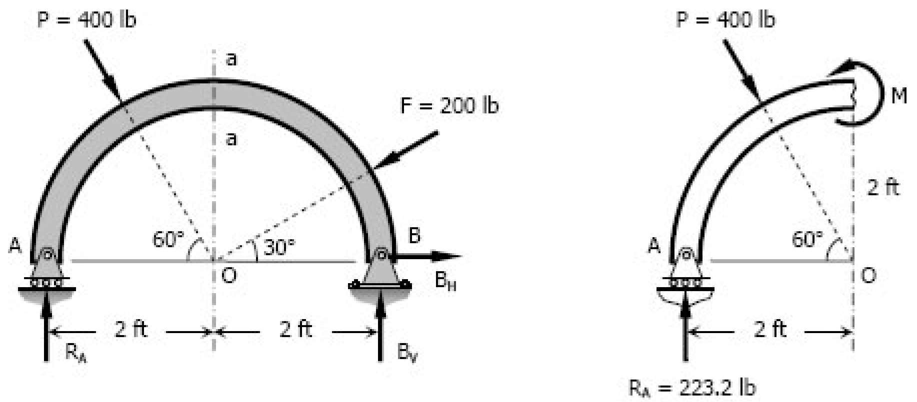


Figure P-512

Solution 512

$$\Sigma M_B = 0$$
$$4R_A = 2(400 \sin 60°) + 2(200 \sin 30°)$$
$$R_A = 223.2 \text{ lb}$$



$$M = 2(223.2) - 2(400 \cos 60°)$$
$$M = 46.4 \text{ lb·ft}$$

$$(f_b)_{max} = \frac{Mc}{I} = \frac{Mr}{\pi r^4/4}$$
$$(f_b)_{max} = \frac{4M}{\pi r^3} = \frac{4(46.4)(12)}{\pi(0.5)^3}$$
$$(f_b)_{max} = \mathbf{5671.52 \text{ psi}}$$

Problem 513

A rectangular steel beam, 2 in wide by 3 in deep, is loaded as shown in Fig. P-513. Determine the magnitude and the location of the maximum flexural stress.

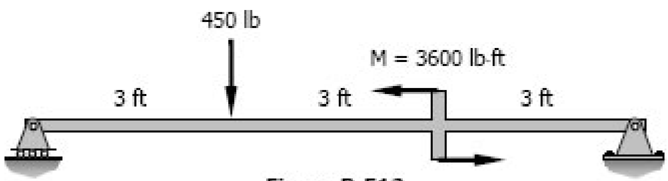
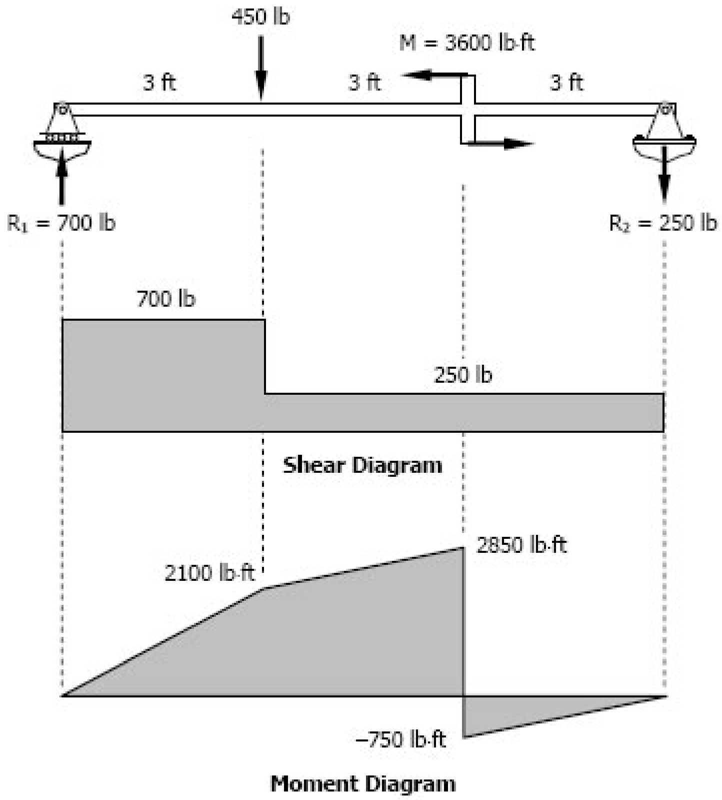


Figure P-513

Solution 513

$$\Sigma M_{R2} = 0$$
$$9R_1 = 6(450) + 3600$$
$$R_1 = 700 \text{ lb}$$

$$\Sigma M_{R1} = 0$$
$$9R_2 + 3(450) = 3600$$
$$R_2 = 250 \text{ lb}$$



$$(f_b)_{max} = \frac{Mc}{I}$$

where $M = 2850$ lb·ft

$c = h/2 = 3/2$

$= 1.5$ in

$$I = \frac{bh^3}{12} = \frac{2(3^3)}{12}$$

$= 4.5 \text{ in}^4$

$$(f_b)_{max} = \frac{2850(12)(1.5)}{4.5}$$

$(f_b)_{max} =$ **11400 psi @ 3 ft from right support**

Problem 514

The right-angled frame shown in Fig. P-514 carries a uniformly distributed loading equivalent to 200 N for each horizontal projected meter of the frame; that is, the total load is 1000 N. Compute the maximum flexural stress at section a-a if the cross-section is 50 mm square.

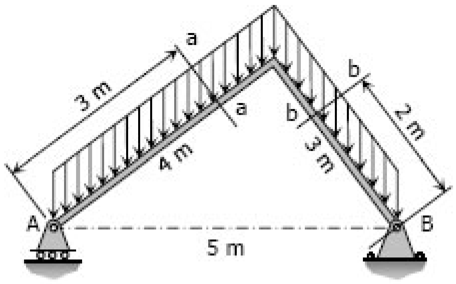


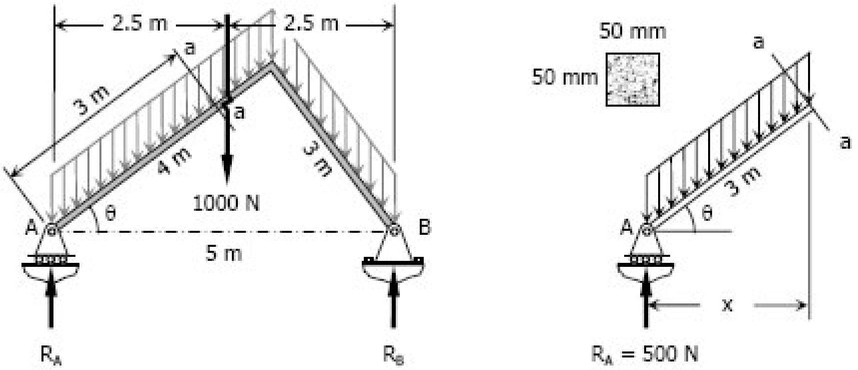
Figure P-514 and P-515

Solution 514

By symmetry

$R_A = 500$ N

$R_B = 500$ N



At section *a-a*:

$$\cos\theta = \frac{x}{3} = \frac{4}{5}$$

$x = 2.4$ m

$M = xR_A - 200x\,(x/2)$

$M = 2.4(500) - 200(2.4)(2.4/2)$

$M = 624$ N·m

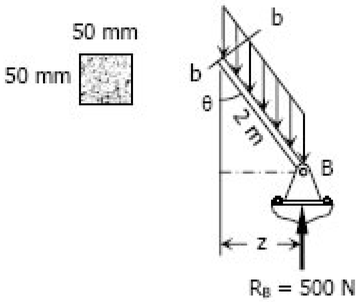
$$f_b = \frac{Mc}{I} = \frac{624(1000)(50/2)}{\dfrac{50(50^3)}{12}}$$

f_b = **29.952 MPa**

Problem 515

Repeat Prob. 524 to find the maximum flexural stress at section b-b.

Solution 515



At section *b-b*:

$$\sin\theta = \frac{z}{2} = \frac{3}{5}$$

$$z = 1.5\text{ m}$$

$$M = zR_B - 200z\,(z/2)$$

$$M = 1.5(500) - 200(1.5)(1.5/2)$$

$$M = 525\text{ N·m}$$

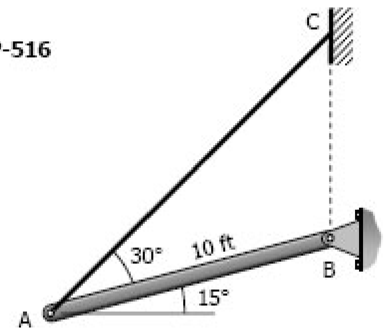
$$f_b = \frac{Mc}{I} = \frac{525(1000)(50/2)}{\dfrac{50(50)^3}{12}}$$

$$f_b = 25.2\text{ MPa}$$

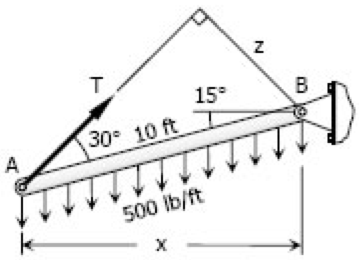
Problem 516

A timber beam AB, 6 in wide by 10 in deep and 10 ft long, is supported by a guy wire AC in the position shown in Fig. P-516. The beam carries a load, including its own weight, of 500 lb for each foot of its length. Compute the maximum flexural stress at the middle of the beam.

Figure P-516



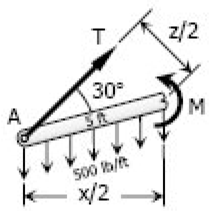
Solution 516



$x = 10\cos 15°$
$x = 9.66$ ft

$z = 10\sin 30°$
$z = 5$ ft

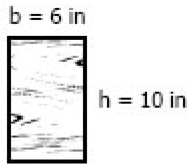
$\Sigma M_B = 0$
$zT = 500(10)(x/2)$
$5T = 500(10)(9.66/2)$
$T = 4829.63$ lb



At midspan:

$M = T(z/2) - 500(5)(x/4)$
$M = 4829.63(5/2) - 500(5)(9.66/4)$
$M = 6036.58$ lb·ft

$$f_b = \frac{Mc}{I} = \frac{M(h/2)}{\frac{bh^3}{12}}$$



$$f_b = \frac{6036.58(12)(10/2)}{\frac{6(10^3)}{12}}$$

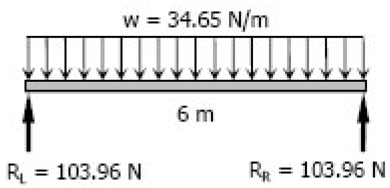
$f_b = \mathbf{724.39\ psi}$

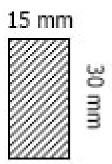
Problem 517

A rectangular steel bar, 15 mm wide by 30 mm high and 6 m long, is simply supported at its ends. If the density of steel is 7850 kg/m^3, determine the maximum bending stress caused by the weight of the bar.

Solution 517

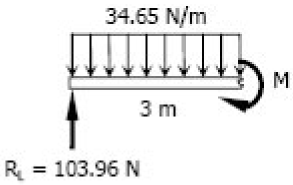
$w = (7850\ \text{kg/m}^3)(0.015\ \text{m} \times 0.03\ \text{m})$
$= (3.5325\ \text{kg/m})(9.81\ \text{m/s}^2)$
$= 34.65\ \text{N/m}$





$R_L = R_R = 6w/2$
$= 6(34.65)/2$
$= 103.96$ N

For simply supported beam subjected to uniformly distributed load, the maximum moment will occur at the midspan. At midspan:



$$M = 3(103.96) - 34.65(3)(3/2)$$

$$M = 155.955 \text{ N·m}$$

$$(f_b)_{max} = \frac{Mc}{I} = \frac{M(h/2)}{\frac{bh^3}{12}}$$

$$(f_b)_{max} = \frac{155.955(1000)(30/2)}{\frac{15(30^3)}{12}}$$

$$(f_b)_{max} = \mathbf{69.31\ MPa}$$

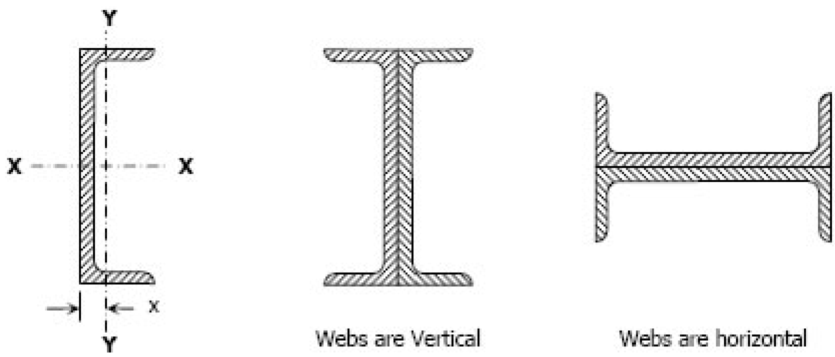
Problem 518

A cantilever beam 4 m long is composed of two C200 × 28 channels riveted back to back. What uniformly distributed load can be carried, in addition to the weight of the beam, without exceeding a flexural stress of 120 MPa if (a) the webs are vertical and (b) the webs are horizontal? Refer to Appendix B of text book for channel properties.

Solution 518

Relevant data from Appendix B, Table B-4 Properties of Channel Sections: SI Units, of text book.

Designation………	C200 × 28
Area……………….	3560 mm²
Width …………….	64 mm
$S_{X\text{-}X}$ ……………….	180×10^3 mm³
$I_{Y\text{-}Y}$ ……………….	0.825×10^6 mm⁴
x …………………..	14.4 mm



Webs are Vertical

Webs are horizontal

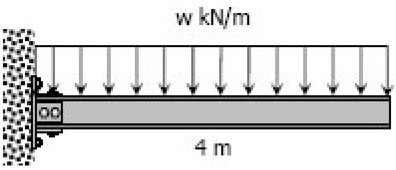
a. Webs are vertical

$$(f_b)_{max} = \frac{M}{S}$$

$$120 = \frac{M}{2(180 \times 10^3)}$$

M = 43,200,000 N·mm

M = 43.2 kN·m



From the figure:

$M = 4w(2)$

$M = 8w$

$43.2 = 8w$

w = 5.4 kN/m

w = 550.46 kg/m

w = dead load, DL + live load, LL

550.46 = 2(28) + LL

LL = **494.46 kg/m**

b. Webs are horizontal

$I_{back} = I_{Y\text{-}Y} + Ax^2$

$I_{back} = (0.825 \times 10^6) + 3560(14.4^2)$

I_{back} = 1 563 201.6 mm^4

$I = 2I_{back}$ = 2(1 563 201.6)

I = 3 126 403.2 mm^4

$$(f_b)_{max} = \frac{Mc}{I}$$

$$120 = \frac{M(64)}{3\,126\,403.2}$$

M = 5 862 006 N·mm

M = 5.862 kN·m

From the figure:

$M = 4w\ (2)$

$M = 8w$

$5.862 = 8w$

w = 0.732 75 kN/m

w = 74.69 kg/m

w = dead load, DL + live load, LL

74.69 = 2(28) + LL

LL = **18.69 kg/m**

Problem 519

A 30-ft beam, simply supported at 6 ft from either end carries a uniformly distributed load of intensity w_o over its entire length. The beam is made by welding two S18 × 70 (see appendix B of text book) sections along their flanges to form the section shown in Fig. P-519. Calculate the maximum value of wo if the flexural stress is limited to 20 ksi. Be sure to include the weight of the beam.

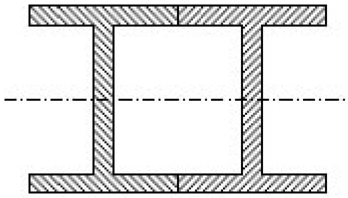


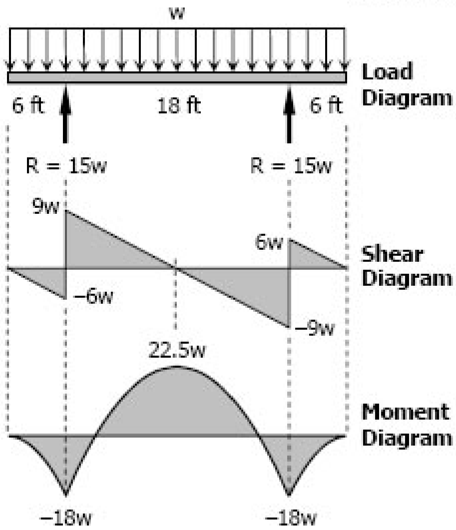
Figure P-519

Solution 519

Relevant data from Appendix B, Table B-8 Properties of I-Beam Sections (S-Shapes): US Customary Units, of text book.

Designation……… S18 × 70

S ………………….. 103 in³



$$(f_b)_{max} = \frac{M}{S}$$

$$20 = \frac{M}{2(103)}$$

$M = 4120$ kip·in

$M = \frac{1030}{3}$ kip·ft

From the moment diagram:

$M = 22.5w$

$\frac{1030}{3} = 22.5w$

$w = 15.26$ kip/ft

w = dead load, DL + live load, w_o

$15.26(1000) = 2(70) + w_o$

$w_o = 15\ 120$ lb/ft

$w_o =$ **15.12 kip/ft**

Problem 520

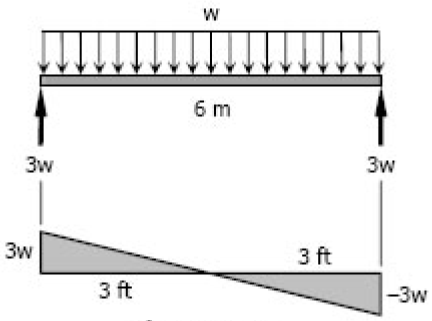
A beam with an S310 × 74 section (see Appendix B of textbook) is used as a simply supported beam 6 m long. Find the maximum uniformly distributed load that can be applied over the entire length of the beam, in addition to the weight of the beam, if the flexural stress is not to exceed 120 MPa.

Solution 520

Relevant data from Appendix B, Table B-4 Properties of I-Beam Sections (S-Shapes): SI Units, of text book.

Designation......... S310 × 74

S .... 833×10^3 mm^3



Shear Diagram

From the shear diagram:

$M_{max} = \frac{1}{2}(3)(3w)$

$M_{max} = 4.5w$ N·m

$(f_b)_{max} = \dfrac{M}{S}$

$120 = \dfrac{4.5w(1000)}{833 \times 10^3}$

$w = 22{,}213.33$ N/m

$w = 2{,}264.36$ kg/m

$w = DL + LL$

$2\,264.36 = 74 + LL$

$LL = 2{,}190.36$ kg/m

$LL = 21.5$ kN/m

Problem 521

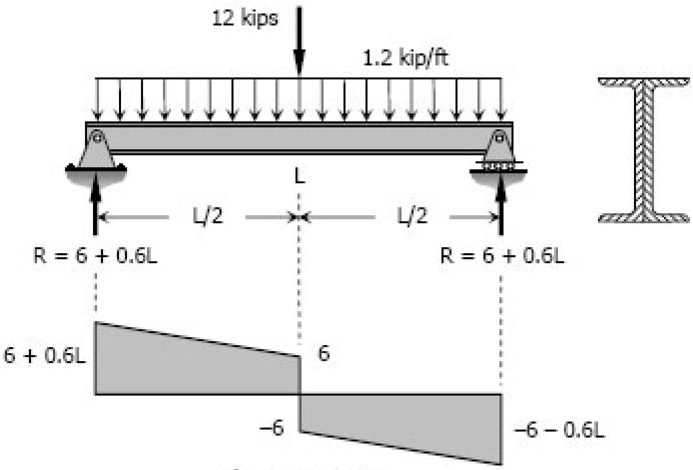
A beam made by bolting two C10 × 30 channels back to back, is simply supported at its ends. The beam supports a central concentrated load of 12 kips and a uniformly distributed load of 1200 lb/ft, including the weight of the beam. Compute the maximum length of the beam if the flexural stress is not to exceed 20 ksi.

Solution 521

Relevant data from Appendix B, Table B-9 Properties of Channel Sections: US Customary Units, of text book.

Designation......... C10 × 30

S .... 20.7 in³



Shear Diagram

From the shear diagram:

$$M_{max} = \frac{1}{2}[(6 + 0.6L) + 6](L/2)$$

$$M_{max} = 3L + 0.15L^2$$

$$(f_b)_{max} = \frac{M}{S}$$

$$20(1000) = \frac{(3L + 0.15L^2)(1000)(12)}{2(20.7)}$$

$$0.15L^2 + 3L - 69 = 0$$

$L = 13.66$ and -33.66 (meaningless)

Use L = **13.66 ft**

Problem 522

A box beam is composed of four planks, each 2 inches by 8 inches, securely spiked together to form the section shown in Fig. P-522. Show that I_{NA} = 981.3 in^4. If w_o = 300 lb/ft, find P to cause a maximum flexural stress of 1400 psi.

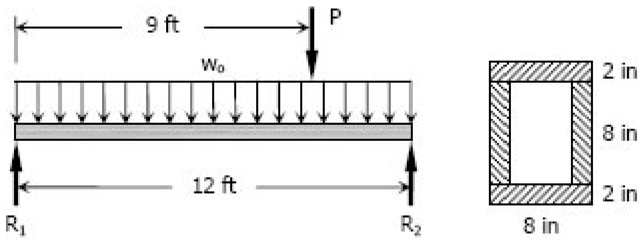
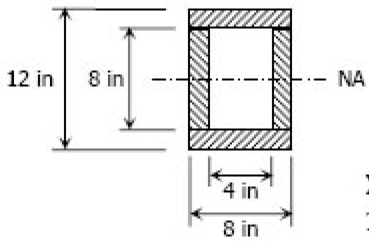


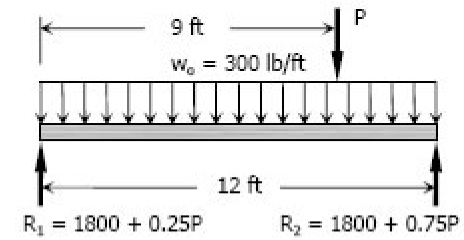
Figure P-522 and P-523

Solution 522



$$I_{NA} = \frac{8(12^3)}{12} - \frac{4(8^3)}{12}$$

$$I_{NA} = 981.33 \text{ in}^4$$



$$\Sigma M_{R2} = 0$$

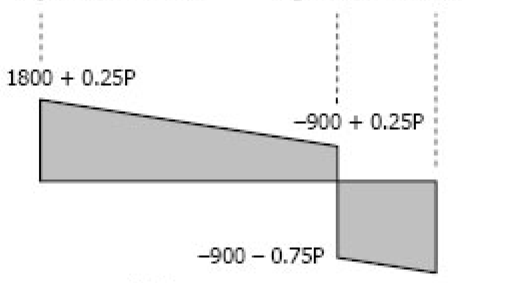
$$12R_1 = 300(12)(6) + 3P$$

$$R_1 = 1800 + 0.25P$$

$$\Sigma M_{R1} = 0$$

$$12R_2 = 300(12)(6) + 9P$$

$$R_2 = 1800 + 0.75P$$



Assumed Shear Diagram

$$M = \tfrac{1}{2}[(1800 + 0.25P) + (-900 + 0.25P)](9)$$

$$M = 4050 + 2.25P \text{ lb·ft}$$

$$(f_b)_{max} = \frac{Mc}{I}$$

$$1400 = \frac{(4050 + 2.25P)(6)(12)}{981.33}$$

$$P = 6680.63 \text{ lb}$$

Check if the shear at P is positive as assumed

$$-900 + 0.25P = -900 + 0.25(6680.63)$$

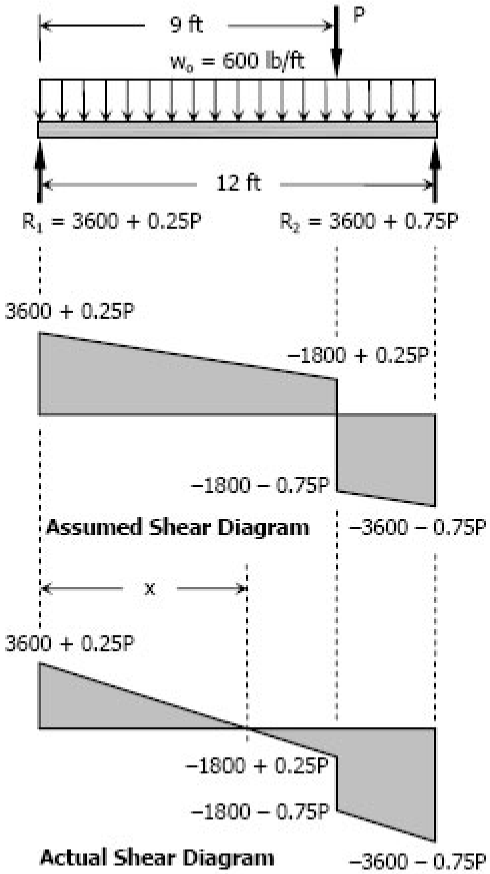
$$= 770.16 \text{ lb} \quad (ok!)$$

Thus, P = **6680.63 lb**

Problem 523

Solve Prob. 522 if w_o = 600 lb/ft.

Solution 523



$\Sigma M_{R2} = 0$

$12R_1 = 600(12)(6) + 3P$

$R_1 = 3600 + 0.25P$

$\Sigma M_{R1} = 0$

$12R_2 = 600(12)(6) + 9P$

$R_2 = 3600 + 0.75P$

$M = \frac{1}{2}[(3600 + 0.25P)$
$\quad + (-1800 + 0.25P)](9)$

$M = 8100 + 2.25P \text{ lb·ft}$

$$(f_b)_{max} = \frac{Mc}{I}$$

$$1400 = \frac{(8100 + 2.25P)(6)(12)}{981.33}$$

$P = 4880.63 \text{ lb}$

Check if the shear at P is positive as assumed

$-1800 + 0.25P$
$\quad = -1800 + 0.25(4880.63)$
$\quad = -579.84 \text{ lb}$ (*not ok!*)

From the actual shear diagram:

$(3600 + 0.25P) - 600x = 0$

$$x = \frac{3600 + 0.25P}{600}$$

$$M_{max} = \tfrac{1}{2}x\,(3600 + 0.25P)$$

$$M_{max} = \tfrac{1}{2}\left(\frac{3600 + 0.25P}{600}\right)(3600 + 0.25P)$$

$$M_{max} = \frac{(3600 + 0.25P)^2}{1200}$$

$$(f_b)_{max} = \frac{Mc}{I}$$

$$1400 = \frac{\dfrac{(3600 + 0.25P)^2}{1200}(6)(12)}{981.33}$$

$22\,897\,700 = (3600 + 0.25P)^2$

$P =$ **4740.62 lb**

Problem 524

A beam with an S380 &times 74 section carries a total uniformly distributed load of 3W and a concentrated load W, as shown in Fig. P-524. Determine W if the flexural stress is limited to 120 MPa.

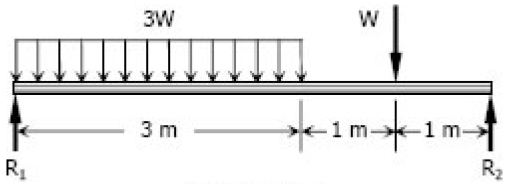
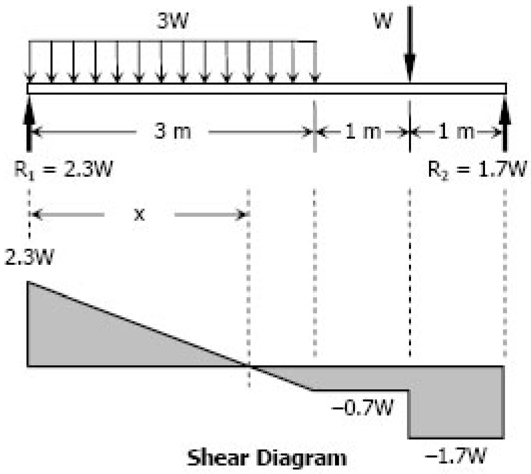


Figure P-524

Solution 524



Shear Diagram

$\Sigma M_{R2} = 0$
$5R_1 = 3W(3.5) + W(1)$
$R_1 = 2.3W$

$\Sigma M_{R1} = 0$
$5R_2 = 3W(1.5) + W(4)$
$R_2 = 1.7W$

$2.3W - Wx = 0$
$x = 2.3\text{ m}$

$M_{max} = \frac{1}{2}x\,(2.3W)$
$M_{max} = \frac{1}{2}(2.3)(2.3W)$
$M_{max} = 2.645W$

From Appendix B, Table B-3 Properties of I-Beam Sections (S-Shapes): SI Units, of text book.

Designation................ S380 × 74
S.............................. $1\,060 \times 10^3\text{ mm}^3$

$$(f_b)_{max} = \frac{M}{S}$$

$$120 = \frac{2.645W(1000)}{1060 \times 10^3}$$

$W = 48\,090.74$ **N**

Problem 525

A square timber beam used as a railroad tie is supported by a uniformly distributed loads and carries two uniformly distributed loads each totaling 48 kN as shown in Fig. P-525. Determine the size of the section if the maximum stress is limited to 8 MPa.

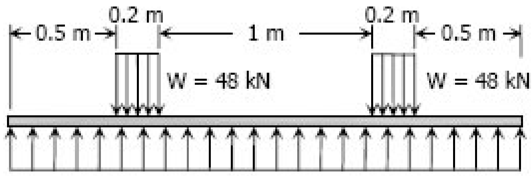
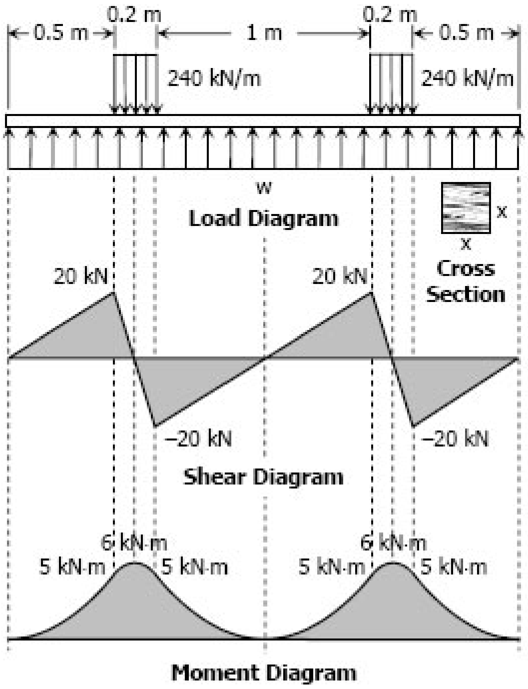


Figure P-524

Solution 525



$\Sigma F_V = 0$

$2.4w = 240(0.2) + 240(0.2)$

$w = 40 \text{ kN/m}$

$$(f_b)_{max} = \frac{Mc}{I}$$

Where: $f_b = 8 \text{ MPa}$

$M = 6 \text{ kN·m}$

$c = \frac{1}{2}x$

$$I = \frac{bh^3}{12} = \frac{x(x^3)}{12} = \frac{1}{12}x^4$$

$$8 = \frac{6(\frac{1}{2}x)(1000^2)}{\frac{1}{12}x^4}$$

$x^3 = 4\ 500\ 000$

$x = 165.1$ **mm square**

Problem 526

A wood beam 6 in wide by 12 in deep is loaded as shown in Fig. P-526. If the maximum flexural stress is 1200 psi, find the maximum values of w_o and P which can be applied simultaneously?

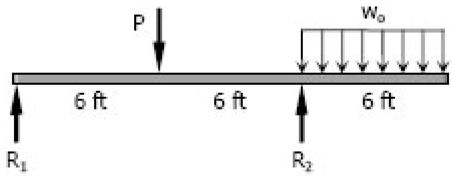
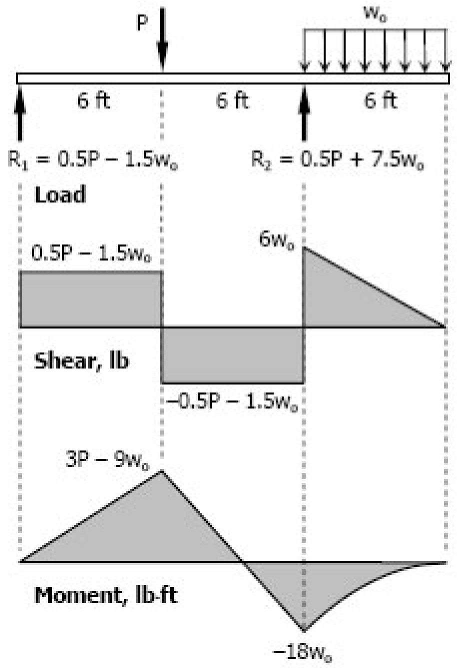


Figure P-526 and P-527

Solution 526



$\Sigma M_{R2} = 0$

$12R_1 + 3(6w_o) = 6P$

$R_1 = 0.5P - 1.5w_o$

$\Sigma M_{R1} = 0$

$12R_2 = 6P + 15(6w_o)$

$R_2 = 0.5P + 7.5w_o$

$$(f_b)_{max} = \frac{Mc}{I}$$

Where: $f_b = 1200$ psi

$c = \frac{1}{2}h = \frac{1}{2}(12) = 6$ in

$$I = \frac{bh^3}{12} = \frac{6\,(12^3)}{12}$$

$= 864\text{ in}^4$

For moment at R_2:

$$1200 = \frac{18w_o(6)(12)}{864}$$

w_o = **800 lb/ft**

For moment under P:

$$1200 = \frac{(3P - 9w_o)(6)(12)}{864}$$

$14\,400 = 3P - 9w_o$

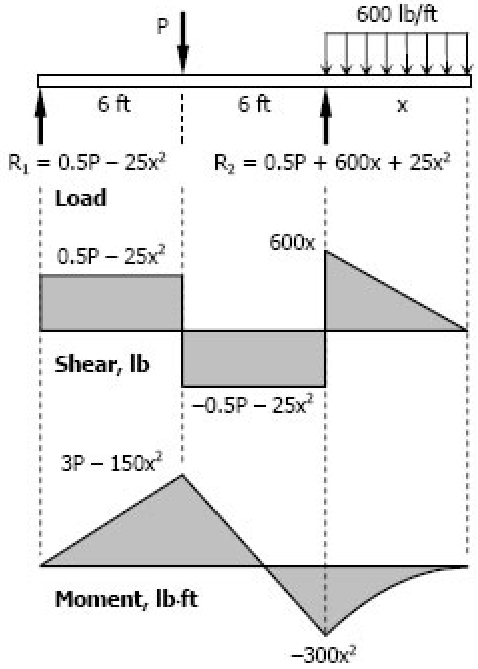
$14\,400 = 3P - 9(800)$

P = **7200 lb**

Problem 527

In Prob. 526, if the load on the overhang is 600 lb/ft and the overhang is x ft long, find the maximum values of P and x that can be used simultaneously.

Solution 527



$\Sigma M_{R2} = 0$

$12R_1 + 600x\,(x/2) = 6P$

$R_1 = 0.5P - 25x^2$

$12R_2 = 6P + 600x\,(12 + \frac{1}{2}x)$

$R_2 = 0.5P + 600x + 25x^2$

$(f_b)_{max} = \dfrac{Mc}{I}$

Refer to Solution 526 for values of c and I.

For moment at R_2:

$1200 = \dfrac{(300x^2)(6)(12)}{864}$

$x^2 = 48$

$x = \mathbf{6.93\ ft}$

For moment under P:

$1200 = \dfrac{(3P - 150x^2)(6)(12)}{864}$

$14\,400 = 3P - 150x^2$

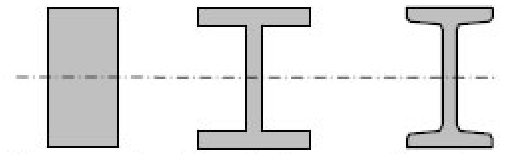
$14\,400 = 3P - 150(6.93^2)$

$P = \mathbf{7\,201.245\ lb}$

Economic Sections

From the flexure formula $f_b = My / I$, it can be seen that the bending stress at the neutral axis, where y = 0, is zero and increases linearly outwards. This means that for a rectangular or circular section a large portion of the cross section near the middle section is understressed.

For steel beams or composite beams, instead of adopting the rectangular shape, the area may be arranged so as to give more area on the outer fiber and maintaining the same overall depth, and saving a lot of weight.



When using a wide flange or I-beam section for long beams, the compression flanges tend to buckle horizontally sidewise. This buckling is a column effect, which may be prevented by providing lateral support such as a floor system so that the full allowable stresses may be used, otherwise the stress should be reduced. The reduction of stresses for these beams will be discussed in steel design. In selecting a structural section to be used as a beam, the resisting moment must be equal or greater than the applied bending moment. Note: $(f_b)_{max} = M/S$.

$$S_{\text{required}} \geq S_{\text{live-load}} \text{ or } S_{\text{required}} \geq \frac{M_{\text{live-load}}}{(f_b)_{\text{max}}}$$

The equation above indicates that the required section modulus of the beam must be equal or greater than the ratio of bending moment to the maximum allowable stress. A check that includes the weight of the selected beam is necessary to complete the calculation. In checking, the beams resisting moment must be equal or greater than the sum of the live-load moment caused by the applied loads and the dead-load moment caused by dead weight of the beam.

$$M_{\text{resisting}} \geq M_{\text{live-load}} + M_{\text{dead-load}}$$

Dividing both sides of the above equation by $(f_b)_{max}$, we obtain the checking equation

$$S_{\text{resisting}} \geq S_{\text{live-load}} + S_{\text{dead-load}}$$

Assume that the beams in the following problems are properly braced against lateral deflection. Be sure to include the weight of the beam itself.