Dividing throughout by 20×10^3,

$$0.305 (r_1 + 0.15) = (r_1)^2 + 0.15\, r_1 + 0.0225$$

$$(r_1)^2 - 0.155\, r_1 - 0.0233 = 0$$

Solving this as a quadratic equation,

$$r_1 = \frac{0.155 \pm \sqrt{(0.155)^2 + 4 \times 0.0233}}{2} = \frac{0.155 \pm 0.342}{2}$$

$$= 0.2485 \text{ m} = 248.5 \text{ mm} \qquad \text{...(Taking + ve sign)}$$

$\therefore$ $d_1 = 2\, r_1 = 2 \times 248.5 = 497$ mm **Ans.**

Number of collars

Let n = Number of collars.

We know that intensity of pressure (p),

$$0.3 = \frac{W}{n\pi[(r_1)^2 - (r_2)^2]} = \frac{200 \times 10^3}{n\pi[(248.5)^2 - (150)^2]} = \frac{1.62}{n}$$

$\therefore$ $n = 1.62 / 0.3 = 5.4$ or 6 **Ans.**

10.31. Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces. In automobiles, friction clutch is used to connect the engine to the driven shaft. In operating such a clutch, care should be taken so that the friction surfaces engage easily and gradually brings the driven shaft up to proper speed. The proper alignment of the bearing must be maintained and it should be located as close to the clutch as possible. It may be noted that

1. The contact surfaces should develop a frictional force that may pick up and hold the load with reasonably low pressure between the contact surfaces.

2. The heat of friction should be rapidly dissipated and tendency to grab should be at a minimum.

3. The surfaces should be backed by a material stiff enough to ensure a reasonably uniform distribution of pressure.

The friction clutches of the following types are important from the subject point of view :

1. Disc or plate clutches (single disc or multiple disc clutch),

2. Cone clutches, and

3. Centrifugal clutches.

We shall now discuss, these clutches, in detail, in the following pages. It may be noted that the disc and cone clutches are based on the same theory as the pivot and collar bearings.

10.32. Single Disc or Plate Clutch

A single disc or plate clutch, as shown in Fig. 10.21, consists of a clutch plate whose both sides are faced with a friction material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel. Both the pressure plate and the flywheel rotate with the engine crankshaft or the driving shaft. The pressure plate pushes the clutch plate towards the flywheel by a set of strong springs which are arranged radially inside the body. The three levers (also known as release levers or fingers) are carried on pivots suspended from the case of the body. These are arranged in such a manner so that the pressure plate moves away from the flywheel by the inward movement of a thrust bearing. The bearing is mounted upon a forked shaft and moves forward when the clutch pedal is pressed.

Single disc clutch

When the clutch pedal is pressed down, its linkage forces the thrust release bearing to move in towards the flywheel and pressing the longer ends of the levers inward. The levers are forced to turn on their suspended pivot and the pressure plate moves away from the flywheel by the knife edges, thereby compressing the clutch springs. This action removes the pressure from the clutch plate and thus moves back from the flywheel and the driven shaft becomes stationary. On the other hand, when the foot is taken off from the clutch pedal, the thrust bearing moves back by the levers. This allows the springs to extend and thus the pressure plate pushes the clutch plate back towards the flywheel.

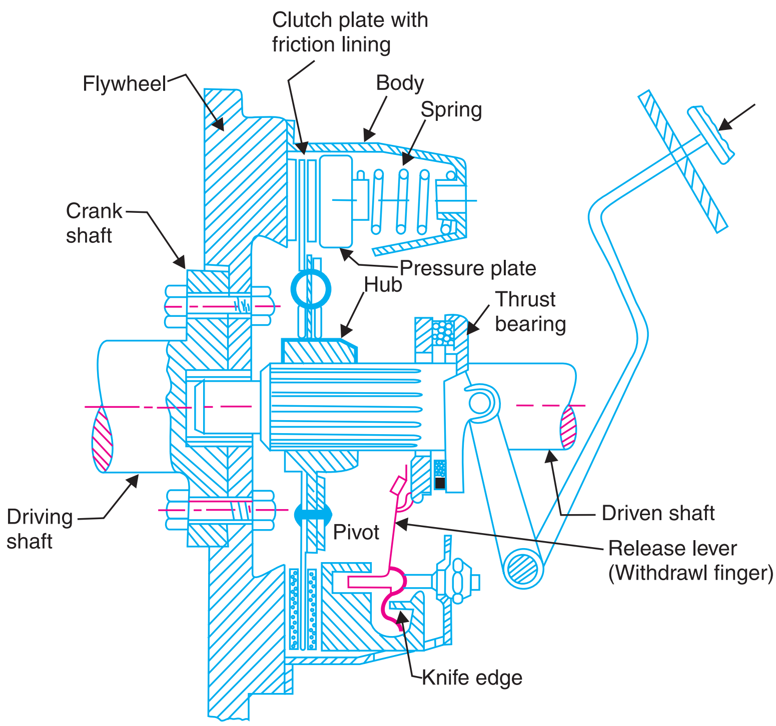


Fig. 10.21. Single disc or plate clutch.

The axial pressure exerted by the spring provides a frictional force in the circumferential direction when the relative motion between the driving and driven members tends to take place. If the torque due to this frictional force exceeds the torque to be transmitted, then no slipping takes place and the power is transmitted from the driving shaft to the driven shaft.

Now consider two friction surfaces, maintained in contact by an axial thrust W, as shown in Fig. 10.22 (*a*).

Let T = Torque transmitted by the clutch,

p = Intensity of axial pressure with which the contact surfaces are held together,

r_1 and r_2 = External and internal radii of friction faces, and

μ = Coefficient of friction.

Consider an elementary ring of radius r and thickness dr as shown in Fig. 10.22 (*b*).

We know that area of contact surface or friction surface,

$$= 2\,\pi\, r.dr$$

∴ Normal or axial force on the ring,

$$\delta W = \text{Pressure} \times \text{Area} = p \times 2\,\pi\, r.dr$$

and the frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W = \mu.p \times 2\,\pi\, r.dr$$

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p \times 2\,\pi\, r.dr \times r = 2\,\pi \times \mu\,.p.r^2\,dr$$

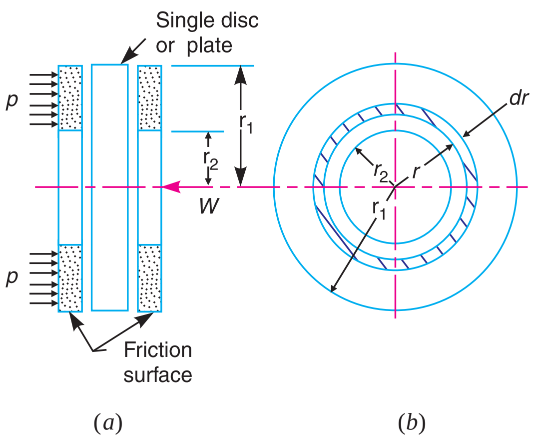


Fig. 10.22. Forces on a single disc or plate clutch.

We shall now consider the following two cases :

1. When there is a uniform pressure, and

2. When there is a uniform wear.

1. *Considering uniform pressure*

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity of pressure,

$$p = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad \ldots(i)$$

where W = Axial thrust with which the contact or friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is

$$T_r = 2\,\pi\,\mu.p.r^2\,dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque.

∴ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_1}^{r_2} 2\pi\mu.p.r^2.dr = 2\,\pi\mu\,p\left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\pi\mu\,p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p from equation **(*i*)**,

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \frac{(r_1)^3 - (r_2)^3}{3}$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu.W.R$$

where R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. *Considering uniform wear*

In Fig. 10.22, let p be the normal intensity of pressure at a distance r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$$p.r. = C \text{ (a constant)} \quad \text{or} \quad p = C/r \qquad \text{...}(i)$$

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi C.dr = 2\pi C.dr$$

∴ Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi C\,dr = 2\pi C\,[r]_{r_2}^{r_1} = 2\pi C\,(r_1 - r_2)$$

or
$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\mu.p\,r^2.dr = 2\pi\mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr$$

...($\because$ $p = C/r$)

∴ Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2] = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}\left[(r_1)^2 - (r_2)^2\right]$$

$$= \frac{1}{2} \times \mu.W\,(r_1 + r_2) = \mu.W.R$$

where R = Mean radius of the friction surface $= \dfrac{r_1 + r_2}{2}$

Notes : 1. In general, total frictional torque acting on the friction surface (or on the clutch) is given by

$$T = n.\mu.W.R$$

where n = Number of pairs of friction or contact surfaces, and

R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad \text{...(For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \qquad \text{...(For uniform wear)}$$

2. For a single disc or plate clutch, normally both sides of the disc are effective. Therefore, a single disc clutch has two pairs of surfaces in contact, *i.e.* $n = 2$.

3. Since the intensity of pressure is maximum at the inner radius (r_2) of the friction or contact surface, therefore equation ***(i)*** may be written as

$$p_{max} \times r_2 = C \quad \text{or} \quad p_{max} = C/r_2$$

4. Since the intensity of pressure is minimum at the outer radius (r_1) of the friction or contact surface, therefore equation ***(i)*** may be written as

$$p_{min} \times r_1 = C \quad \text{or} \quad p_{min} = C/r_1$$

5. The average pressure (p_{av}) on the friction or contact surface is given by

$$p_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

6. In case of a new clutch, the intensity of pressure is approximately uniform but in an old clutch the uniform wear theory is more approximate.

7. The uniform pressure theory gives a higher frictional torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

10.33. Multiple Disc Clutch

A multiple disc clutch, as shown in Fig. 10.23, may be used when a large torque is to be transmitted. The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion

Dual Disc Clutches.

(except for the last disc). The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft. The multiple disc clutches are extensively used in motor cars, machine tools etc.

Let n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

∴ Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

and total frictional torque acting on the friction surfaces or on the clutch,

$$T = n.\mu.W.R$$

where R = Mean radius of the friction surfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad \text{...(For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \qquad \text{...(For uniform wear)}$$

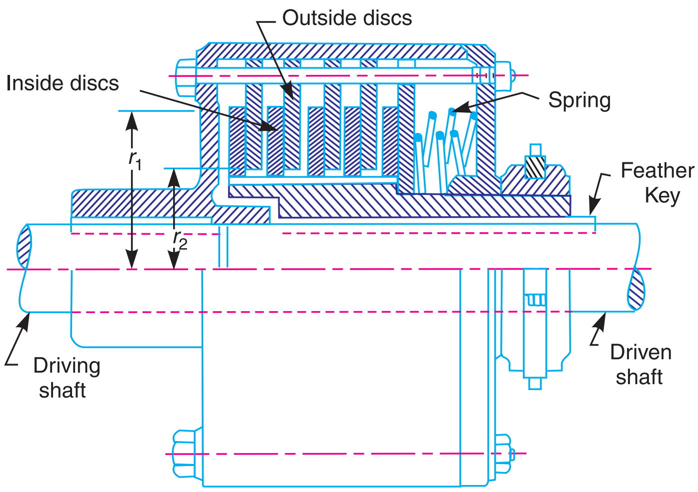


Fig. 10.23. Multiple disc clutch.

Example 10.22. *Determine the maximum, minimum and average pressure in plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.*

Solution. Given : $W = 4$ kN $= 4 \times 10^3$ N ; $r_2 = 50$ mm ; $r_1 = 100$ mm

Maximum pressure

Let p_{max} = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 50\, p_{max}$$

We know that the total force on the contact surface (W),

$$4 \times 10^3 = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 50\, p_{max}\,(100 - 50) = 15\,710\, p_{max}$$

∴ $p_{max} = 4 \times 10^3/15\,710 = 0.2546$ N/mm² **Ans.**

Minimum pressure

Let p_{min} = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), therefore

$$p_{min} \times r_1 = C \quad \text{or} \quad C = 100\, p_{min}$$

We know that the total force on the contact surface (W),

$$4 \times 10^3 = 2\,\pi\,C\,(r_1 - r_2) = 2\pi \times 100\, p_{min}\,(100 - 50) = 31\,420\, p_{min}$$

$\therefore$ $$p_{min} = 4 \times 10^3/31\,420 = 0.1273 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surfaces}}$$

$$= \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{4 \times 10^3}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Example 10.23. *A single plate clutch, with both sides effective, has outer and inner diameters 300 mm and 200 mm respectively. The maximum intensity of pressure at any point in the contact surface is not to exceed 0.1 N/mm². If the coefficient of friction is 0.3, determine the power transmitted by a clutch at a speed 2500 r.p.m.*

Solution. Given : d_1 = 300 mm or r_1 = 150 mm ; d_2 = 200 mm or r_2 = 100 mm ; p = 0.1 N/mm^2 ; μ = 0.3 ; N = 2500 r.p.m. or $\omega = 2\pi \times 2500/60 = 261.8$ rad/s

Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform wear,

$$p.r_2 = C \quad \text{or} \quad C = 0.1 \times 100 = 10 \text{ N/mm}$$

We know that the axial thrust,

$$W = 2\,\pi\,C\,(r_1 - r_2) = 2\,\pi \times 10\,(150 - 100) = 3142 \text{ N}$$

and mean radius of the friction surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 100}{2} = 125 \text{ mm} = 0.125 \text{ m}$$

We know that torque transmitted,

$$T = n.\mu.W.R = 2 \times 0.3 \times 3142 \times 0.125 = 235.65 \text{ N-m}$$

...($\because$ $n = 2$, for both sides of plate effective)

$\therefore$ Power transmitted by a clutch,

$$P = T.\omega = 235.65 \times 261.8 = 61\,693 \text{ W} = 61.693 \text{ kW} \quad \textbf{Ans.}$$

Example 10.24. *A single plate clutch, effective on both sides, is required to transmit 25 kW at 3000 r.p.m. Determine the outer and inner radii of frictional surface if the coefficient of friction is 0.255, the ratio of radii is 1.25 and the maximum pressure is not to exceed 0.1 N/mm². Also determine the axial thrust to be provided by springs. Ass ume the theory of uniform wear.*

Solution. Given: n = 2 ; P = 25 kW = 25 × 10^3 W ; N = 3000 r.p.m. or $\omega = 2\pi \times 3000/60$ = 314.2 rad/s ; μ = 0.255 ; r_1/r_2 = 1.25 ; p = 0.1 N/mm^2

Outer and inner radii of frictional surface

Let r_1 and r_2 = Outer and inner radii of frictional surfaces, and

T = Torque transmitted.

Since the ratio of radii (r_1/r_2) is 1.25, therefore

$$r_1 = 1.25\, r_2$$

We know that the power transmitted (P),

$$25 \times 10^3 = T.\omega = T \times 314.2$$

$\therefore$ $$T = 25 \times 10^3/314.2 = 79.6 \text{ N-m} = 79.6 \times 10^3 \text{ N-mm}$$

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p.r_2 = C \qquad \text{or} \qquad C = 0.1\, r_2 \text{ N/mm}$$

and the axial thrust transmitted to the frictional surface,

$$W = 2\pi C (r_1 - r_2) = 2\pi \times 0.1\, r_2 (1.25\, r_2 - r_2) = 0.157\, (r_2)^2 \quad \text{...}(i)$$

We know that mean radius of the frictional surface for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\, r_2 + r_2}{2} = 1.125\, r_2$$

We know that torque transmitted (T),

$$79.6 \times 10^3 = n.\mu.W.R = 2 \times 0.255 \times 0.157\, (r_2)^2 \times 1.125\, r_2 = 0.09\, (r_2)^3$$

$\therefore$ $(r_2)^3 = 79.6 \times 10^3/0.09 = 884 \times 10^3$ or $r_2 = 96$ mm **Ans.**

and $r_1 = 1.25\, r_2 = 1.25 \times 96 = 120$ mm **Ans.**

Axial thrust to be provided by springs

We know that axial thrust to be provided by springs,

$$W = 2\pi C (r_1 - r_2) = 0.157\, (r_2)^2 \quad \text{...[From equation } (i)]$$
$$= 0.157\, (96)^2 = 1447 \text{ N} \textbf{ Ans.}$$

Example 10.25. *A single dry plate clutch transmits 7.5 kW at 900 r.p.m. The axial pressure is limited to 0.07 N/mm². If the coefficient of friction is 0.25, find* **1.** *Mean radius and face width of the friction lining assuming the ratio of the mean radius to the face width as 4, and* **2.** *Outer and inner radii of the clutch plate.*

Solution. Given : $P = 7.5$ kW $= 7.5 \times 10^3$ W ; $N = 900$ r.p.m or $\omega = 2\pi \times 900/60 = 94.26$ rad/s ; $p = 0.07$ N/mm² ; $\mu = 0.25$

1. Mean radius and face width of the friction lining

Let R = Mean radius of the friction lining in mm, and

w = Face width of the friction lining in mm,

Ratio of mean radius to the face width,

$$R/w = 4 \quad \text{...(Given)}$$

We know that the area of friction faces,

$$A = 2\pi R.w$$

$\therefore$ Normal or the axial force acting on the friction faces,

$$W = A \times p = 2\pi R.w.p$$

We know that torque transmitted (considering uniform wear),

$$T = n.\mu.W.R = n.\mu\,(2\pi R.w.p)\,R$$

$$= n.\mu\left(2\pi R \times \frac{R}{4} \times p\right) R = \frac{\pi}{2} \times n.\mu.p.R^3 \quad \text{...}(\because w = R/4)$$

$$= \frac{\pi}{2} \times 2 \times 0.25 \times 0.07\, R^3 = 0.055\, R^3 \text{ N-mm} \quad \text{...}(i)$$

...($\because$ $n = 2$, for single plate clutch)

We also know that power transmitted (P),

$$7.5 \times 10^3 = T.\omega = T \times 94.26$$

$$\therefore \quad T = 7.5 \times 10^3/94.26 = 79.56 \text{ N-m} = 79.56 \times 10^3 \text{ N-mm} \quad ...(ii)$$

From equations **(*i*)** and **(*ii*),**

$$R^3 = 79.56 \times 10^3/0.055 = 1446.5 \times 10^3 \text{ or } R = 113 \text{ mm } \textbf{Ans.}$$

and $$w = R/4 = 113/4 = 28.25\text{mm } \textbf{Ans.}$$

2. Outer and inner radii of the clutch plate

Let r_1 and r_2 = Outer and inner radii of the clutch plate respectively.

Since the width of the clutch plate is equal to the difference of the outer and inner radii, therefore

$$w = r_1 - r_2 = 28.25 \text{ mm} \quad ...(iii)$$

Also for uniform wear, the mean radius of the clutch plate,

$$R = \frac{r_1 + r_2}{2} \quad \text{or} \quad r_1 + r_2 = 2R = 2 \times 113 = 226 \text{ mm} \quad ...(iv)$$

From equations **(*iii*)** and **(*iv*),**

$$r_1 = 127.125 \text{ mm ; and } r_2 = 98.875 \;\; \textbf{Ans.}$$

Example 10.26. *A dry single plate clutch is to be designed for an automotive vehicle whose engine is rated to give 100 kW at 2400 r.p.m. and maximum torque 500 N-m. The outer radius of friction plate is 25% more than the inner radius. The intensity of pressure between the plate is not to exceed 0.07 N/mm². The coefficient of friction may be assumed equal to 0.3. The helical springs required by this clutch to provide axial force necessary to engage the clutch are eight. If each spring has stiffness equal to 40 N /mm, determine the initial compression in the springs and dimensions of the friction plate.*

Solution. Given : P = 100 kW = 100×10^3 W ; T = 500 N-m = 500×10^3 N-mm ; p = 0.07 N/mm^2 ; μ = 0.3 ; Number of springs = 8 ; Stiffness = 40 N/mm

Dimensions of the friction plate

Let r_1 and r_2 = Outer and inner radii of the friction plate respectively.

Since the outer radius of the friction plate is 25% more than the inner radius, therefore

$$r_1 = 1.25\, r_2$$

We know that, for uniform wear,

$$p.r_2 = C \quad \text{or} \quad C = 0.07\, r_2 \text{ N/mm}$$

and load transmitted to the friction plate,

$$W = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 0.07\, r^2\,(1.125\, r_2 - r_2) = 0.11\,(r_2)^2 \text{ N} \quad ...(i)$$

We know that mean radius of the plate for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\, r_2 + r_2}{2} = 1.125\, r_2$$

$\therefore$ Torque transmitted (T),

$$500 \times 10^3 = n.\mu.W.R = 2 \times 0.3 \times 0.11\,(r_2)^2 \times 1.125\, r_2 = 0.074\,(r_2)^3 \quad ...(\because \; n = 2)$$

$$\therefore \quad (r_2)^3 = 500 \times 10^3/0.074 = 6757 \times 10^3 \text{ or } r_2 = 190 \text{ mm } \textbf{Ans.}$$

and $r_1 = 1.25\ r_2 = 1.25 \times 190 = 273.5$ mm **Ans.**

Initial compression of the springs

We know that total stiffness of the springs,

$$s = \text{Stiffness per spring} \times \text{No. of springs} = 40 \times 8 = 320 \text{ N/mm}$$

Axial force required to engage the clutch,

$$W = 0.11\,(r_2)^2 = 0.11\,(190)^2 = 3970 \text{ N} \qquad \text{...[From equation } (i)]$$

∴ Initial compression in the springs

$$= W/s = 3970/320 = 12.5 \text{ mm} \quad \textbf{Ans.}$$

Example 10.27. *A rotor is driven by a co-axial motor through a single plate clutch, both sides of the plate being effective. The external and internal diameters of the plate are respectively 220 mm and 160 mm and the total spring load pressing the plates together is 570 N. The motor armature and shaft has a mass of 800 kg with an effective radius of gyration of 200 mm. The rotor has a mass of 1300 kg with an effective radius of gyration of 180 mm. The coefficient of friction for the clutch is 0.35.*

The driving motor is brought up to a speed of 1250 r.p.m. when the current is switched off and the clutch suddenly engaged. Determine

1. *The final speed of motor and rotor,* **2.** *The time to reach this speed, and* **3.** *The kinetic energy lost during the period of slipping.*

How long would slipping continue if it is assumed that a constant resisting torque of 60 N-m were present? If instead of a resisting torque, it is assumed that a constant driving torque of 60 N-m is maintained on the armature shaft, what would then be slipping time?

Solution. Given : $d_1 = 220$ mm or $r_1 = 110$ mm ; $d_2 = 160$ mm or $r_2 = 80$ mm ; $W = 570$ N ; $m_1 = 800$ kg ; $k_1 = 200$ mm $= 0.2$ m ; $m_2 = 1300$ kg ; $k_2 = 180$ mm $= 0.18$ m ; $\mu = 0.35$; $N_1 = 1250$ r.p.m. or $\omega_1 = \pi \times 1250/60 = 131$ rad/s

1. Final speed of the motor and rotor

Let ω_3 = Final speed of the motor and rotor in rad/s.

We know that moment of inertia for the motor armature and shaft,

$$I_1 = m_1\,(k_1)^2 = 800\,(0.2)^2 = 32 \text{ kg-m}^2$$

and moment of inertia for the rotor,

$$I_2 = m_2\,(k_2)^2 = 1300\,(0.18)^2 = 42.12 \text{ kg-m}^2$$

Since the angular momentum before slipping is equal to the angular momentum after slipping, therefore

$$I_1.\omega_1 + I_2.\omega_2 = (I_1 + I_2)\,\omega_3$$

$$32 \times 131 + I_2 \times 0 = (32 + 42.12)\,\omega_3 = 74.12\,\omega_3 \qquad \text{...}(\because \omega_2 = 0)$$

$$\therefore \quad \omega_3 = 32 \times 131 / 74.12 = 56.56 \text{ rad/s} \quad \textbf{Ans.}$$

2. Time to reach this speed

Let t = Time to reach this speed *i.e.* 56.56 rad/s.

We know that mean radius of the friction plate,

$$R = \frac{r_1 + r_2}{2} = \frac{110 + 80}{2} = 95 \text{ mm} = 0.095 \text{ m}$$

and total frictional torque,

$$T = n.\mu.W.R = 2 \times 0.35 \times 570 \times 0.095 = 37.9 \text{ N-m} \qquad ...(\because \; n = 2)$$

Considering the rotor, let α_2, ω_I and ω_F be the angular acceleration, initial angular speed and the final angular speed of the rotor respectively.

We know that the torque (T),

$$37.9 = I_2.\alpha_2 = 42.12\ \alpha_2 \quad \text{or} \quad \alpha_2 = 37.9/42.12 = 0.9 \text{ rad/s}^2$$

Since the angular acceleration is the rate of change of angular speed, therefore

$$\alpha_2 = \frac{\omega_F - \omega_I}{t} \quad \text{or} \quad t = \frac{\omega_F - \omega_I}{\alpha_2} = \frac{56.56 - 0}{0.9} = 62.8 \text{ s} \quad \textbf{Ans.}$$

$$...(\because \; \omega_F = \omega_3 = 56.56 \text{ rad/s, and } \omega_1 = 0)$$

3. *Kinetic energy lost during the period of slipping*

We know that angular kinetic energy before impact,

$$E_1 = \frac{1}{2} I_1 (\omega_1)^2 + \frac{1}{2} I_2 (\omega_2)^2 = \frac{1}{2} I_1 (\omega_1)^2 \qquad ...(\because \; \omega_2 = 0)$$

$$= \frac{1}{2} \times 32 (131)^2 = 274\ 576 \text{ N-m}$$

and angular kinetic energy after impact,

$$E_2 = \frac{1}{2} (I_1 + I_2)(\omega_3)^2 = \frac{1}{2}(32 + 42.12)(56.56)^2 = 118\ 556 \text{ N-m}$$

$\therefore$ Kinetic energy lost during the period of slipping,

$$= E_1 - E_2 = 274\ 576 - 118\ 556 = 156\ 020 \text{ N-m} \quad \textbf{Ans.}$$

Time of slipping assuming constant resisting torque

Let t_1 = Time of slipping, and

ω_2 = Common angular speed of armature and rotor shaft = 56.56 rad/s

When slipping has ceased and there is exerted a constant torque of 60 N-m on the armature shaft, then

Torque on armature shaft,

$$T_1 = -60 - 37.9 = -97.9 \text{ N-m}$$

Torque on rotor shaft,

$$T_2 = T = 37.9 \text{ N-m}$$

Considering armature shaft,

$$\omega_3 = \omega_1 + \alpha_1.t_1 = \omega_1 + \frac{T_1}{I_1} \times t_1 = 131 - \frac{97.9}{32} \times t_1 = 131 - 3.06\ t_1 \qquad ...(i)$$

Considering rotor shaft,

$$\omega_3 = \alpha_2.t_1 = \frac{T_2}{I_2} \times t_1 = \frac{37.9}{42.12} \times t_1 = 0.9\ t_1 \qquad ...(ii)$$

From equations ***(i)*** and ***(ii)***,

$$131 - 3.06\ t_1 = 0.9\ t_1 \quad \text{or} \quad 3.96\ t_1 = 131$$

$$\therefore \qquad t_1 = 131/3.96 = 33.1 \text{ s} \quad \textbf{Ans.}$$

Time of slipping assuming constant driving torque of 60 N-m

In this case, $T_1 = 60 - 37.9 = 22.1$ N-m

Since $\omega_1 + \dfrac{T_1}{I_1} \times t_1 = \dfrac{T_2}{I_2} \times t_1$, therefore

$$131 + \frac{22.1}{32} \times t_1 = \frac{37.9}{42.12} \times t_1 \quad \text{or} \quad 131 + 0.69\, t_1 = 0.9\, t_1$$

$\therefore$ $0.9\, t_1 - 0.69\, t_1 = 131$ or $t_1 = 624$ s **Ans.**

Example 10.28. *A multiple disc clutch has five plates having four pairs of active friction surfaces. If the intensity of pressure is not to exceed 0.127 N/mm², find the power transmitted at 500 r.p.m. The outer and inner radii of friction surfaces are 125 mm and 75 mm respectively. Assume uniform wear and take coefficient of friction = 0.3.*

Solution. Given : $n_1 + n_2 = 5$; $n = 4$; $p = 0.127$ N/mm² ; $N = 500$ r.p.m. or $\omega = 2\pi \times 500/60 = 52.4$ rad/s ; $r_1 = 125$ mm ; $r_2 = 75$ mm ; $\mu = 0.3$

Since the intensity of pressure is maximum at the inner radius r_2, therefore

$$p.r_2 = C \quad \text{or} \quad C = 0.127 \times 75 = 9.525 \text{ N/mm}$$

We know that axial force required to engage the clutch,

$$W = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 9.525\,(125 - 75) = 2990 \text{ N}$$

and mean radius of the friction surfaces,

$$R = \frac{r_1 + r_2}{2} = \frac{125 + 75}{2} = 100 \text{ mm} = 0.1 \text{ m}$$

We know that torque transmitted,

$$T = n.\mu.W.R = 4 \times 0.3 \times 2990 \times 0.1 = 358.8 \text{ N-m}$$

$\therefore$ Power transmitted,

$$P = T.\omega = 358.8 \times 52.4 = 18\,800 \text{ W} = 18.8 \text{ kW}$$ **Ans.**

Example 10.29. *A multi-disc clutch has three discs on the driving shaft and two on the driven shaft. The outside diameter of the contact surfaces is 240 mm and inside diameter 120 mm. Assuming uniform wear and coefficient of friction as 0.3, find the maximum axial intensity of pressure between the discs for transmitting 25 kW at 1575 r.p.m.*

Solution. Given : $n_1 = 3$; $n_2 = 2$; $d_1 = 240$ mm or $r_1 = 120$ mm ; $d_2 = 120$ mm or $r_2 = 60$ mm ; $\mu = 0.3$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 1575$ r.p.m. or $\omega = 2\,\pi \times 1575/60 = 165$ rad/s

Let T = Torque transmitted in N-m, and

W = Axial force on each friction surface.

We know that the power transmitted (P),

$$25 \times 10^3 = T.\omega = T \times 165 \quad \text{or} \quad T = 25 \times 10^3/165 = 151.5 \text{ N-m}$$

Number of pairs of friction surfaces,

$$n = n_1 + n_2 - 1 = 3 + 2 - 1 = 4$$

and mean radius of friction surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{120 + 60}{2} = 90 \text{ mm} = 0.09 \text{ m}$$

We know that torque transmitted (T),

$$151.5 = n.\mu.W.R = 4 \times 0.3 \times W \times 0.09 = 0.108\ W$$

$$\therefore \quad W = 151.5/0.108 = 1403 \text{ N}$$

Let $\quad p$ = Maximum axial intensity of pressure.

Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform wear

$$p.r_2 = C \quad \text{or} \quad C = p \times 60 = 60\ p \text{ N/mm}$$

We know that the axial force on each friction surface (W),

$$1403 = 2\,\pi.C\,(r_1 - r_2) = 2\,\pi \times 60\ p\,(120 - 60) = 22\ 622\ p$$

$$\therefore \quad p = 1403/22\ 622 = 0.062 \text{ N/mm}^2$$ **Ans.**

Example 10.30. *A plate clutch has three discs on the driving shaft and two discs on the driven shaft, providing four pairs of contact surfaces. The outside diameter of the contact surfaces is 240 mm and inside diameter 120 mm. Assuming uniform pressure and μ = 0.3; find the total spring load pressing the plates together to transmit 25 kW at 1575 r.p.m.*

If there are 6 springs each of stiffness 13 kN/m and each of the contact surfaces has worn away by 1.25 mm, find the maximum power that can be transmitted, assuming uniform wear.

Solution. Given : $n_1 = 3$; $n_2 = 2$; $n = 4$; $d_1 = 240$ mm or $r_1 = 120$ mm ; $d_2 = 120$ mm or $r_2 = 60$ mm ; $\mu = 0.3$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 1575$ r.p.m. or $\omega = 2\,\pi \times 1575/60 = 165$ rad/s

Total spring load

Let $\quad W$ = Total spring load, and

$\quad T$ = Torque transmitted.

We know that power transmitted (P),

$$25 \times 10^3 = T.\omega = T \times 165 \quad \text{or} \quad T = 25 \times 10^3/165 = 151.5 \text{ N-m}$$

Mean radius of the contact surface, for uniform pressure,

$$R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3}\left[\frac{(120)^3 - (60)^3}{(120)^2 - (60)^2}\right] = 93.3 \text{ mm} = 0.0933 \text{ m}$$

and torque transmitted (T),

$$151.5 = n.\mu.W.R = 4 \times 0.3\ W \times 0.0933 = 0.112\ W$$

$$\therefore \quad W = 151.5/0.112 = 1353 \text{ N}$$ **Ans.**

Maximum power transmitted

Given : No of springs = 6

$\therefore$ Contact surfaces of the spring

= 8

Wear on each contact surface

= 1.25 mm

$\therefore$ Total wear = 8 × 1.25 = 10 mm = 0.01 m

Stiffness of each spring = 13 kN/m = 13×10^3 N/m

$\therefore$ Reduction in spring force

= Total wear × Stiffness per spring × No. of springs

$= 0.01 \times 13 \times 10^3 \times 6 = 780$ N

∴ New axial load, $W = 1353 - 780 = 573$ N

We know that mean radius of the contact surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{120 + 60}{2} = 90 \text{ mm} = 0.09 \text{ m}$$

∴ Torque transmitted,

$$T = n.\mu.W.R. = 4 \times 0.3 \times 573 \times 0.09 = 62 \text{ N-m}$$

and maximum power transmitted,

$$P = T.\,\omega = 62 \times 155 = 10\,230 \text{ W} = 10.23 \text{ kW} \quad \textbf{Ans.}$$

10.34. Cone Clutch

A cone clutch, as shown in Fig. 10.24, was extensively used in automobiles but now-a-days it has been replaced completely by the disc clutch.

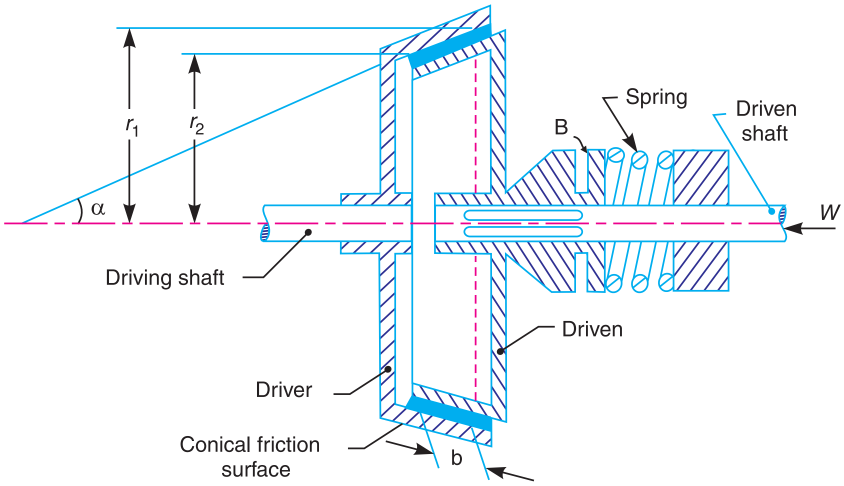


Fig. 10.24. Cone clutch.

It consists of one pair of friction surface only. In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven. The driven member resting on the feather key in the driven shaft, may be shifted along the shaft by a forked lever provided at B, in order to engage the clutch by bringing the two conical surfaces in contact. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another. In some cases, a spring is placed around the driven shaft in contact with the hub of the driven. This spring holds the clutch faces in contact and maintains the pressure between them, and the forked lever is used only for disengagement of the clutch. The contact surfaces of the clutch may be metal to metal contact, but more often the driven member is lined with some material like wood, leather, cork or asbestos etc. The material of the clutch faces (*i.e.* contact surfaces) depends upon the allowable normal pressure and the coefficient of friction.

Consider a pair of friction surface as shown in Fig. 10.25 (*a*). Since the area of contact of a pair of friction surface is a frustrum of a cone, therefore the torque transmitted by the cone clutch may be determined in the similar manner as discussed for conical pivot bearings in Art. 10.28.

Let p_n = Intensity of pressure with which the conical friction surfaces are held together (*i.e.* normal pressure between contact surfaces),

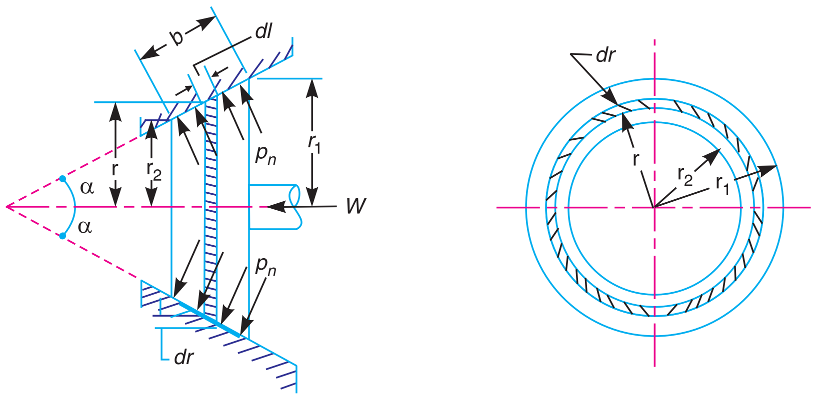
r_1 and r_2 = Outer and inner radius of friction surfaces respectively.

R = Mean radius of the friction surface $= \dfrac{r_1 + r_2}{2}$,

α = Semi angle of the cone (also called face angle of the cone) or the angle of the friction surface with the axis of the clutch,

μ = Coefficient of friction between contact surfaces, and

b = Width of the contact surfaces (also known as face width or clutch face).



(*a*) (*b*)

Fig. 10.25. Friction surfaces as a frustrum of a cone.

Consider a small ring of radius *r* and thickness *dr*, as shown in Fig. 10.25 (*b*). Let *dl* is length of ring of the friction surface, such that

$$dl = dr.\text{cosec}\ \alpha$$

$\therefore$ Area of the ring,

$$A = 2\pi\ r.dl = 2\pi r.dr\ \text{cosec}\ \alpha$$

We shall consider the following two cases :

1. When there is a uniform pressure, and
2. When there is a uniform wear.

1. *Considering uniform pressure*

We know that normal load acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_n \times 2\ \pi\ r.dr.\text{cosec}\ \alpha$$

and the axial load acting on the ring,

$$\delta W = \text{Horizontal component of } \delta W_n \text{ (i.e. in the direction of } W)$$

$$= \delta W_n \times \sin\alpha = p_n \times 2\pi\ r.dr.\ \text{cosec}\ \alpha \times \sin\alpha = 2\pi \times p_n.r.dr$$

$\therefore$ Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi\, p_n.r.dr = 2\pi\, p_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\, p_n \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\, p_n \left[(r_1)^2 - (r_2)^2\right]$$

$$\therefore \qquad p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad \text{...}(i)$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_n \times 2\,\pi\, r.dr.\text{cosec}\,\alpha$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_n \times 2\,\pi\, r.dr.\ \text{cosec}\,\alpha.r = 2\,\pi\,\mu.p_n.\text{cosec}\,\alpha.r^2\, dr$$

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

$\therefore$ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p_n.\text{cosec}\,\alpha.r^2.dr = 2\pi\mu\, p_n.\text{cosec}\,\alpha\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu\, p_n.\text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p_n from equation (*i*), we get

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W.\text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad ..(ii)$$

2. *Considering uniform wear*

In Fig. 10.25, let p_r be the normal intensity of pressure at a distance r from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$\therefore$ $p_r.r = C$ (a constant) or $p_r = C / r$

We know that the normal load acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_r \times 2\pi r.dr\ \text{cosec}\,\alpha$$

and the axial load acting on the ring ,

$$\delta W = \delta W_n \times \sin\alpha = p_r.2\,\pi\, r.dr.\text{cosec}\,\alpha\,.\sin\alpha = p_r \times 2\,\pi\, r.dr$$

$$= \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr \qquad ...(\because\ p_r = C / r)$$

$\therefore$ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

or
$$C = \frac{W}{2\pi(r_1 - r_2)} \qquad ...(iii)$$

We know that frictional force acting on the ring,

$$F_r = \mu.\delta W_n = \mu.p_r \times 2\,\pi\, r \times dr\ \text{cosec}\,\alpha$$

and frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_r \times 2\,\pi\, r.dr.\text{cosec}\,\alpha \times r$$

$$= \mu \times \frac{C}{r} \times 2\pi r^2.dr.\text{cosec}\,\alpha = 2\pi\mu.C\ \text{cosec}\,\alpha \times r\ dr$$

∴ Total frictional torque acting on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.\text{cosec}\,\alpha.r\,dr = 2\pi\mu.C.\text{cosec}\,\alpha\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu.C.\text{cosec}\,\alpha\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

Substituting the value of C from equation **(*i*)**, we have

$$T = 2\pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \text{cosec}\,\alpha\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

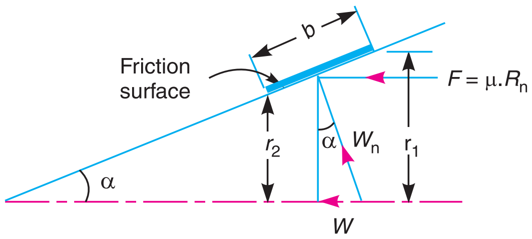
$$= \mu.W\ \text{cosec}\,\alpha\left(\frac{r_1 + r_2}{2}\right) = \mu.W.R\ \text{cosec}\,\alpha \qquad \text{...}(iv)$$

where $R = \dfrac{r_1 + r_2}{2}$ = Mean radius of friction surface

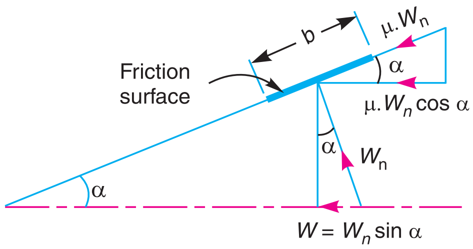
Since the normal force acting on the friction surface, $W_n = W/\sin\alpha$, therefore the equation **(*iv*)** may be written as

$$T = \mu.W_n.R \qquad \text{...}(v)$$

The forces on a friction surface, for steady operation of the clutch and after the clutch is engaged, is shown in Fig. 10.26.



(*a*) For steady operation of the clutch.



(*b*) During engagement of the clutch.

Fig. 10.26. Forces on a friction surface.

From Fig. 10.26 (*a*), we find that

$$r_1 - r_2 = b\sin\alpha;\ \text{and}\ R = \frac{r_1 + r_2}{2} \quad \text{or} \quad r_1 + r_2 = 2R$$

∴ From equation, *(i)*, normal pressure acting on the friction surface,

$$p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{W}{\pi(r_1 + r_2)(r_1 - r_2)} = \frac{W}{2\pi R.b.\sin\alpha}$$

or $W = p_n \times 2\pi R.b \sin\alpha = W_n \sin\alpha$

where W_n = Normal load acting on the friction surface = $p_n \times 2\pi R.b$

Now the equation *(iv)* may be written as,

$$T = \mu(p_n \times 2\pi R.b \sin\alpha)\, R \operatorname{cosec}\alpha = 2\pi\mu.p_n.R^2 b$$

The following points may be noted for a cone clutch :

1. The above equations are valid for steady operation of the clutch and after the clutch is engaged.

2. If the clutch is engaged when one member is stationary and the other rotating (*i.e.* during engagement of the clutch) as shown in Fig. 10.26 (*b*), then the cone faces will tend to slide on each other due to the presence of relative motion. Thus an additional force (of magnitude equal to $\mu.W_n.\cos\alpha$) acts on the clutch which resists the engagement and the axial force required for engaging the clutch increases.

∴ Axial force required for engaging the clutch,

$$W_e = W + \mu.W_n\cos\alpha = W_n\sin\alpha + \mu.W_n\cos\alpha$$

$$= W_n(\sin\alpha + \mu\cos\alpha)$$

3. Under steady operation of the clutch, a decrease in the semi-cone angle (α) increases the torque produced by the clutch (*T*) and reduces the axial force (*W*). During engaging period, the axial force required for engaging the clutch (W_e) increases under the influence of friction as the angle α decreases. The value of α can not be decreased much because smaller semi-cone angle (α) requires larger axial force for its disengagement.

For free disengagement of the clutch, the value of tan α must be greater than μ. In case the value of tan α is less than μ, the clutch will not disengage itself and the axial force required to disengage the clutch is given by

$$W_d = W_n(\mu\cos\alpha - \sin\alpha)$$

Example 10.31. *A conical friction clutch is used to transmit 90 kW at 1500 r.p.m. The semi-cone angle is 20º and the coefficient of friction is 0.2. If the mean diameter of the bearing surface is 375 mm and the intensity of normal pressure is not to exceed 0.25 N/mm², find the dimensions of the conical bearing surface and the axial load required.*

Solution. Given : P = 90 kW = 90×10^3 W ; N = 1500 r.p.m. or $\omega = 2\pi \times 1500/60 = 156$ rad/s ; α = 20º ; μ = 0.2 ; D = 375 mm or R = 187.5 mm ; p_n = 0.25 N/mm²

Dimensions of the conical bearing surface

Let r_1 and r_2 = External and internal radii of the bearing surface respectively,

b = Width of the bearing surface in mm, and

T = Torque transmitted.

We know that power transmitted (*P*),

$$90 \times 10^3 = T.\omega = T \times 156$$

∴ $T = 90 \times 10^3/156 = 577$ N-m $= 577 \times 10^3$ N-mm and the torque transmitted (T),

$$577 \times 10^3 = 2\,\pi\,\mu\,p_n.R^2.b = 2\pi \times 0.2 \times 0.25\,(187.5)^2\,b = 11\,046\,b$$

$$\therefore \quad b = 577 \times 10^3/11\,046 = 52.2 \text{ mm} \quad \textbf{Ans.}$$

We know that $r_1 + r_2 = 2R = 2 \times 187.5 = 375$ mm ...(*i*)

and $r_1 - r_2 = b \sin \alpha = 52.2 \sin 20^\circ = 18$ mm ...(*ii*)

From equations (*i*) and (*ii*),

$$r_1 = 196.5 \text{ mm, and } r_2 = 178.5 \text{ mm} \quad \textbf{Ans.}$$

Axial load required

Since in case of friction clutch, uniform wear is considered and the intensity of pressure is maximum at the minimum contact surface radius (r_2), therefore

$$p_n.r_2 = C \text{ (a constant) or } C = 0.25 \times 178.5 = 44.6 \text{ N/mm}$$

We know that the axial load required,

$$W = 2\pi C\,(r_1 - r_2) = 2\pi \times 44.6\,(196.5 - 178.5) = 5045 \text{ N} \quad \textbf{Ans.}$$

Example 10.32. *An engine developing 45 kW at 1000 r.p.m. is fitted with a cone clutch built inside the flywheel. The cone has a face angle of 12.5º and a maximum mean diameter of 500 mm. The coefficient of friction is 0.2. The normal pressure on the clutch face is not to exceed 0.1 N/mm². Determine :* **1.** *the axial spring force necessary to engage to clutch, and* **2.** *the face width required.*

Solution. Given : $P = 45$ kW $= 45 \times 10^3$ W ; $N = 1000$ r.p.m. or $\omega = 2\pi \times 1000/60 = 104.7$ rad/s ; $\alpha = 12.5^\circ$; $D = 500$ mm or $R = 250$ mm $= 0.25$ m ; $\mu = 0.2$; $p_n = 0.1$ N/mm²

1. ***Axial spring force necessary to engage the clutch***

First of all, let us find the torque (T) developed by the clutch and the normal load (W_n) acting on the friction surface.

We know that power developed by the clutch (P),

$$45 \times 10^3 = T.\omega = T \times 104.7 \quad \text{or} \quad T = 45 \times 10^3/104.7 = 430 \text{ N-m}$$

We also know that the torque developed by the clutch (T),

$$430 = \mu.W_n.R = 0.2 \times W_n \times 0.25 = 0.05\,W_n$$

$$\therefore \quad W_n = 430/0.05 = 8600 \text{ N}$$

and axial spring force necessary to engage the clutch,

$$W_e = W_n\,(\sin \alpha + \mu \cos \alpha)$$
$$= 8600\,(\sin 12.5^\circ + 0.2 \cos 12.5^\circ) = 3540 \text{ N} \quad \textbf{Ans.}$$

2. ***Face width required***

Let b = Face width required.

We know that normal load acting on the friction surface (W_n),

$$8600 = p_n \times 2\,\pi\,R.b = 0.1 \times 2\pi \times 250 \times b = 157\,b$$

$$\therefore \quad b = 8600/157 = 54.7 \text{ mm} \quad \textbf{Ans.}$$

Example 10.33. *A leather faced conical clutch has a cone angle of 30º. If the intensity of pressure between the contact surfaces is limited to 0.35 N/mm² and the breadth of the conical surface is not to exceed one-third of the mean radius, find the dimensions of the contact surfaces to transmit 22.5 kW at 2000 r.p.m. Assume uniform rate of wear and take coefficient of friction as 0.15.*

Solution. Given : $2\,\alpha = 30^\circ$ or $\alpha = 15^\circ$; $p_n = 0.35$ N/mm²; $b = R/3$; $P = 22.5$ kW $= 22.5 \times 10^3$ W ; $N = 2000$ r.p.m. or $\omega = 2\,\pi \times 2000/60 = 209.5$ rad/s ; $\mu = 0.15$

Let r_1 = Outer radius of the contact surface in mm,

r_2 = Inner radius of the contact surface in mm,
R = Mean radius of the the contact surface in mm,
b = Face width of the contact surface in mm = $R/3$, and
T = Torque transmitted by the clutch in N-m.

We know that power transmitted (P),

$$22.5 \times 10^3 = T.\omega = T \times 209.5$$

$$\therefore \quad T = 22.5 \times 10^3/209.5 = 107.4 \text{ N-m} = 107.4 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted (T),

$$107.4 \times 10^3 = 2\pi\,\mu\, p_n.R^2.\, b = 2\pi \times 0.15 \times 0.35 \times R^2 \times R/3 = 0.11\, R^3$$

$$\therefore \quad R^3 = 107.4 \times 10^3/0.11 = 976.4 \times 10^3 \quad \text{or} \quad R = 99 \text{ mm}$$ **Ans.**

The dimensions of the contact surface are shown in Fig. 10.27.

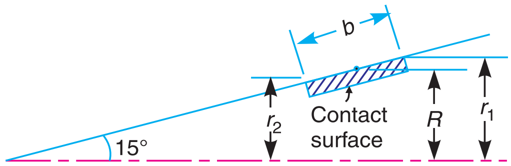


Fig. 10.27

From Fig. 10.27, we find that

$$r_1 - r_2 = b \sin \alpha = \frac{R}{3} \times \sin \alpha = \frac{99}{3} \times \sin 15^\circ = 8.54 \text{ mm} \quad \text{...}(i)$$

and

$$r_1 + r_2 = 2R = 2 \times 99 = 198 \text{ mm} \quad \text{...}(ii)$$

From equations *(i)* and *(ii)*,

$$r_1 = 103.27 \text{ mm, and } r_2 = 94.73 \text{ mm}$$ **Ans.**

Example 10.34. *The contact surfaces in a cone clutch have an effective diameter of 75 mm. The semi-angle of the cone is 15º. The coefficient of friction is 0.3. Find the torque required to produce slipping of the clutch if an axial force applied is 180 N.*

This clutch is employed to connect an electric motor running uniformly at 1000 r.p.m. with a flywheel which is initially stationary. The flywheel has a mass of 13.5 kg and its radius of gyration is 150 mm. Calculate the time required for the flywheel to attain full speed and also the energy lost in the slipping of the clutch.

Solution. Given : D = 75 mm or R = 37.5 mm = 0.0375 m ; α = 15º ; μ = 0.3 ; W = 180 N ; N_F = 1000 r.p.m. or $\omega_F = 2\pi \times 1000/60 = 104.7$ rad/s ; m = 13.5 kg ; k = 150 mm = 0.15 m

Torque required to produce slipping

We know that torque required to produce slipping,

$$T = \mu.W.R.\text{cosec}\,\alpha = 0.3 \times 180 \times 0.0375 \times \text{cosec}\, 15^\circ = 7.8 \text{ N-m}$$ **Ans.**

Time required for the flywheel to attain full speed

Let t_F = Time required for the flywheel to attain full speed in seconds, and
α_F = Angular acceleration of the flywheel in rad/s^2.

We know that the mass moment of inertia of the flywheel,

$$I_F = m.k^2 = 13.5 \times (0.15)^2 = 0.304 \text{ kg-m}^2$$

∴ Torque required (T),

$$7.8 = I_F.\alpha_F = 0.304\ \alpha_F \quad \text{or} \quad \alpha_F = 7.8/0.304 = 25.6\ \text{rad/s}^2$$

and angular speed of the flywheel (ω_F),

$$104.7 = \alpha_F.t_F = 25.6\ t_F \quad \text{or} \quad t_F = 104.7/25.6 = 4.1\ \text{s}$$ **Ans.**

Energy lost in slipping of the clutch

We know that the angle turned through by the motor and flywheel (*i.e.* clutch) in time 4.1 s from rest,

$$\theta = \text{Average angular velocity} \times \text{time} = \frac{1}{2} \times W_F \times t_F = \frac{1}{2} \times 104.7 \times 4.1 = 214.6\ \text{rad}$$

∴ Energy lost in slipping of the clutch,

$$= T.\theta = 7.8 \times 214.6 = 1674\ \text{N-m}$$ **Ans.**

10.35. Centrifugal Clutch

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in Fig. 10.28. The outer surface of the shoes are covered with a friction material. These shoes, which can move radially in guides, are held

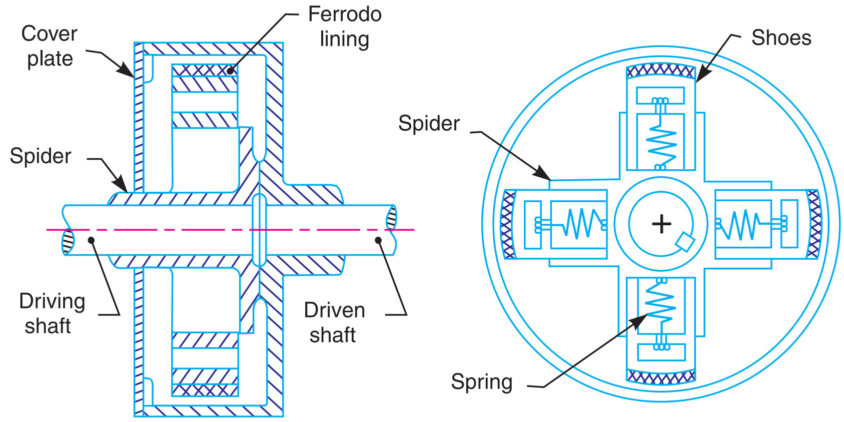


Fig. 10.28. Centrifugal clutch.

against the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The mass of the shoe, when revolving, causes it to exert a radially outward force (*i.e.* centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder

Centrifugal clutch.

and enables more torque to be transmitted.

In order to determine the mass and size of the shoes, the following procedure is adopted :

1. *Mass of the shoes*

Consider one shoe of a centrifugal clutch as shown in Fig. 10.29.

Let m = Mass of each shoe,

n = Number of shoes,

r = Distance of centre of gravity of the shoe from the centre of the spider,

R = Inside radius of the pulley rim,

N = Running speed of the pulley in r.p.m.,

ω = Angular running speed of the pulley in rad/s = $2\pi N/60$ rad/s,

ω_1 = Angular speed at which the engagement begins to take place, and

μ = Coefficient of friction between the shoe and rim.

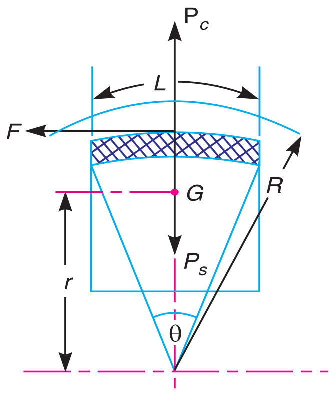


Fig. 10.29. Forces on a shoe of centrifugal clutch.

We know that the centrifugal force acting on each shoe at the running speed,

$$*P_c = m.\omega^2.r$$

and the inward force on each shoe exerted by the spring at the speed at which engagement begins to take place,

$$P_s = m\,(\omega_1)^2\,r$$

$\therefore$ The net outward radial force (*i.e.* centrifugal force) with which the shoe presses against the rim at the running speed

$$= P_c - P_s$$

and the frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s)$$

$\therefore$ Frictional torque acting on each shoe,

$$= F \times R = \mu\,(P_c - P_s)\,R$$

and total frictional torque transmitted,

$$T = \mu\,(P_c - P_s)\,R \times n = n.F.R$$

From this expression, the mass of the shoes (m) may be evaluated.

2. *Size of the shoes*

Let l = Contact length of the shoes,

b = Width of the shoes,

* The radial clearance between the shoe and the rim being very small as compared to r, therefore it is neglected. If, however, the radial clearance is given, then the operating radius of the mass centre of the shoe from the axis of the clutch,

$$r_1 = r + c, \text{ where } c = \text{Radial clearance.}$$

Then $P_c = m.\omega^2.r_1$, and $P_s = m\,(\omega_1)^2\,r_1$

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley.

θ = Angle subtended by the shoes at the centre of the spider in radians.

p = Intensity of pressure exerted on the shoe. In order to ensure reasonable life, the intensity of pressure may be taken as 0.1 N/mm^2.

We know that $\theta = l/R$ rad or $l = \theta.R$

$\therefore$ Area of contact of the shoe,

$$A = l.b$$

and the force with which the shoe presses against the rim

$$= A \times p = l.b.p$$

Since the force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$, therefore

$$l.b.p = P_c - P_s$$

From this expression, the width of shoe (b) may be obtained.

Example 10.35. *A centrifugal clutch is to transmit 15 kW at 900 r.p.m. The shoes are four in number. The speed at which the engagement begins is 3/4th of the running speed. The inside radius of the pulley rim is 150 mm and the centre of gravity of the shoe lies at 120 mm from the centre of the spider. The shoes are lined with Ferrodo for which the coefficient of friction may be taken as 0.25. Determine :* ***1.*** *Mass of the shoes, and* ***2.*** *Size of the shoes, if angle subtended by the shoes at the centre of the spider is 60° and the pressure exerted on the shoes is 0.1 N/mm*^{2}.

Solution. Given : P = 15 kW = 15 × 10^3 W ; N = 900 r.p.m. or ω = 25 × 900/60 = 94.26 rad/s ; n = 4 ; R = 150 mm = 0.15 m ; r = 120 mm = 0.12 m ; μ = 0.25

Since the speed at which the engagement begins (*i.e.* ω_1) is 3/4th of the running speed (*i.e.* ω), therefore

$$\omega_1 = \frac{3}{4}\,\omega = \frac{3}{4} \times 94.26 = 70.7 \text{ rad/s}$$

Let T = Torque transmitted at the running speed.

We know that power transmitted (P),

$$15 \times 10^3 = T.\omega = T \times 94.26 \quad \text{or} \quad T = 15 \times 10^3/94.26 = 159 \text{ N-m}$$

1. *Mass of the shoes*

Let m = Mass of the shoes in kg.

We know that the centrifugal force acting on each shoe,

$$P_c = m.\omega^2.r = m\,(94.26)^2 \times 0.12 = 1066\,m \text{ N}$$

and the inward force on each shoe exerted by the spring *i.e.* the centrifugal force at the engagement speed ω_1,

$$P_s = m\,(\omega_1)^2\,r = m\,(70.7)^2 \times 0.12 = 600\,m \text{ N}$$

$\therefore$ Frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s) = 0.25\,(1066\,m - 600\,m) = 116.5\,m \text{ N}$$

We know that the torque transmitted (T),

$$159 = n.F.R = 4 \times 116.5\,m \times 0.15 = 70\,m \quad \text{or} \quad m = 2.27 \text{ kg}$$ **Ans.**

2. *Size of the shoes*

Let l = Contact length of shoes in mm,

b = Width of the shoes in mm,

θ = Angle subtended by the shoes at the centre of the spider in radians
= $60° = \pi/3$ rad, and ...(Given)

p = Pressure exerted on the shoes in $N/mm^2 = 0.1\ N/mm^2$ ...(Given)

We know that $l = \theta.\ R = \frac{\pi}{3} \times 150 = 157.1$ mm

and $l.b.p = P_c - P_s = 1066\ m - 600\ m = 466\ m$

$\therefore$ $157.1 \times b \times 0.1 = 466 \times 2.27 = 1058$

or $b = 1058/157.1 \times 0.1 = 67.3$ mm **Ans.**

Example 10.36. *A centrifugal clutch has four shoes which slide radially in a spider keyed to the driving shaft and make contact with the internal cylindrical surface of a rim keyed to the driven shaft. When the clutch is at rest, each shoe is pulled against a stop by a spring so as to leave a radial clearance of 5 mm between the shoe and the rim. The pull exerted by the spring is then 500 N. The mass centre of the shoe is 160 mm from the axis of the clutch.*

If the internal diameter of the rim is 400 mm, the mass of each shoe is 8 kg, the stiffness of each spring is 50 N/mm and the coefficient of friction between the shoe and the rim is 0.3 ; find the power transmitted by the clutch at 500 r.p.m.

Solution. Given : $n = 4$; $c = 5$ mm ; $S = 500$ N ; $r = 160$ mm ; $D = 400$ mm or $R = 200$ mm $= 0.2$ m ; $m = 8$ kg ; $s = 50$ N/mm ; $\mu = 0.3$; $N = 500$ r.p.m. or $\omega = 2\pi \times 500/60 = 52.37$ rad/s

We know that the operating radius,

$$r_1 = r + c = 160 + 5 = 165 \text{ mm} = 0.165 \text{ m}$$

Centrifugal force on each shoe,

$$P_c = m.\omega^2.r_1 = 8\ (52.37)^2 \times 0.165 = 3620 \text{ N}$$

and the inward force exerted by the spring,

$$P_4 = S + c.s = 500 + 5 \times 50 = 750 \text{ N}$$

$\therefore$ Frictional force acting tangentially on each shoe,

$$F = \mu\ (P_c - P_s) = 0.3\ (3620 - 750) = 861 \text{ N}$$

We know that total frictional torque transmitted by the clutch,

$$T = n.F.R = 4 \times 861 \times 0.2 = 688.8 \text{ N-m}$$

$\therefore$ Power transmitted,

$$P = T.\omega = 688.8 \times 52.37 = 36\ 100 \text{ W} = 36.1 \text{ kW}$$ **Ans.**

EXERCISES

1. Find the force required to move a load of 300 N up a rough plane, the force being applied parallel to the plane. The inclination of the plane is such that a force of 60 N inclined at 30° to a similar smooth plane would keep the same load in equilibrium. The coefficient of friction is 0.3. **[Ans. 146 N]**

2. A square threaded screw of mean diameter 25 mm and pitch of thread 6 mm is utilised to lift a weight of 10 kN by a horizontal force applied at the circumference of the screw. Find the magnitude of the force if the coefficient of friction between the nut and screw is 0.02. **[Ans. 966 N]**

3. A bolt with a square threaded screw has mean diameter of 25 mm and a pitch of 3 mm. It carries an axial thrust of 10 kN on the bolt head of 25 mm mean radius. If $\mu = 0.12$, find the force required at the end of a spanner 450 mm long, in tightening up the bolt. **[Ans. 110.8 N]**

4. A turn buckle, with right and left hand threads is used to couple two railway coaches. The threads which are square have a pitch of 10 mm and a mean diameter of 30 mm and are of single start type. Taking the coefficient of friction as 0.1, find the work to be done in drawing the coaches together a distance of 200 mm against a steady load of 20 kN. **[Ans. 3927 N-m]**

5. A vertical two start square threaded screw of a 100 mm mean diameter and 20 mm pitch supports a vertical load of 18 kN. The axial thrust on the screw is taken by a collar bearing of 250 mm outside diameter and 100 mm inside diameter. Find the force required at the end of a lever which is 400 mm long in order to lift and lower the load. The coefficient of friction for the vertical screw and nut is 0.15 and that for collar bearing is 0.20.

[Ans. 1423 N ; 838 N]

6. A sluice gate weighing 18 kN is raised and lowered by means of square threaded screws, as shown in Fig.10.30. The frictional resistance induced by water pressure against the gate when it is in its lowest position is 4000 N.

The outside diameter of the screw is 60 mm and pitch is 10 mm. The outside and inside diameter of washer is 150 mm and 50 mm respectively. The coefficient of friction between the screw and nut is 0.1 and for the washer and seat is 0.12. Find :

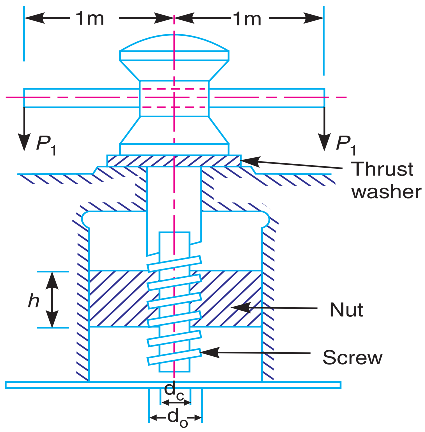


Fig. 10.30

1. The maximum force to be exerted at the ends of the lever for raising and lowering the gate, and
2. Efficiency of the arrangement. **[Ans. 114 N ; 50 N ; 15.4%]**

7. The spindle of a screw jack has single start square threads with an outside diameter of 45 mm and a pitch of 10 mm. The spindle moves in a fixed nut. The load is carried on a swivel head but is not free to rotate. The bearing surface of the swivel head has a mean diameter of 60 mm. The coefficient of friction between the nut and screw is 0.12 and that between the swivel head and the spindle is 0.10. Calculate the load which can be raised by efforts of 100 N each applied at the end of two levers each of effective length of 350 mm. Also determine the velocity ratio and the efficiency of the lifting arrangement. **[Ans. 9943 N ; 218.7 N ; 39.6%]**

8. The lead screw of a lathe has acme threads of 50 mm outside diameter and 10 mm pitch. The included angle of the thread is 29°. It drives a tool carriage and exerts an axial pressure of 2500 N. A collar bearing with outside diameter 100 mm and inside diameter 50 mm is provided to take up the thrust. If the lead screw rotates at 30 r.p.m., find the efficiency and the power required to drive the screw. The coefficient of friction for screw threads is 0.15 and for the collar is 0.12. **[Ans. 16.3% ; 75.56 W]**

9. A flat foot step bearing 225 mm in diameter supports a load of 7.5 kN. If the coefficient of friction is 0.09 and r.p.m is 60, find the power lost in friction, assuming 1. Uniform pressure, and 2. Uniform wear. **[Ans. 318 W ; 239 W]**

10. A conical pivot bearing 150 mm in diameter has a cone angle of 120º. If the shaft supports an axial load of 20 kN and the coefficient of friction is 0.03, find the power lost in friction when the shaft rotates at 200 r.p.m., assuming 1. Uniform pressure, and 2. uniform wear.

[Ans. 727.5 W ; 545.6 W]

11. A vertical shaft supports a load of 20 kN in a conical pivot bearing. The external radius of the cone is 3 times the internal radius and the cone angle is 120º. Assuming uniform intensity of pressure as 0.35 MN/m^2, determine the dimensions of the bearing.

If the coefficient of friction between the shaft and bearing is 0.05 and the shaft rotates at 120 r.p.m., find the power absorbed in friction. **[Ans. 47.7 mm ; 143 mm ; 1.50 kW]**

12. A plain collar type thrust bearing having inner and outer diameters of 200 mm and 450 mm is subjected to an axial thrust of 40 kN. Assuming coefficient of friction between the thrust surfaces as 0.025, find the power absorbed in overcoming friction at a speed of 120 r.p.m. The rate of wear is considered to be proportional to the pressure and rubbing speed. **[Ans. 4.1 kW]**

13. The thrust on the propeller shaft of a marine engine is taken up by 8 collars whose external and internal diameters are 660 mm and 420 mm respectively. The thrust pressure is 0.4 MN/m^2 and may be assumed uniform. The coefficient of friction between the shaft and collars is 0.04. If the shaft rotates at 90 r.p.m. ; find 1. total thrust on the collars ; and 2. power absorbed by friction at the bearing.

[Ans. 651 kN ; 68 kW]

14. A shaft has a number of collars integral with it. The external diameter of the collars is 400 mm and the shaft diameter is 250 mm. If the uniform intensity of pressure is 0.35 N/mm^2 and its coefficient of friction is 0.05, estimate : 1. power absorbed in overcoming friction when the shaft runs at 105 r.p.m. and carries a load of 150 kN, and 2. number of collars required. **[Ans. 13.4 kW ; 6]**

15. A car engine has its rated output of 12 kW. The maximum torque developed is 100 N-m. The clutch used is of single plate type having two active surfaces. The axial pressure is not to exceed 85 kN/m^2. The external diameter of the friction plate is 1.25 times the internal diameter. Determine the dimensions of the friction plate and the axial force exerted by the springs. Coefficient of friction = 0.3.

[Ans. 129.5 mm ; 103.6 mm ; 1433 N]

16. A single plate clutch (both sides effective) is required to transmit 26.5 kW at 1600 r.p.m. The outer diameter of the plate is limited to 300 mm and intensity of pressure between the plates is not to exceed 68.5 kN/m^2. Assuming uniform wear and a coefficient of friction 0.3, show that the inner diameter of the plates is approximately 90 mm.

17. A multiplate clutch has three pairs of contact surfaces. The outer and inner radii of the contact surfaces are 100 mm and 50 mm respectively. The maximum axial spring force is limited to 1 kN. If the coefficient of friction is 0.35 and assuming uniform wear, find the power transmitted by the clutch at 1500 r.p.m. **[Ans. 12.37 kW]**

18. A cone clutch is to transmit 7.5 kW at 900 r.p.m. The cone has a face angle of 12º. The width of the face is half of the mean radius and the normal pressure between the contact faces is not to exceed 0.09 N/mm^2. Assuming uniform wear and the coefficient of friction between contact faces as 0.2, find the main dimensions of the clutch and the axial force required to engage the clutch.

[Ans. R = 112 mm, b = 56 mm, r_1 = 117.8 mm, r_2 = 106.2 mm ; 1433 N]

19. A cone clutch with cone angle 20º is to transmit 7.5 kW at 750 r.p.m. The normal intensity of pressure between the contact faces is not to exceed 0.12 N/mm^2. The coefficient of friction is 0.2. If face width is $\frac{1}{5}$th of mean diameter, find : 1. the main dimensions of the clutch, and 2. axial force required while running. **[Ans. R = 117 mm ; b = 46.8 mm ; r_1 = 125 mm ; r_2 = 109 mm ; 1395 N]**

20. A centrifugal friction clutch has a driving member consisting of a spider carrying four shoes which are kept from contact with the clutch case by means of flat springs until increase of centrifugal force overcomes the resistance of the springs and the power is transmitted by friction between the shoes and the case.

Determine the necessary mass of each shoe if 22.5 kW is to be transmitted at 750 r.p.m. with engagement beginning at 75% of the running speed. The inside diameter of the drum is 300 mm and the radial distance of the centre of gravity of each shoe from the shaft axis is 125 mm. Assume μ = 0.25. **[Ans. 5.66 kg]**

DO YOU KNOW ?

1. Discuss briefly the various types of friction experienced by a body.

2. State the laws of

(*i*) Static friction ; (*ii*) Dynamic friction ;

(*iii*) Solid friction ; and (*iv*) Fluid friction.

3. Explain the following :

(*i*) Limiting friction, (*ii*) Angle of friction, and

(*iii*) Coefficient of friction.

4. Derive from first principles an expression for the effort required to raise a load with a screw jack taking friction into consideration.

5. Neglecting collar friction, derive an expression for mechanical advantage of a square threaded screw moving in a nut, in terms of helix angle of the screw and friction angle.

6. In a screw jack, the helix angle of thread is α and the angle of friction is ϕ. Show that its efficiency is maximum, when $2\alpha = (90^o - \phi)$.

7. For a screw jack having the nut fixed, derive the equation (with usual notations),

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi) + \mu.r_m.r}.$$

8. Neglecting collar friction, from first principles, prove that the maximum efficiency of a square threaded screw moving in a nut is $\dfrac{1-\sin\phi}{1+\sin\phi}$, where ϕ is the friction angle.

9. Write a short note on journal bearing.

10. What is meant by the expression 'friction circle'? Deduce an expression for the radius of friction circle in terms of the radius of the journal and the angle of friction.

11. From first principles, deduce an expression for the friction moment of a collar thrust bearing, stating clearly the assumptions made.

12. Derive an expression for the friction moment for a flat collar bearing in terms of the inner radius r_1, outer radius r_2, axial thrust W and coefficient of friction μ. Assume uniform intensity of pressure.

13. Derive from first principles an expression for the friction moment of a conical pivot assuming (*i*) Uniform pressure, and (*ii*) Uniform wear.

14. A truncated conical pivot of cone angle ϕ rotating at speed N supports a load W. The smallest and largest diameter of the pivot over the contact area are '*d*' and '*D*' respectively. Assuming uniform wear, derive the expression for the frictional torque.

15. Describe with a neat sketch the working of a single plate friction clutch.

16. Establish a formula for the maximum torque transmitted by a single plate clutch of external and internal radii r_1 and r_2, if the limiting coefficient of friction is μ and the axial spring load is W. Assume that the pressure intensity on the contact faces is uniform.

17. Which of the two assumptions-uniform intensity of pressure or uniform rate of wear, would you make use of in designing friction clutch and why ?

18. Describe with a neat sketch a centrifugal clutch and deduce an equation for the total torque transmitted.

OBJECTIVE TYPE QUESTIONS

1. The angle of inclination of the plane, at which the body begins to move down the plane, is called

(*a*) angle of friction (*b*) angle of repose (*c*) angle of projection

2. In a screw jack, the effort required to lift the load W is given by

(*a*) $P = W \tan(\alpha - \phi)$ (*b*) $P = W \tan(\alpha + \phi)$

(*c*) $P = W \cos(\alpha - \phi)$ (*d*) $P = W \cos(\alpha + \phi)$

where α = Helix angle, and

ϕ = Angle of friction.

3. The efficiency of a screw jack is given by

(*a*) $\dfrac{\tan(\alpha+\phi)}{\tan\alpha}$ (*b*) $\dfrac{\tan\alpha}{\tan(\alpha+\phi)}$

(*c*) $\dfrac{\tan(\alpha-\phi)}{\tan\alpha}$ (*d*) $\dfrac{\tan\alpha}{\tan(\alpha-\phi)}$

4. The radius of a friction circle for a shaft of radius r rotating inside a bearing is

(*a*) $r\sin\phi$ (*b*) $r\cos\phi$ (*c*) $r\tan\phi$ (*d*) $r\cot\phi$

5. The efficiency of a screw jack is maximum, when

(*a*) $\alpha = 45^\circ + \frac{\phi}{2}$ (*b*) $\alpha = 45^\circ - \frac{\phi}{2}$ (*c*) $\alpha = 90^\circ + \phi$ (*d*) $\alpha = 90^\circ - \phi$

6. The maximum efficiency of a screw jack is

(*a*) $\frac{1 - \sin\phi}{1 + \sin\phi}$ (*b*) $\frac{1 + \sin\phi}{1 - \sin\phi}$ (*c*) $\frac{1 - \tan\phi}{1 + \tan\phi}$ (*d*) $\frac{1 + \tan\phi}{1 - \tan\phi}$

7. The frictional torque transmitted in a flat pivot bearing, considering uniform pressure, is

(*a*) $\frac{1}{2} \times \mu.W.R$ (*b*) $\frac{2}{3} \times \mu.W.R$ (*c*) $\frac{3}{4} \times \mu.W.R$ (*d*) $\mu.W.R$

where μ = Coefficient of friction,

W = Load over the bearing, and

R = Radius of the bearing surface.

8. The frictional torque transmitted in a conical pivot bearing, considering uniform wear, is

(*a*) $\frac{1}{2} \times \mu.W.R \text{ cosec } \alpha$ (*b*) $\frac{2}{3} \times \mu.W.R \text{ cosec } \alpha$

(*c*) $\frac{3}{4} \times \mu.W.R \text{ cosec } \alpha$ (*d*) $\mu . W.R \text{ cosec } \alpha$

where R = Radius of the shaft, and

α = Semi-angle of the cone.

9. The frictional torque transmitted by a disc or plate clutch is same as that of

(*a*) flat pivot bearing (*b*) flat collar bearing

(*c*) conical pivot bearing (*d*) trapezoidal pivot bearing

10. The frictional torque transmitted by a cone clutch is same as that of

(*a*) flat pivot bearing (*b*) flat collar bearing

(*c*) conical pivot bearing (*d*) trapezoidal pivot bearing

ANSWERS

1. (*a*)	**2.** (*b*)	**3.** (*b*)	**4.** (*a*)	**5.** (*b*)
6. (*a*)	**7.** (*b*)	**8.** (*a*)	**9.** (*b*)	**10.** (*d*)

GO To FIRST