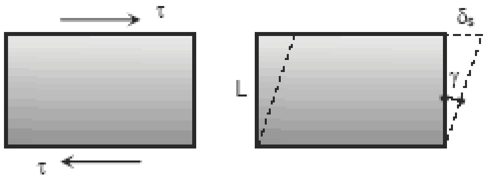
Shearing Deformation

Shearing forces cause shearing deformation. An element subject to shear does not change in length but undergoes a change in shape.



The change in angle at the corner of an original rectangular element is called the **shear strain** and is expressed as

$$\gamma = \frac{\delta_s}{L}$$

The ratio of the shear stress τ and the shear strain γ is called the **modulus of elasticity** in shear or modulus of rigidity and is denoted as G, in MPa.

$$G = \frac{\tau}{\gamma}$$

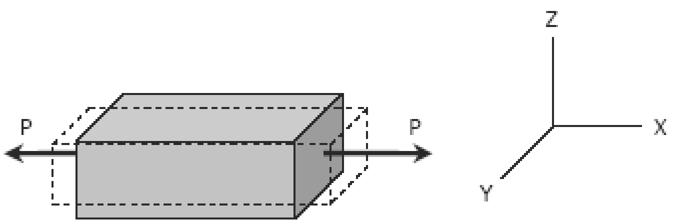
The relationship between the shearing deformation and the applied shearing force is

$$\delta_s = \frac{VL}{A_s G} = \frac{\tau L}{G}$$

where V is the shearing force acting over an area A_s.

Poisson's Ratio

When a bar is subjected to a tensile loading there is an increase in length of the bar in the direction of the applied load, but there is also a decrease in a lateral dimension perpendicular to the load. The ratio of the sidewise deformation (or strain) to the longitudinal deformation (or strain) is called the Poisson's ratio and is denoted by ν. For most steel, it lies in the range of 0.25 to 0.3, and 0.20 for concrete.



$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

where ε_x is strain in the x-direction and ε_y and ε_z are the strains in the perpendicular direction. The negative sign indicates a decrease in the transverse dimension when ε_x is positive.

BIAXIAL DEFORMATION

If an element is subjected simultaneously by ensile stresses, σ_x and σ_y, in the x and y directions, the strain in the x-direction is σ_x / E and the strain in the y direction is σ_y / E. Simultaneously, the stress in the y direction will produce a lateral contraction on the x direction of the amount -ν ε_y or -ν σ_y/E. The resulting strain in the x direction will be

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} \text{ or } \sigma_x = \frac{(\varepsilon_x + \nu\varepsilon_y)E}{1-\nu^2}$$

and

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} \text{ or } \sigma_y = \frac{(\varepsilon_y + \nu\varepsilon_x)E}{1-\nu^2}$$

TRIAXIAL DEFORMATION

If an element is subjected simultaneously by three mutually perpendicular normal stresses σ_x, σ_y, and σ_z, which are accompanied by strains ε_x, ε_y, and ε_z, respectively,

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$
$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$
$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

Tensile stresses and elongation are taken as positive. Compressive stresses and contraction are taken as negative.

Relationship Between E, G, and ν

The relationship between modulus of elasticity E, shear modulus G and Poisson's ratio ν is:

$$G = \frac{E}{2(1+\nu)}$$

Bulk Modulus of Elasticity or Modulus of Volume Expansion, K

The bulk modulus of elasticity K is a measure of a resistance of a material to change in volume without change in shape or form. It is given as

$$K = \frac{E}{3(1-2\nu)} = \frac{\sigma}{\Delta V / V}$$

where V is the volume and ΔV is change in volume. The ratio ΔV / V is called volumetric strain and can be expressed as

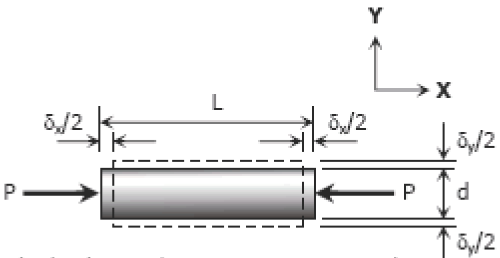
$$\frac{\Delta V}{V} = \frac{\sigma}{K} = \frac{3(1-2\nu)}{E}$$

Solved Problems in Shearing Deformation

Problem 222

A solid cylinder of diameter d carries an axial load P. Show that its change in diameter is 4Pν / πEd.

Solution 222



The load P can be compressive or tensile

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$\varepsilon_y = -\nu\varepsilon_x$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E}$$

$$\frac{\delta_y}{d} = -\nu\frac{-P}{AE}$$

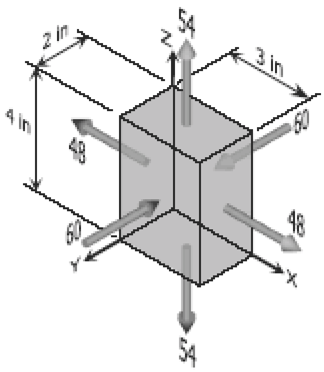
$$\delta_y = \nu\frac{Pd}{\frac{1}{4}\pi d^2 E}$$

$$\delta_y = \frac{4P\nu}{\pi E d} \qquad ok!$$

Problem 223

A rectangular steel block is 3 inches long in the x direction, 2 inches long in the y direction, and 4 inches long in the z direction. The block is subjected to a triaxial loading of three uniformly distributed forces as follows: 48 kips tension in the x direction, 60 kips compression in the y direction, and 54 kips tension in the z direction. If $\nu = 0.30$ and $E = 29 \times 10^6$ psi, determine the single uniformly distributed load in the x direction that would produce the same deformation in the y direction as the original loading.

Solution 223



For triaxial deformation (tensile triaxial stresses):
(compressive stresses are negative stresses)

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$\sigma_x = \frac{P_x}{A_{yz}} = \frac{48}{4(2)} = 6.0 \text{ ksi (tension)}$$

$$\sigma_y = \frac{P_y}{A_{xz}} = \frac{60}{4(3)} = 5.0 \text{ ksi (compression)}$$

$$\sigma_z = \frac{P_z}{A_{xy}} = \frac{54}{2(3)} = 9.0 \text{ ksi (tension)}$$

$$\varepsilon_y = \frac{1}{29 \times 10^6}\left[-5000 - 0.30(6000 + 9000)\right]$$

$$\varepsilon_y = -3.276 \times 10^{-4}$$

ε_y is negative, thus tensile force is required in the x-direction to produce the same deformation in the y-direction as the original forces.

For equivalent single force in the x-direction:
(uniaxial stress)

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$-\nu\varepsilon_x = \varepsilon_y$$

$$-\nu\frac{\sigma_x}{E} = \varepsilon_y$$

$$-0.30\left(\frac{\sigma_x}{29 \times 10^6}\right) = -3.276 \times 10^{-4}$$

$$\sigma_x = 31\,666.67 \text{ psi}$$

$$\sigma_x = \frac{P_x}{4(2)} = 31\,666.67$$

$$P_x = 253\,333.33 \text{ lb (tension)}$$

$$P_x = \mathbf{253.33 \text{ kips (tension)}}$$

Problem 224

For the block loaded triaxially as described in Prob. 223, find the uniformly distributed load that must be added in the x direction to produce no deformation in the z direction.

Solution 224

$\varepsilon_z = \dfrac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$

$\sigma_x = 6.0$ ksi (tension)

$\sigma_y = 5.0$ ksi (compression)

$\sigma_z = 9.0$ ksi (tension)

$\varepsilon_z = \dfrac{1}{29\times10^6}[9000 - 0.3(6000 - 5000)]$

$\varepsilon_z = 2.07 \times 10^{-5}$

ε_z is positive, thus positive stress is needed in the x-direction to eliminate deformation in z-direction.

The application of loads is still simultaneous:
(No deformation means zero strain)

$\varepsilon_z = \dfrac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] = 0$

$\sigma_z = \nu(\sigma_x + \sigma_y)$

$\sigma_y = 5.0$ ksi → (compression)

$\sigma_z = 9.0$ ksi → (tension)

$9000 = 0.30(\sigma_x - 5000)$

$\sigma_x = 35\ 000$ psi

$\sigma_{added} + 6000 = 35\ 000$

$\sigma_{added} = 29\ 000$ psi

$\dfrac{P_{added}}{2(4)} = 29\ 000$

$P_{added} = 232\ 000$ lb

$P_{added} = \mathbf{232\ kips}$

Problem 225

A welded steel cylindrical drum made of a 10-mm plate has an internal diameter of 1.20 m. Compute the change in diameter that would be caused by an internal pressure of 1.5 MPa. Assume that Poisson's ratio is 0.30 and E = 200 GPa.

Solution 225

σ_y = longitudinal stress

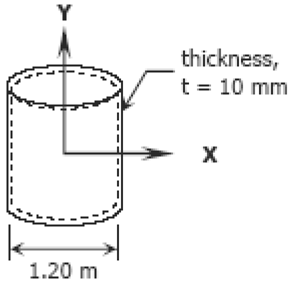
$\sigma_y = \dfrac{pD}{4t} = \dfrac{1.5(1200)}{4(10)}$

$\sigma_y = 45$ MPa

σ_x = tangential stress

$\sigma_x = \dfrac{pD}{2t} = \dfrac{1.5(1200)}{2(10)}$

$\sigma_x = 90$ MPa



$\varepsilon_x = \dfrac{\sigma_x}{E} - \nu\dfrac{\sigma_y}{E}$

$\varepsilon_x = \dfrac{90}{200\,000} - 0.3\left(\dfrac{45}{200\,000}\right)$

$\varepsilon_x = 3.825 \times 10^{-4}$

$\varepsilon_x = \dfrac{\Delta D}{D}$

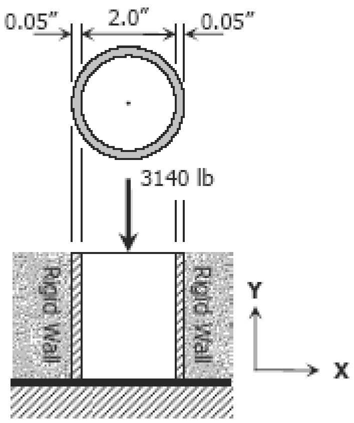
$\Delta D = \varepsilon_x D = (3.825 \times 10^{-4})(1200)$

$\Delta D = \mathbf{0.459\ mm}$

Problem 226

A 2-in.-diameter steel tube with a wall thickness of 0.05 inch just fits in a rigid hole. Find the tangential stress if an axial compressive load of 3140 lb is applied. Assume ν = 0.30 and neglect the possibility of buckling.

Solution 226



$\varepsilon_x = \dfrac{\sigma_x}{E} - \nu\dfrac{\sigma_y}{E} = 0$

$\sigma_x = \nu\sigma_y$

where σ_x = tangential stress

σ_y = longitudinal stress

$\sigma_y = \dfrac{P_y}{A} = \dfrac{3140}{\pi(2)(0.05)}$

$\sigma_y = 31400/\pi$ psi

$\sigma_x = 0.30(31400/\pi)$

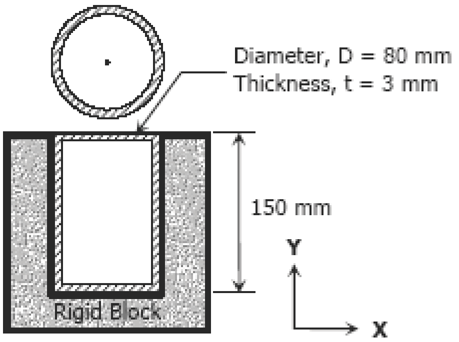
$\sigma_x = 9430/\pi$ psi

$\sigma_x = \mathbf{2298.5\ psi}$

Problem 227

A 150-mm-long bronze tube, closed at its ends, is 80 mm in diameter and has a wall thickness of 3 mm. It fits without clearance in an 80-mm hole in a rigid block. The tube is then subjected to an internal pressure of 4.00 MPa. Assuming ν = 1/3 and E = 83 GPa, determine the tangential stress in the tube.

Solution 227



Longitudinal stress:

$$\sigma_y = \frac{pD}{4t} = \frac{4(80)}{4(3)}$$

$$\sigma_y = \frac{80}{3}\ \text{MPa}$$

The strain in the x-direction is:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0$$

$\sigma_x = \nu\sigma_y =$ tangential stress

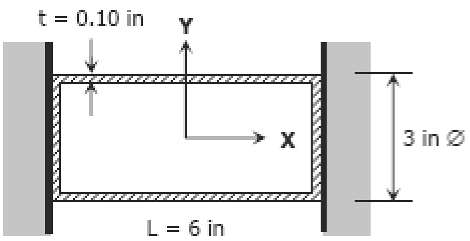
$$\sigma_x = \frac{1}{3}\left(\frac{80}{3}\right)$$

$$\sigma_x = \mathbf{8.89\ MPa}$$

Problem 228

A 6-in.-long bronze tube, with closed ends, is 3 in. in diameter with a wall thickness of 0.10 in. With no internal pressure, the tube just fits between two rigid end walls. Calculate the longitudinal and tangential stresses for an internal pressure of 6000 psi. Assume ν = 1/3 and E = 12 × 10^6 psi.

Solution 228



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0$$

$\sigma_x = \nu\sigma_y = \sigma_l \rightarrow$ longitudinal stress

$\sigma_t = \sigma_y \rightarrow$ tangential stress

$$\sigma_t = \frac{pD}{2t} = \frac{6000(3)}{2(0.10)}$$

$$\sigma_t = \mathbf{90{,}000\ psi}$$

$$\sigma_l = \nu\sigma_y = \tfrac{1}{3}(90{,}000)$$

$$\sigma_l = \mathbf{30{,}000\ psi}$$

Statically Indeterminate Members

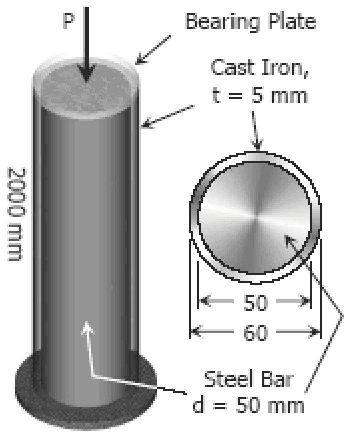
When the reactive forces or the internal resisting forces over a cross section exceed the number of independent equations of equilibrium, the structure is called **statically indeterminate**. These cases require the use of additional relations that depend on the elastic deformations in the members.

Solved Problems in Statically Indeterminate Members

Problem 233

A steel bar 50 mm in diameter and 2 m long is surrounded by a shell of a cast iron 5 mm thick. Compute the load that will compress the combined bar a total of 0.8 mm in the length of 2 m. For steel, E = 200 GPa, and for cast iron, E = 100 GPa.

Solution 233



$$\delta = \frac{PL}{AE}$$

$$\delta = \delta_{cast\ iron} = \delta_{steel} = 0.8 \text{ mm}$$

$$\delta_{cast\ iron} = \frac{P_{cast\ iron}(2000)}{[\frac{1}{4}\pi(60^2 - 50^2)](100\,000)} = 0.8$$

$$P_{cast\ iron} = 11\,000\pi \text{ N}$$

$$\delta_{steel} = \frac{P_{steel}(2000)}{[\frac{1}{4}\pi(50^2)](200\,000)} = 0.8$$

$$P_{steel} = 50\,000\pi \text{ N}$$

$$\Sigma F_V = 0$$

$$P = P_{cast\ iron} + P_{steel}$$

$$P = 11\,000\pi + 50\,000\pi$$

$$P = 61\,000\pi \text{ N}$$

$$P = \mathbf{191.64 \text{ kN}}$$