Assistant Prof. Dr.Khitam Abdulhussein Saeed
Al-Mustansiriyah University
Faculty of Civil Engineering
Water Resources Engineering Department

SOIL MECHANICS

CHAPTER FIVE

2020-2021

Third

Stage Students

Undergraduate students (3th stage students)
Faculty of Engineering
Mustansiriyah University
Water Resources Engineering Department

CHAPTER FIVE

SOIL PERMEABILITY AND FLOW

1. The Permeability

A material is permeable if it contains continuous voids. All materials such as rocks, concrete, soils etc. are permeable. The flow of water through all of them obeys approximately the same laws. Hence, the difference between the flow of water through rock or concrete is one of degree. The permeability of soils has a decisive effect on the stability of foundations, seepage loss through embankments of reservoirs, drainage of sub grades, excavation of open cuts in water bearing sand, rate of flow of water into wells and many others.

2. Soil permeability

The soil permeability is a measure indicating the capacity of the soil or rock to allow fluids to pass through it. It is often represented by the permeability coefficient (k) through the Darcy's equation:

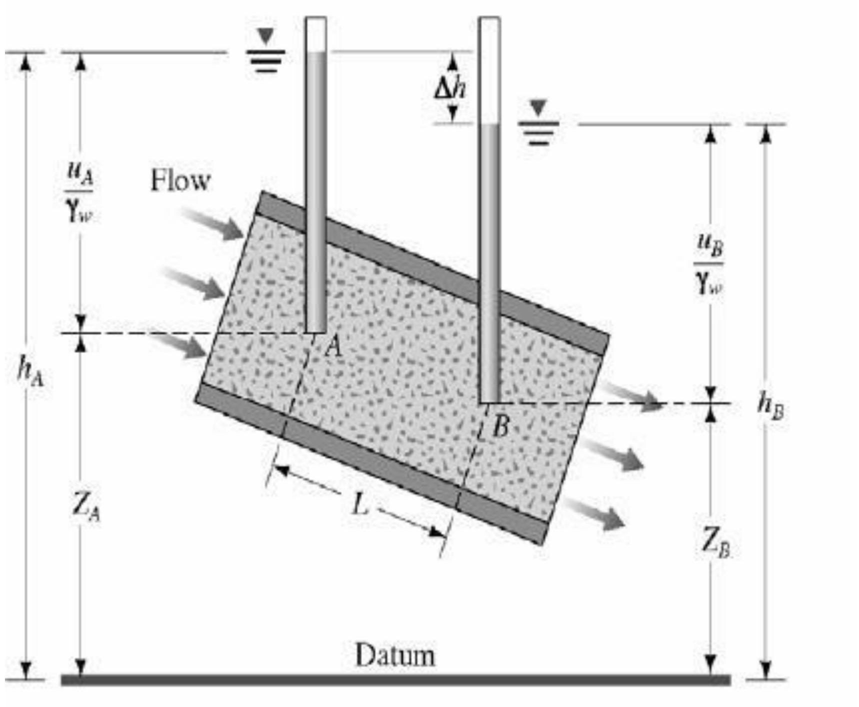
$$V=ki$$

3. Flow of Water Through Soils

3.1 Fluid Mechanics(Hydraulic Gradient)

When water flows through a saturated soil mass there is certain resistance for the flow because of the presence of solid matter. However, the laws of fluid mechanics which are applicable for the flow of fluids through pipes are also applicable to flow of water through soils. As per Bernoulli's equation, the total head at any point in water under steady flow condition may be expressed as

Total head = pressure head + velocity head + elevation head



The flow of water through a sample of soil of length L and cross sectional area A as shown in figure 1

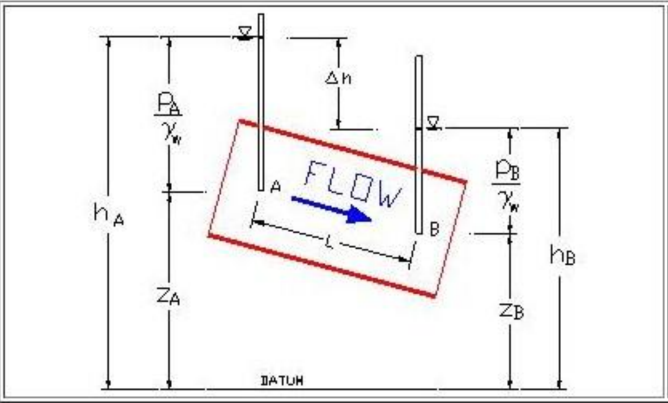
$\mathbf{h} = \frac{\mathbf{p}}{\gamma_w} + \frac{\mathbf{v}^2}{2\mathbf{g}} + \mathbf{z}$	**h = total head,** $\frac{\mathbf{p}}{\gamma_w}$ **= pressure head** $\frac{\mathbf{v}^2}{2\mathbf{g}}$ **= velocity head,** **z = elevation head**

For most soils the velocity of water flow is very small therefore the velocity head term can be neglected

$$h = \frac{u}{\gamma_w} + Z = h_p + h_e$$

$$h = \text{Pressure head} + \text{Elevation head}$$

$$H_C = Z_C + \frac{P_c}{\gamma_w} + \frac{v_c^2}{2g}$$



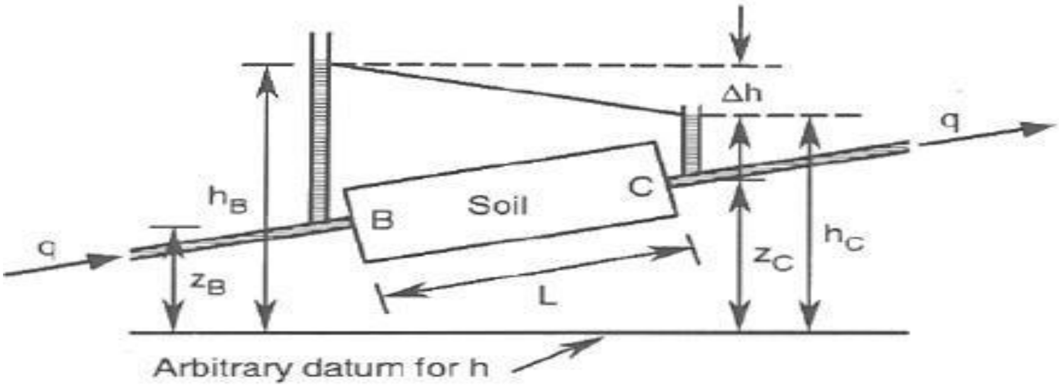
h_A = Total Head at A

h_B = Total Head at B

$\Delta h = h_A - h_B$ = Head Loss from A to B

L = Flow Length from A to B

i = Hydraulic Gradient $= \frac{\Delta h}{L}$



(1) flow of water through a soil sample

For all practical purposes the velocity head is a small quantity and may be neglected.

The water flows from the higher total head to lower total head. So the water will flow from point B to C.

$H_B - H_C = (Z_B + \frac{P_B}{\gamma_w})$ - $(Z_C + \frac{P_C}{\gamma_w})$

Where, Z_B and $Z_C = Elevation\ head,$ P_B and P_C = Pressure Head.

The loss of head per unit length of flow may be expresses as :

$$i = \frac{h}{L}$$

Where i is the hydraulic gradient.

Which is define as the potential drop between two adjacent equipotential divided by the distance between them is known as the

Hydraulic Gradient (i)

- Head loss per unit length is:

$$i = \frac{\Delta h}{L}$$

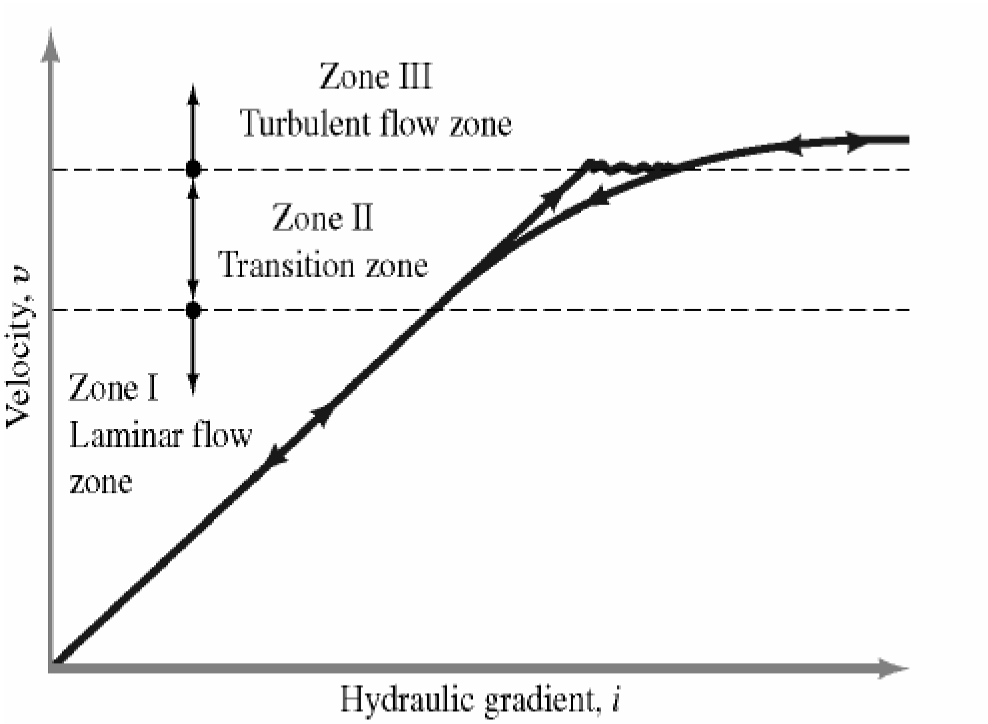
where:

i = hydraulic gradient

Δh = head loss between points of interest

L = distance between points of interest

hydraulic gradient



In Most Soils

Flow though most soils can be considered to be laminar. Therefore a linear relationship between velocity and hydraulic gradient **v α i**

Flow conditions may be turbulent in fractured rock, stones, gravel, and very coarse sands

4. Darcy's Law and Other Principles

Darcy's law is a constitutive equation that describes the flow of a fluid through a porous medium. The law was formulated by Henry Darcy based on the results of experiments on the flow of water through beds of sand, forming the basis of hydrogeology, a branch of earth sciences.

Darcy in 1856 derived an empirical formula for the behavior of flow through saturated soils. He found that the quantity of water q per sec flowing through a cross-sectional area of soil under hydraulic gradient / can be expressed by the formula

q = kiA

or the velocity of flow can be written as

$$V = \frac{q}{A}$$

Where k is termed the hydraulic conductivity (or coefficient of permeability) with units of velocity. The coefficient of permeability is inversely proportional to the viscosity of water which decreases with increasing temperature; therefore, permeability measurement at laboratory temperatures should be corrected to the values at standard temperature of 200C using the following equation.

$$k_{20} = \frac{\mu_T}{\mu_{20}} k_T$$

Where k_{20} : Coefficient of permeability at 20^0 C

k_T : Cofficient of permeability at Lab. Temperture^0 C

μ_T Viscosity of water at lab. Temperature

μ_{20} Viscosity of water at 20^0C

Temperature T(°C)	μ_T/μ_{20}	Temperature T(°C)	μ_T/μ_{20}
10	1.298	21	0.975
11	1.263	22	0.952
12	1.228	23	0.930
13	1.195	24	0.908
14	1.165	25	0.887
15	1.135	26	0.867
16	1.106	27	0.847
17	1.078	28	0.829
18	1.051	29	0.811
19	1.025	30	0.793
20	1.000		

Table (1) :The of $\frac{\mu_T}{\mu_{20}}$ at different temperature.

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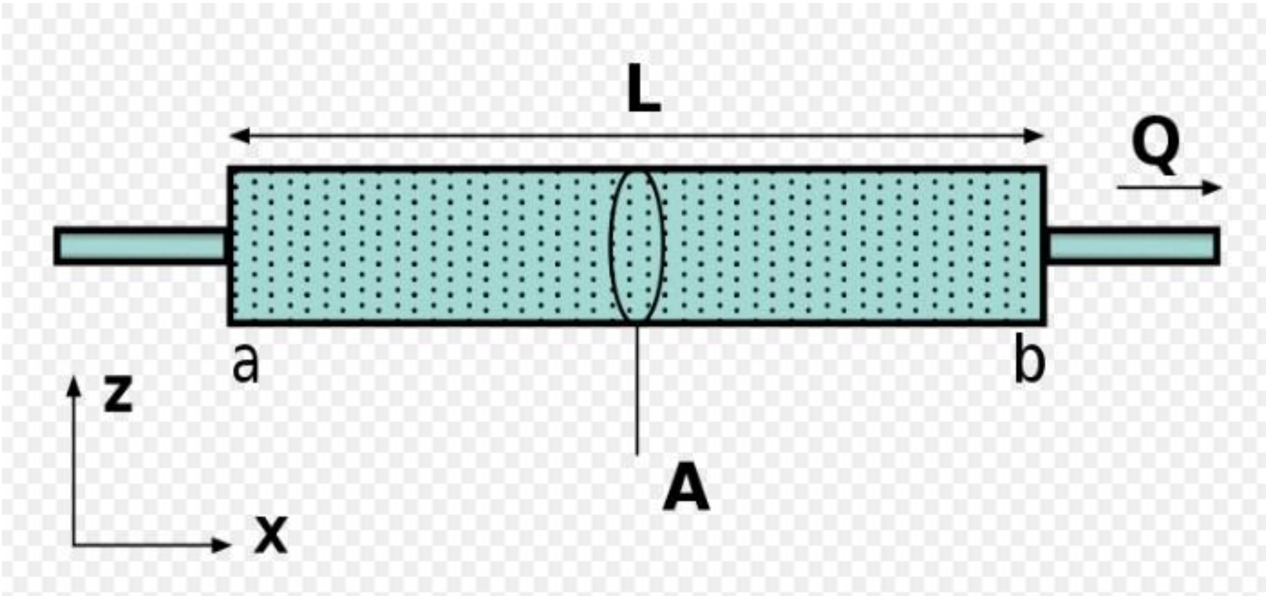


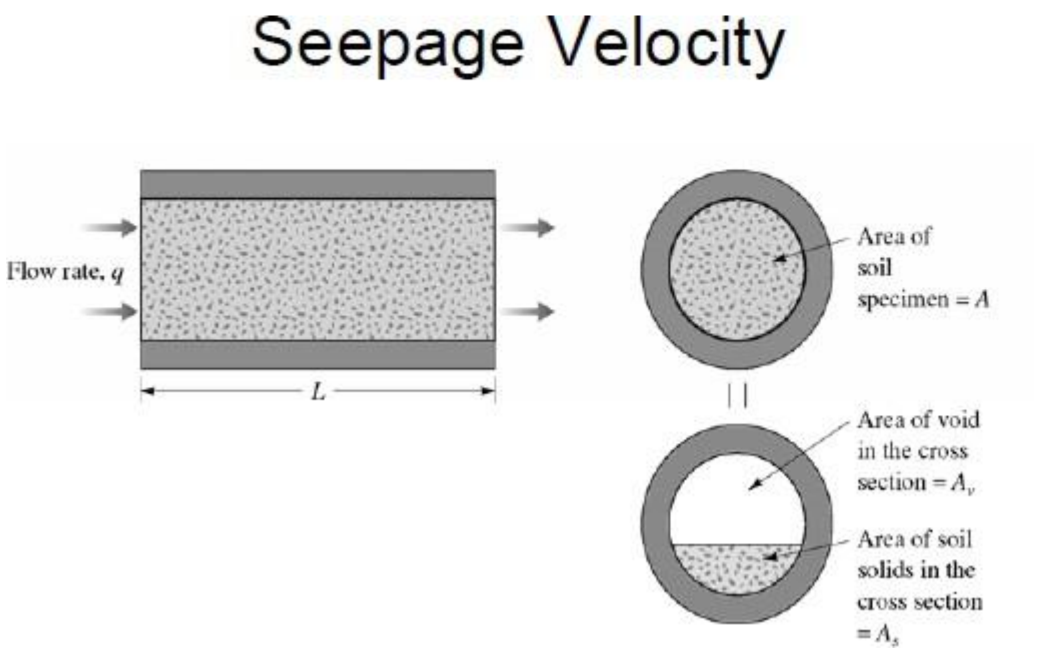
Diagram showing definitions and directions for Darcy's law. A is the cross sectional area (m2) of the cylinder. Q is the flow rate (m3/s) of the fluid flowing through the area A. The flux of fluid through A is q = Q/A. L is the length of the cylinder. Δp = p-outlet - pinlet = pb - pa. = Δp/L = hydraulic gradient applied between the points a and b.

A. Darcy's Equation

v = ki	**v = average velocity, ft / s or m / s** **k = coefficient of permeability, ft / s or m / s** **i = hydraulic gradient**

5- Discharge Velocity

Quantity of water flowing in unit time through a unit gross cross sectional area of soil at right angles to the direction of flow . Does not account for flow through soil voids.

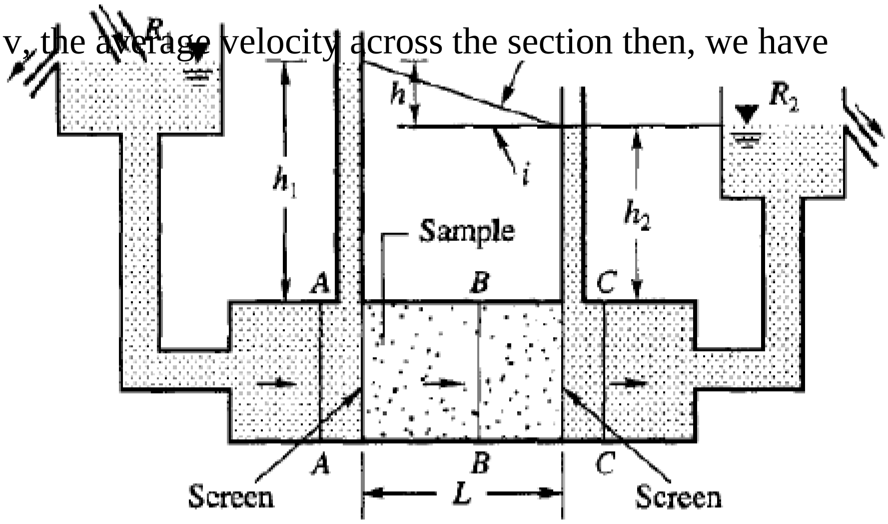


6- DISCHARGE AND SEEPAGE VELOCITIES

Figure below shows a soil sample of length L and cross-sectional area A. The sample is placed in a cylindrical horizontal tube between screens. The tube is connected to two reservoirs R1 and R2 in which the water levels are maintained constant. The difference in head between R1 and R2 is h. This difference in head is responsible for the flow of water. Since Darcy's law assumes no change in the volume of voids and the soil is saturated, the quantity of flow past sections AA, BB and CC should remain the same for steady flow conditions. We may express the equation of continuity as follows

qaa = qbb = qcc

If the soil be represented as divided into solid matter and void space, then the area available for the passage of water is only Av. If vs. is the velocity of flow in the voids, and v, the average velocity across the section then, we have



Where A_v is the area of the void,

v_s is the seepage velocity,

v is the approach velocity

A: is the cross sectional area of the sample

$$v_s = \frac{A * L}{A_v * L} v = \frac{v_t}{v_v} v = \frac{v}{n}$$

Where n : is the porosity of the soil

•degree of saturation

• Size of double layer (clay type)

Hydraulic Conductivity

$$k = \frac{K\rho g}{\eta} = \frac{K\gamma_w}{\eta}$$

where:

K = absolute permeability (L^2)

ρ = density of fluid

g = gravitational constant

η = viscosity of fluid

γ_w = Unit weight of fluid

Table shows the typical values of hydraulic conductivity for saturated soils

	k	k
Soil Type	cm/sec	ft/min
Clean Gravel	1.0 to 100	2.0 to 200
Coarse Sand	0.01 to 1.0	0.02 to 2.0
Fine Sand	0.001 to 0.01	0.002 to 0.02
Silty Clay	0.00001 to 0.001	0.00002 to 0.002
Clay	Less Than 0.000001	Less Than 0.000002

A-Laboratory Measurement of Hydraulic Conductivity

Cohesion less soils (sand and gravel)

•**Constant head test**

•**Falling head test**

Cohesive soils (silt and clay(

• **Triaxial cell**

B- Field methods:

1- Pumping test

2- Bore hole tests

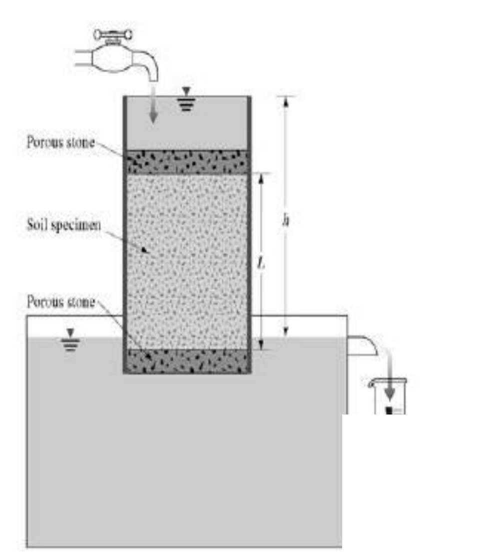
Laboratory methods

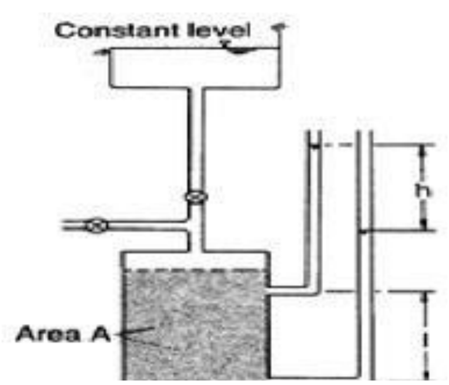
1- Constant head permeability test

The coefficient of permeability for coarse soils can be determined by means of the constant-head permeability test (as shown in figures): A steady vertical flow of water, under a constant total head, is maintained through the soil and the volume of water flowing per unit time (q):

A series of tests should be run, each at different rate of flow. Prior to running the test a vacuum is applied to the specimen to ensure that the degree of saturation under flow

Constant Head Test





Constant Head Test

Q = vAt = kiAt = k(h/L)At

where:
Q = quantity of flow (L^3)
A = cross section area of column (L^2)
t = duration of water collection (T)

Solve for k:

$$k = \frac{QL}{Aht}$$

2- Falling head permeability test:

For fine soils the falling-head test (Figure below) should be used. In the case of fine soils, undisturbed specimens are normally tested. The length of the specimen is l and the cross-sectional area A. the standpipe is filled with water and a measurement is made of the time (t_1) for water level (relative to the water level in the reservoir) to fall from h_0 to h_1. At any intermediate time t the water level in the standpipe is given by h and its rate of change by $-\frac{dh}{dt}$. At time t the difference in total head between the top and bottom of the specimen is h. then applying Darcy's law:

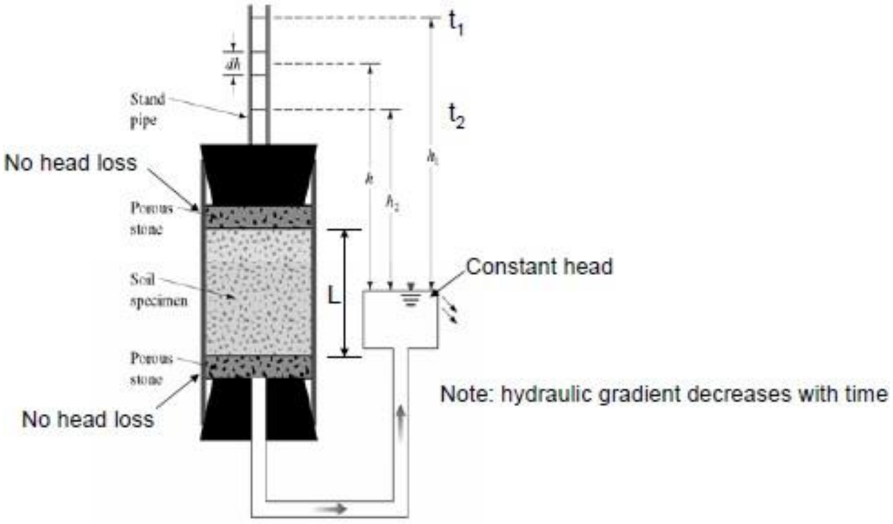
$$-a\frac{dh}{dt} = AK\frac{h}{l}$$

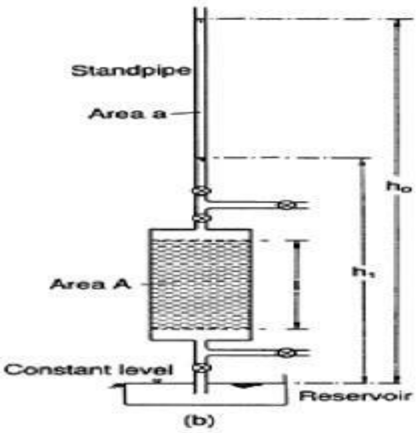
$$-a\int_{h_0}^{h_1}\frac{dh}{h} = \frac{AK}{l}\int_0^{h} dt$$

$$\therefore K = \frac{al}{At_1} ln\frac{h_0}{h_1} = 2.3\frac{al}{At_1}\log\frac{h_0}{h_1}$$

Ensure that the degree of saturation remains close to 100%. A series of tests should be run using different values of h_0 and h_1

Falling Head Test





Falling Head Test

- Record level h_1 at t_1=0
- Head is allowed to flow such that the final head difference is at time t = t_2 is h_2

Falling Head Test

- Record level h_1 at t_1=0
- Head is allowed to flow such that the final head difference is at time t = t_2 is h_2

$$q_{\text{standpipe}} = -a\frac{dh}{dt} = q_{\text{into soil column}}$$

where:

a = cross sectional area of stand pipe

dh/dt = change in head in change in time= velocity of water falling

Minus sign is used to indicate falling head (decreasing head)

Falling Head Test

$$q_{\text{soil}} = k\frac{h}{L}A$$

where:

k = soil hydraulic conductivity

h = total head loss

L = length of soil column resulting in head loss

A = soil column cross sectional area

Continuity:

$$q_{in} = q_{soil}$$

Example 1:

A constant head permeability test was carried out on a cylindrical of sand 4 in. in diameter and 6 in. in height. 10 in^3 Of water was collected in 1.75 min, under a head of 12 in. Compute the hydraulic conductivity in ft/year and the velocity of flow in ft/sec.

Solution:

$$k = \frac{Q}{Ai\,t}$$

$$Q = 10\,in^3\ , A = 3.14 * \frac{4^2}{4} = 12.56\,in^2$$

$$i = \frac{h}{L} = \frac{12}{6} = 2, t = 105\,sec$$

Therefore $k = \frac{10}{12.56*2*105} = 3.79 * \frac{10^{-3} in}{sec} = 31.58 * 10^{-5}\,ft/sec$

Velocity of flow $= ki = 31.58 * 10^{-5} * 2 = 6.316 * 10^{-4}\,ft/sec$

Example 2

A sand sample of 35 yP cross sectional area and 20 cm long was tested in a constant head permeameter. Under a head of 60 cm, the discharge was 120 ml in 6 min. The dry weight of sand used for the test was 1120 g. and Gs = 2.68. Determine (a) the hydraulic conductivity in cm/sec. (b) the discharge velocity, and (c) the seepage velocity.

Solution:

$$k = \frac{QL}{\Delta hAt}$$

Q = 120 ml, t =6 min, $A = 35\ cm^2, L = 20\ cm, and\ h = 60\ cm$

$$k = \frac{120*20}{60*35*6*60} = 3.174 * 10^{-3} cm/sec$$

Discharge velocity, v= ki =$31.74 * 10^{-3} * \frac{60}{20} = 9.52 * 10^{-3} cm/sec$

Seepage velocity v_s

$$\gamma_d = \frac{w_s}{v} = \frac{1120}{35 * 20} = 1.6\ gm/cm^3$$

$$\gamma_d = \frac{\gamma_w G_s}{1+e}\ or\ \ e = \frac{G_s}{\gamma_d} - 1$$

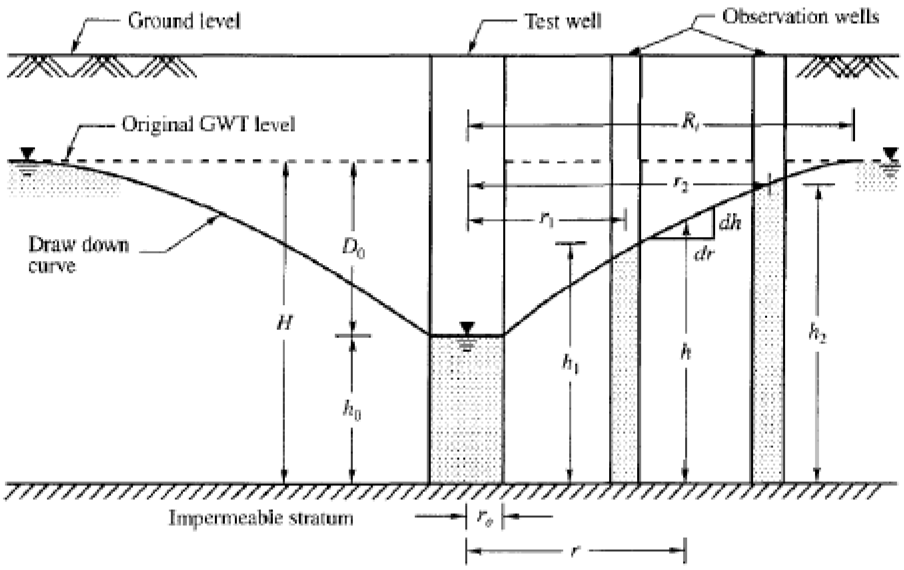
$$e = \frac{2.68}{1.6} - 1 = 0.675$$

$$n = \frac{e}{1+e} = 0.403$$

$$v_s = \frac{v}{n} = \frac{9.52 * 10^{-3}}{0.403} = 2.36 * 10^{-2}\ cm/sec$$

DIRECT DETERMINATION OF K OF SOILS IN FIELD:

1- Field test in unconfined aquifer



Pumping Test in an unconfined aquifer

$$k = \frac{2.3q}{\pi(h_2^2 - h_1^2)} \log\frac{r_2}{r_1}$$

2- Field test in Confined aquifer

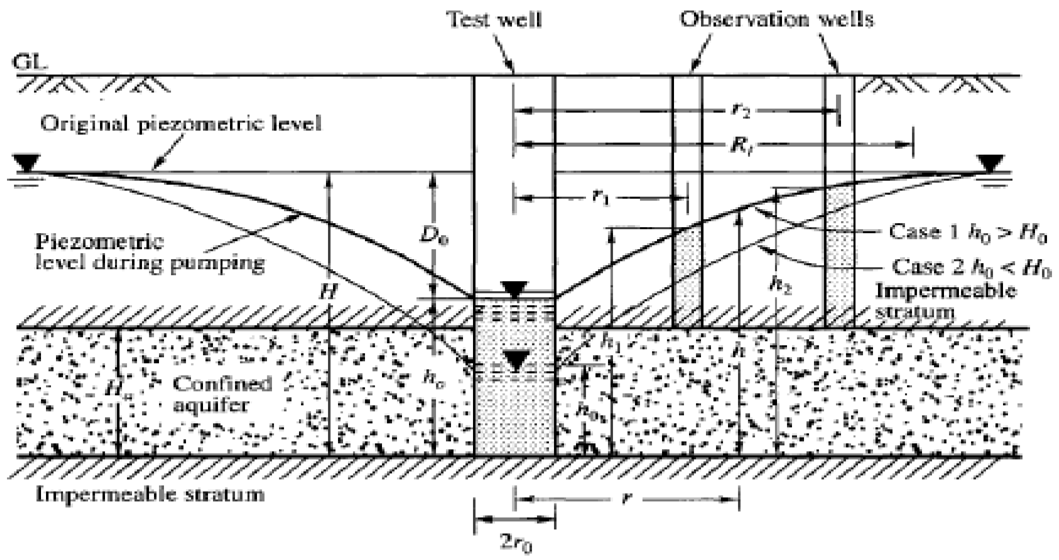
There are two cases :

Case 1- when $h_0 > H_0$

$$k = \frac{2.3q}{2\pi H_0 (h_2 - h_1)} \log\frac{r_2}{r_1}$$

Case 2-when $h_0 < H_0$

$$k = \frac{2.3q}{\pi(2HH_0 - H_0^2 - h_0^2)} \log\frac{R_i}{r_0}$$



Empirical Relations for Hydraulic Conductivity

Several empirical equations for estimating hydraulic conductivity have been proposed over the years.

K Relationships for Granular Soils

Hazen Equation

$$k(\text{cm/s}) = cD^2_{10}$$

where:

c= a constant that varies from 1.0 to 1.5

D_{10} = effective particle size for 10 percent passing (mm)

Equation works OK for clean loose sand.

The above equation is based primarily on Hazen's observations of loose, clean, filter sands. A small quantity of silts and clays, when present in a sandy soil, may change the hydraulic conductivity substantially.

The accuracy of the values of k determined in the laboratory depends on the following factors:

1- Temperature of the fluid

2- Viscosity of fluid

3- Trapped air bubbles present in the specimen

4- Degree of saturation

5- Migration of fines during testing

6- Duplication of field conditions in the laboratory.

The coefficient of consolidation of saturated cohesive soils can be determined by laboratory consolidation tests. This will be listed in details in "consolidation of soil"

Table -1 Typical Values of Hydraulic Conductivity of Saturated Soils

Typical values of permeability	
Gravel	$> 10^{-1}$ m/s
Sands	10^{-1} to 10^{-5} m/s
Fine sands, coarse silts	10^{-5} to 10^{-7} m/s
Silts	10^{-7} to 10^{-9} m/s
Clays	$< 10^{-9}$ m/s

Exercise:

1-

In a constant-head permeability test in the laboratory, the following are given: $L = 300$ mm and $A = 110\ \text{cm}^2$. If the value of $k = 0.02$ cm/sec and a flow rate of $140\ \text{cm}^3/\text{min}$ must be maintained through the soil, what is the head difference, h, across the specimen? Also, determine the discharge velocity under the test conditions.

2-

For a variable-head permeability test, the following are given:

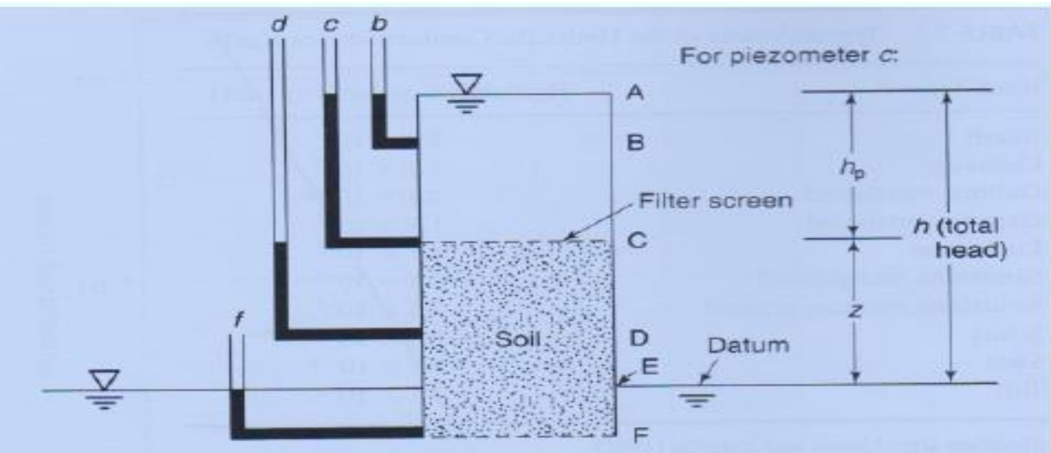
- Length of the soil specimen = 20 in.
- Area of the soil specimen = 2.5 in.^2
- Area of the standpipe = 0.15 in.^2
- Head difference at time $t = 0$ is 30 in.
- Head difference at time $t = 8$ min is 16 in.

a. Determine the hydraulic conductivity of the soil (in./min)

b. What is the head difference at time $t = 6$ min?

HEADS AND ONE-DIMENSIONAL FLOW

There are three heads which must be considered in problem involving fluid flow in soil as shown in the figure below:



Point	Pressure Head	Elevation Head	Total Head	Head Loss through Soil
B	AB	BE	AE	0
C	AC	CE	AE	0
D	CD	DE	CE	½AE
F	EF	−EF	0	AE

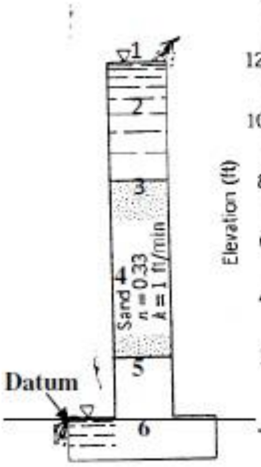
Figure: illustration of types of head (after Taylor, 1948).

1- Pressure head (h_p) is the pizometer reading = pore water pressure /unit weight of water

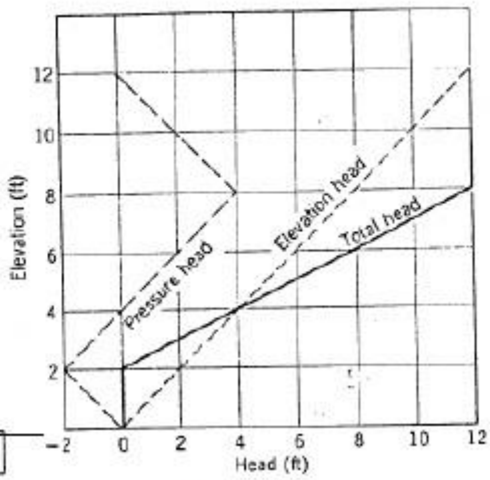
2- Elevation head at any point (h_e) is the vertical distance above or below some reference elevation or datum plane.

3- Total head, $h = h_p + h_e$

Example 1: For the Setup shown (Figure 5a), plot, ht, he, hp and the velocity of flow?



Figure(5a)



Figure(5b)

Figure(5c)

Points	ht(ft) (Figure 5b)	he(ft) (Figure 5b)	hp=ht-he(ft) (Figure 5b)	V(ft/min)*=Ki (Figure5c)
1	12	12	0	2
2	12	10	2	2
3	12	8	4	6
4	$=\frac{12+0}{2}=6$	5	1	6
5	0	2	-2	6
6	0	0	0	2

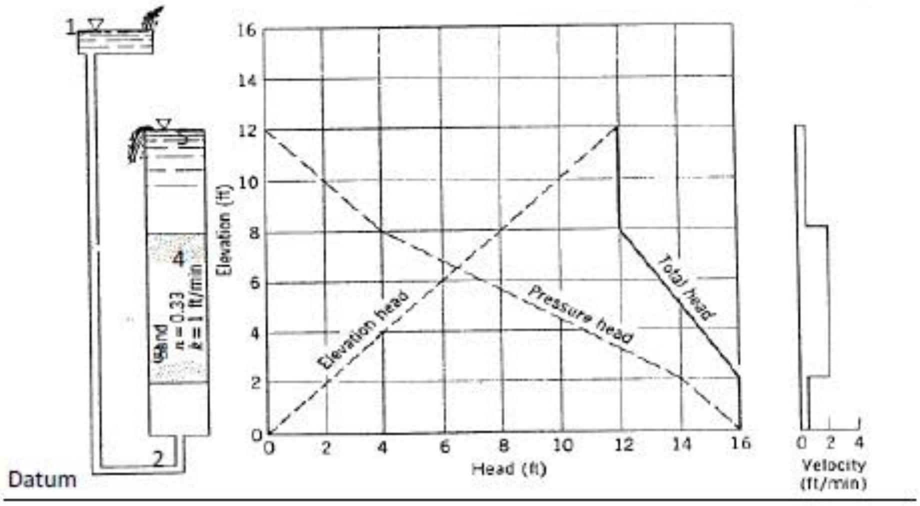
$^{*}i = \frac{h_{t\,at\,3} - h_{t\,at\,5}}{L_{3-5}} = \frac{12-0}{6} = 2$

Approch velocity = ki= 1*2= 2 ft/min

Seepage velocity $= \frac{v}{n} = \frac{2}{0.333} = 6\ ft/min$

Example 2

For the setup shown(Figure 6a) , Draw, ht, he , hp and velocity of flow ?



1- Direction of flow is upward flow (look to the water'symbole usaually water flow from higher one to lower one)
2- List all point with direction of flow
3- Construct a table to solve the problem

Points	ht(ft) (Figure 6b)	he(ft) (Figure 6b)	hp=ht-he(ft) (Figure 6b)	V(ft/min)* =Ki (Figure6c)
1	16	16	0	
2	16	2	14	
3	$\frac{16+12}{2}=14$	5	9	
4	12	8	4	
5	12	12	0	

$^{*}i = \frac{h_{t\,at\,2} - h_{t\,at\,4}}{L_{2-4}} = \frac{16-12}{6} = 0.667$

Approch velocity = ki= 1*0.667= 0.667 ft/min

Seepage velocity $= \frac{v}{n} = \frac{0.667}{0.333} = 2\ ft/min$

Example 3: For the setup shown(Figure below) , Draw, ht, he , hp and velocity of flow ?

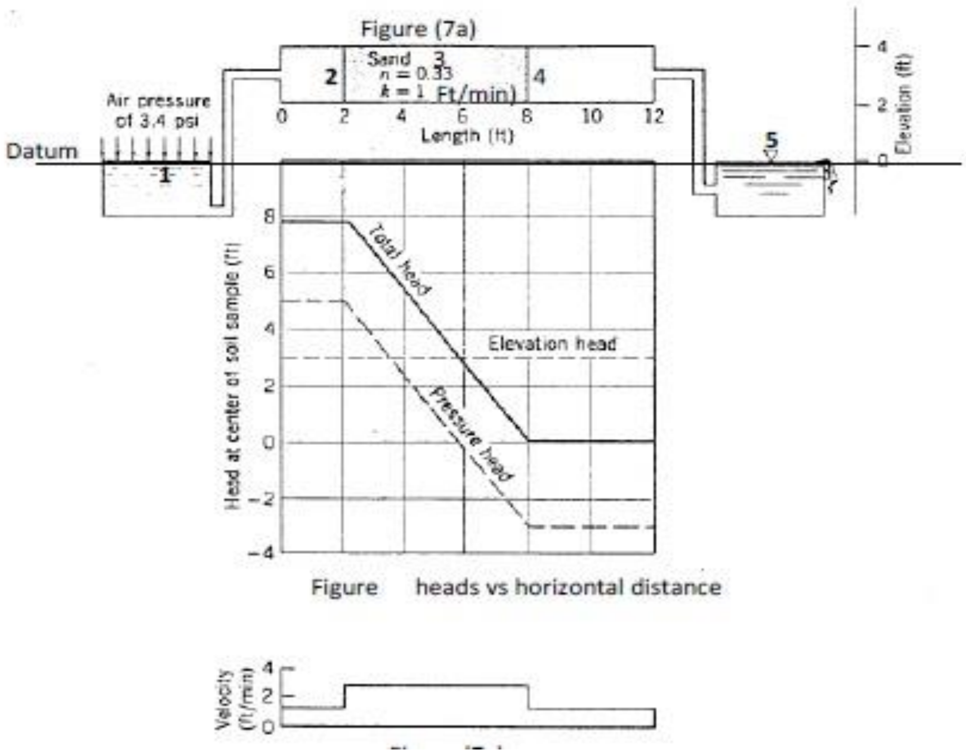


Figure heads vs horizontal distance

Solution:

1- Assume any arbitrary line representing the datum and let it at elevation =0(Figure 7a).
2- The flow will be in horizontal direction (elevation head is constant)
3- Construct the table

Since pressure =3.4 psi=3.4*144= 489.6 lb/ft^2

$$hp = \frac{pressure}{unit\ weight\ of\ water} = \frac{489.6}{62.4} = 7.84\ ft$$

Points	ht(ft)	he(ft)	hp(ft)
1	7.84	0	8
2	7.84	3	5.84
3	3.92	3	0.92
4	0	3	-3
5	0	0	0

$$i = \frac{h_{t\ at\ 2} - h_{t\ at\ 4}}{L_{2-4}} = \frac{7.84 - 0}{6} = 1.3$$

Approch velocity = ki= 1*1.3= 1.3 ft/min

Seepage velocity $= \frac{v}{n} = \frac{1.3}{0.333} = 3.9\ ft/sec$

Example 4

For the setup shown in figure 8:a) - Calculate the pressure head, elevation head, total head and head loss at points B, C,D and F in centimeter of water. b)-Plot the heads versus the elevation.

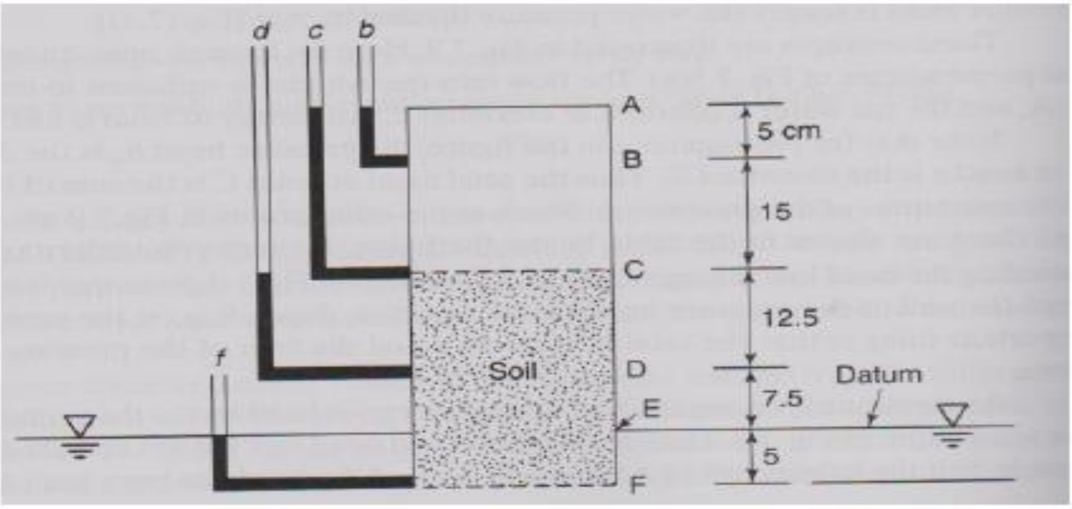
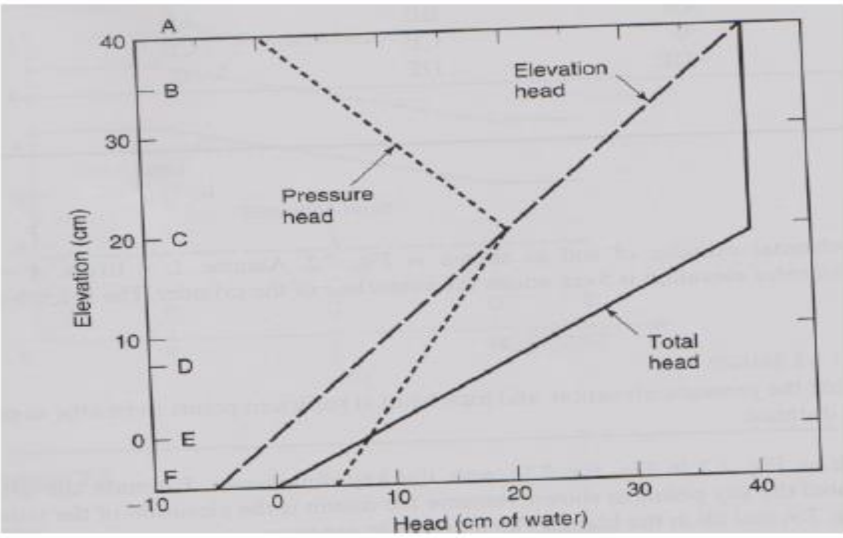


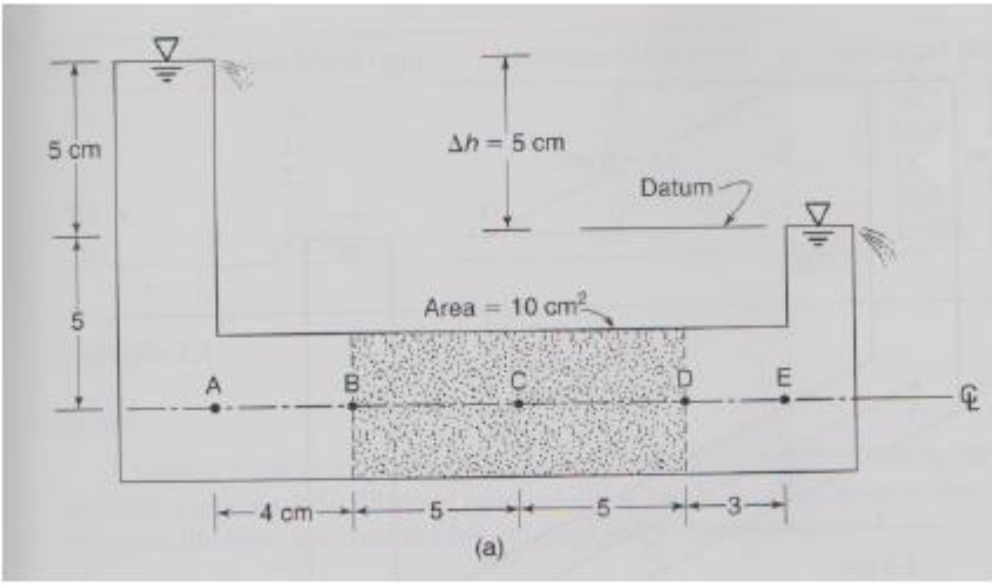
Figure shows the Set up of example 4

Solution (1):

points	ht(cm)	he(cm)	hp(cm)	Head loss
B	40	35	5	0
C	40	20	20	0
D	20	7.5	12.5	20
F	0	-5	5	40

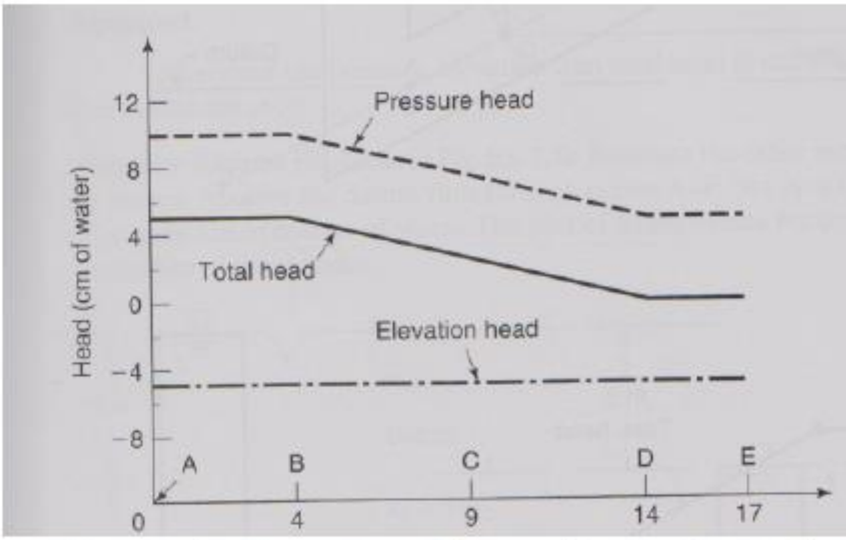


Example (5): for the setup shown Calculate and plot total head, elevation head and pressure head.

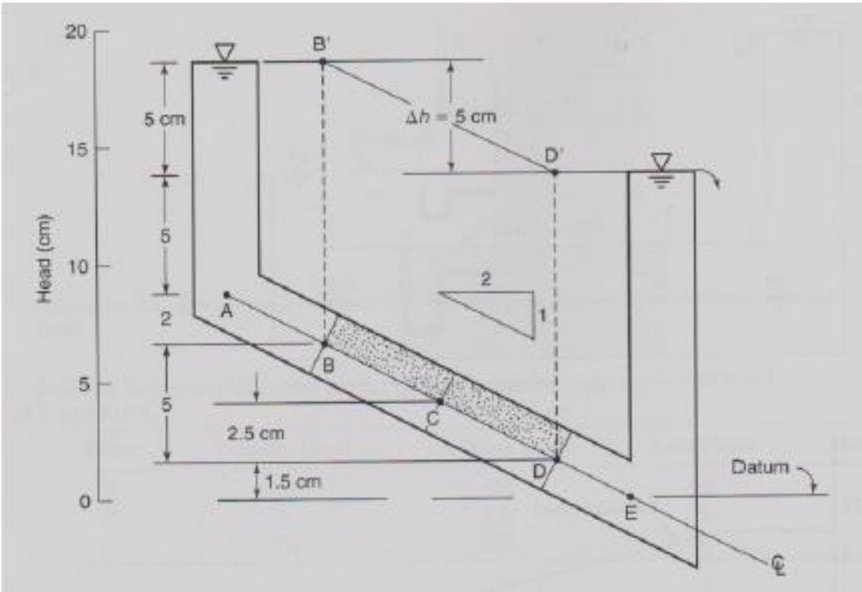


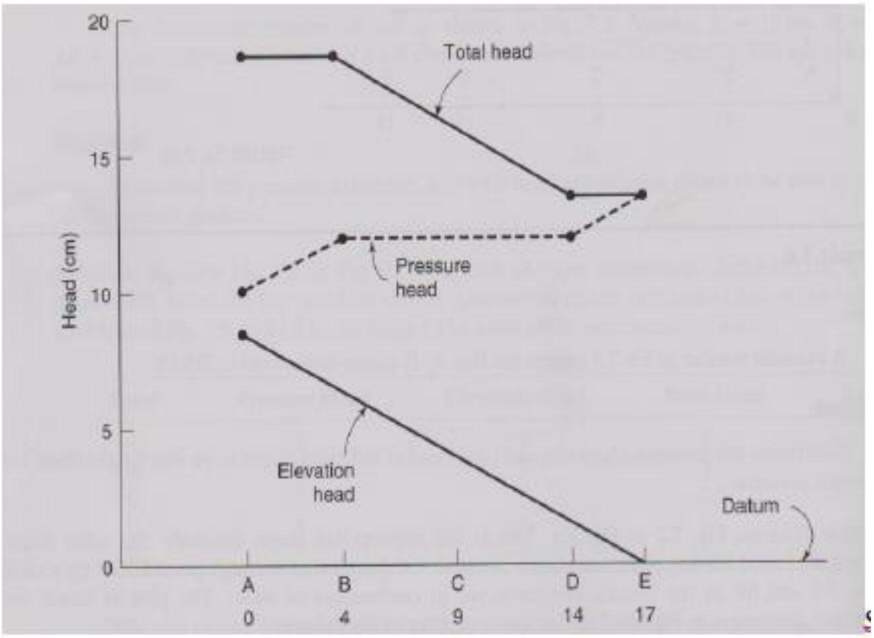
(a)

Points	Ht(cm)	He(cm)	Hp(cm)	Head loss
A	5	-5	10	0
B	5	-5	10	0
C	2.5	-5	7.5	2.5
D	0	-5	5	5



Example -6 : For the set up shown , draw the variation of total head, pressure head and elevation head along points A,B,C,D and E.

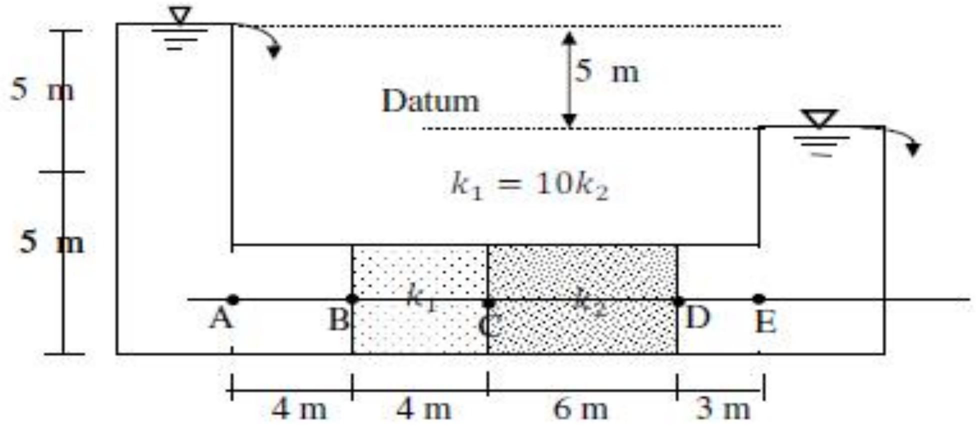




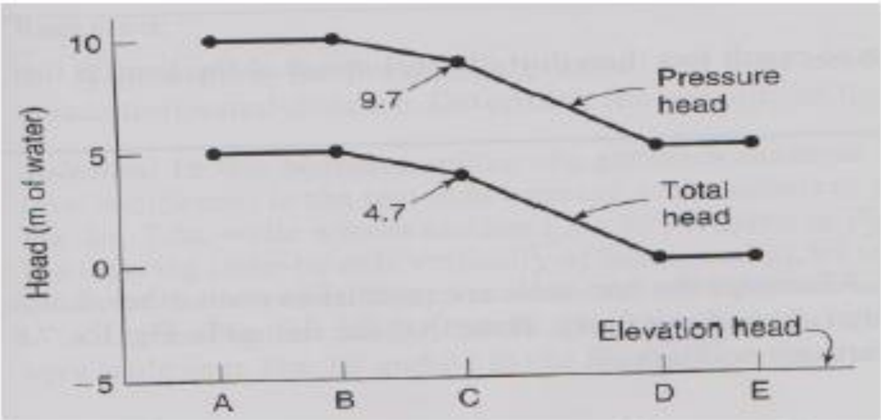
Solution of Example 6

point	Total Head(cm)	Elevation Head(cm)	Pressure Head(cm)	Head Loss (cm)
A	18.5	8.5	10	0
B	18.5	6.5	12	0
C	16	4	12	2.5
D	13.5	1.5	12	5
E	13.5	0	13.5	5

Example 7: For the setup shown, Find total head (ht) , Elevation head (he) and Pressure head(hp) for the soil the setup shown.



Example 7 setup



series. Thus the quantity of flow in one soil has to be the same as in the second soil. So,

$$q_1 = k_1 i_1 A_1 = q_2 = k_2 i_2 A_2$$

Since the areas are the same, $q_{1,2} = k_1 i_1 = k_2 i_2$ with $k_1 = 10k_2$ and $i = \Delta h/l$. Substituting,

$$q_{1,2} = 10k_2 \frac{\Delta h_1}{L_1} = k_2 \frac{\Delta h_2}{L_2}$$

Also, the total head loss, $\Delta h = \Delta h_1 + \Delta h_2$. So, $\Delta h_1 = \Delta h - \Delta h_2$, and we obtain

$$q_{1,2} = 10k_2 \frac{(\Delta h - \Delta h_2)}{L_1} = k_2 \frac{\Delta h_2}{L_2}$$

Rearranging and multiplying out,

$$L_2 10k_2 \Delta h - L_2 10k_2 \Delta h_2 = k_2 \Delta h_2 L_1$$

Rearranging and canceling out the k_2's,

$$10L_2 \Delta h = \Delta h_2 (L_1 + 10L_2)$$

Solving for Δh_2,

$$\Delta h_2 = \frac{10L_2 \Delta h}{L_1 + 10L_2}$$

$$= \frac{10 \times 6\text{ m} \times 5\text{ m}}{(4\text{ m} + 10 \times 6\text{ m})} = \frac{300\text{ m}^2}{64\text{ m}}$$

$$= 4.69\text{ m}$$

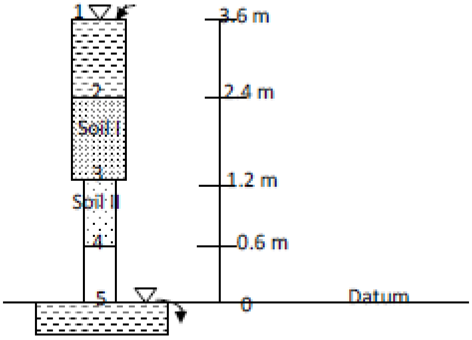
$$= 4.69\text{ m}$$

$$\therefore \Delta h_1 = \Delta h - \Delta h_2 = 5 - 4.69 = 0.31\text{ m}$$

Point	Pressure Head (m)	Elevation Head (m)	Total Head (m)	Head Loss (m)
A	10	−5	5	0
B	10	−5	5	0
C	9.7	−5	4.7	0.31
D	5	−5	0	5
E	5	−5	0	5

Because the permeability of soil 2 is so much less than that of soil 1, most of the head is lost in soil 2.

Example 8:For Setup shown: Soil I , A= 0.37 m^2 , n=0.5 and k= 1 cm/sec, Soil II, A= 0.186 m^2 , n=0.5 and k=0.5 cm/sec.



Points	ht(m)	he(m)	hp(m)
1	3.6	3.6	0
2	3.6	2.4	1.2
3	2.4	1.2	1.2
4	0	0.6	-0.6
5	0	0	0.0

Solution :

1- qI=qII

$$k_I i_I A_I = k_{II} i_{II} A_{II}$$

$$= \frac{1\ cm/sec}{100} * \frac{\Delta h_I}{(2.4-1.2)} * 0.37 = \frac{0.5\ \text{cm/sec}}{100} * \frac{\Delta h_{II}}{(1.2-0.6)} * 0.1 ----(1)$$

2- $\Delta h_I + \Delta h_{II} = 3.6$ ------------------(2)

From Equation(1) -----------$\Delta h_I = 0.502\ \Delta h_{II}$

Substitute in Equation 2--------- $0.502\Delta h_I + \Delta h_{II} = 3.6$

$$\therefore\ \Delta h_{II} = 2.4\ m$$

$$\Delta h_I = 1.2\ m$$

Approach Velocity= ki

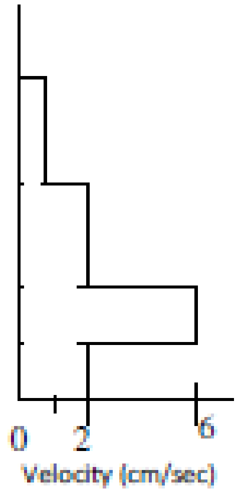
Approach velocity for soil I=$k_I i_I = 1 * \frac{1.2}{(2.4-1.2)} = 1\ cm/sec$

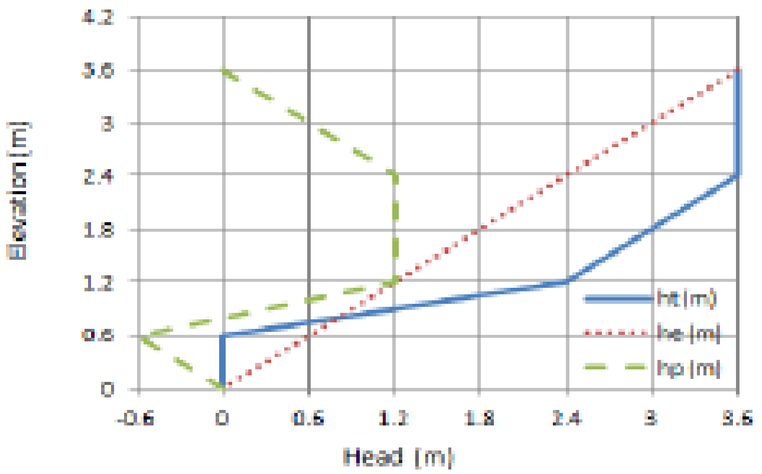
Seepage velocity $= \frac{v}{n} = \frac{1}{0.5} = 2\ cm/sec$

For soil II

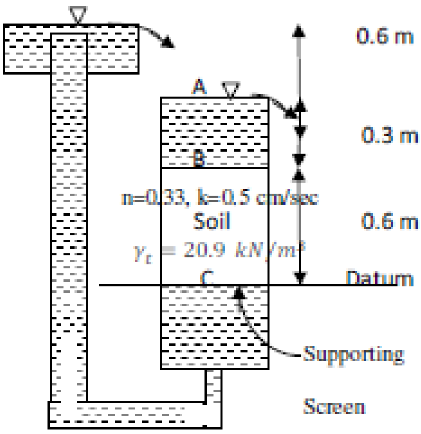
Approach velocity$= k_{II} i_{II} = 0.5 * \frac{2.4}{(1.2-0.6)} = 2\ cm/sec$

Seepage velocity (II) $= \frac{v}{n} = \frac{2}{0.333} = 6 cm/sec$

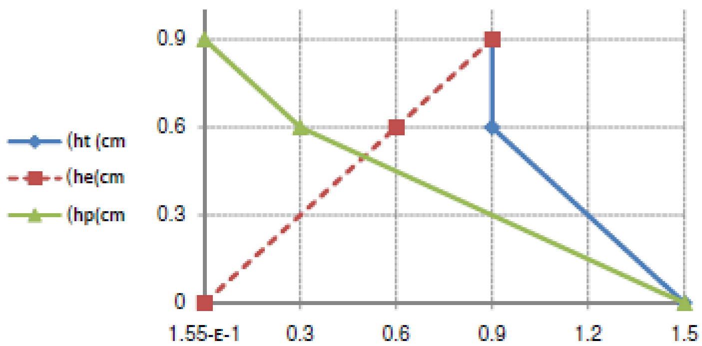




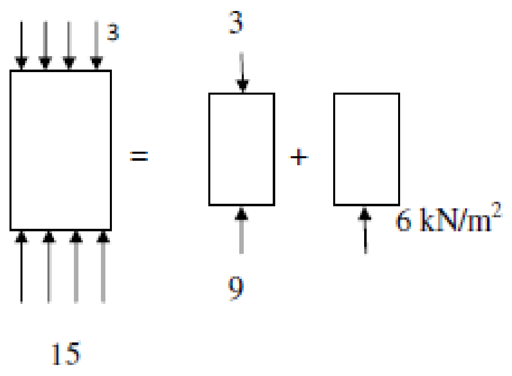
Example 9: For the setup shown draw ht, he, hp and find the seepage force .



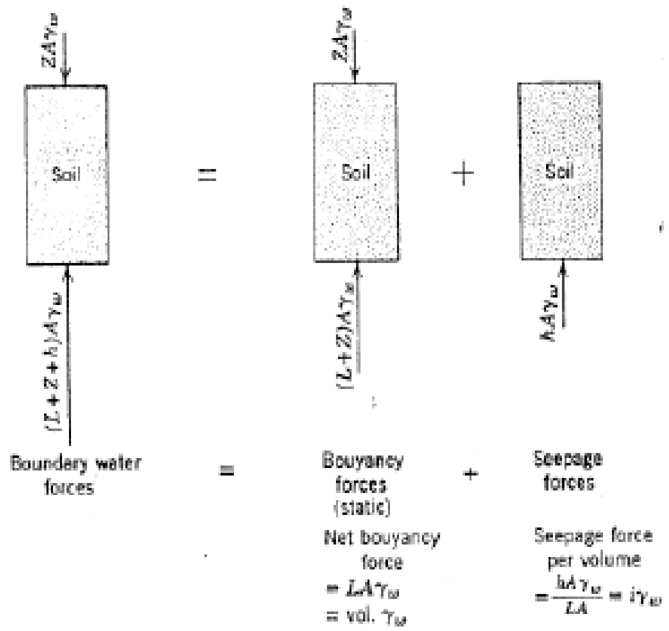
Points	ht(cm)	he(cm)	hp(cm)
A	0.9	0.9	0
B	0.9	0.6	0.3
C	1.5	0.0	1.5



Elevation (cm)	$\Delta\sigma_{v\,(kN/m^2)}$	$\sigma_v(\frac{kN}{m^2})$	$u(\frac{kN}{m^2})$	
0.9		0	0	0
	0.3 *10=3kN/m^2			
0.6		3	0.3*10=3	0
	0.6*20.9 = 12.54			
0		15.54	1.5*10=15	0.54



Water pressure on soil sample (a) Boundary water pressure (b) Buoyancy water pressure(static) (c) Pressure lost in seepage.



Water Force on Soil:

$$j = \frac{Seepage\ force}{Volume\ of\ soil} = \frac{hA\gamma_w}{LA} = i\ \gamma_w$$

Seepage forces usually act with direction of flow.

Quick Condition:

The shear strength of cohesionless soil is directly proportional to the effective stress. When a cohesionless soil is subjected to a water condition that results in zero effective stress, the strength of the soil becomes zero and quick condition exists.

Quick condition: occurs in upward flow(for cohesionless soil) and when the total stress equals to pore water pressure .

$$\acute{\sigma}_{effect} = 0 = LA\gamma_w - hA\gamma_w = 0$$

$$\frac{h}{L} = i = \frac{\gamma_b}{\gamma_w} = i_c$$

i_c: The gradient required to cause a quick condition, termed critical gradient.

$$\frac{h}{L} = i = \frac{\gamma_b}{\gamma_w} = i_c$$

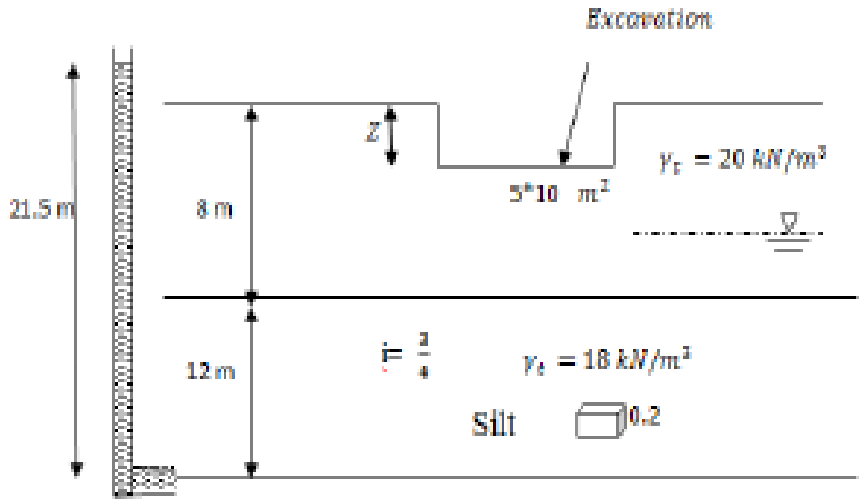
i_c: The gradient required to cause a quick condition, termed critical gradient.

Example 10: Excavation is been carried out as shown in the figure. Find: 1- the depth Z that could caused boiling at the bottom of clay layer.

2-The depth (Z) for the factor of safety against boiling equal to 2 at the bottom of the clay layer.

3- What is the thickness of the raft foundation that should be used before boiling occurs. If an uplift pressure of 60 kN/m^2 at the bottom of excavation exist($\gamma_{concrete} = 25\ kN/m^3$).

4- Find the seepage force at an element of 0.2 m cube located at the center of silt layer.



Solution:

$$ht_1 = 21.5$$

1- $i = \frac{3}{4} = \frac{\Delta h_{1--2}}{10\ m} = \frac{21.5 - ht_2}{10}$ ------ $\therefore\ ht_2 = 14\ m$

$$\therefore\ hp_2 = ht_2 - he_2 = 14 - 10 = 4\ m$$

To find Z

F. down = F. upward

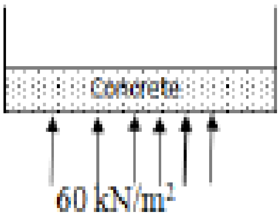
A(8-z) *20 = 4.0 *10 *A ⟶ z= 6 m

2- $F.S = \frac{down\ ward\ force}{up\ ward\ force}$

$$2 = \frac{(8 - z) * 20 * A}{4 * 10 * A}$$

$$\therefore Z = 4\ m$$

3- F down ward = F upward



t*5*10*25 = 60*5*10

$\therefore t = \frac{60}{25}$= 2.4 m (thickness of concrete)

4- Seepage force = iγ_w $volume$

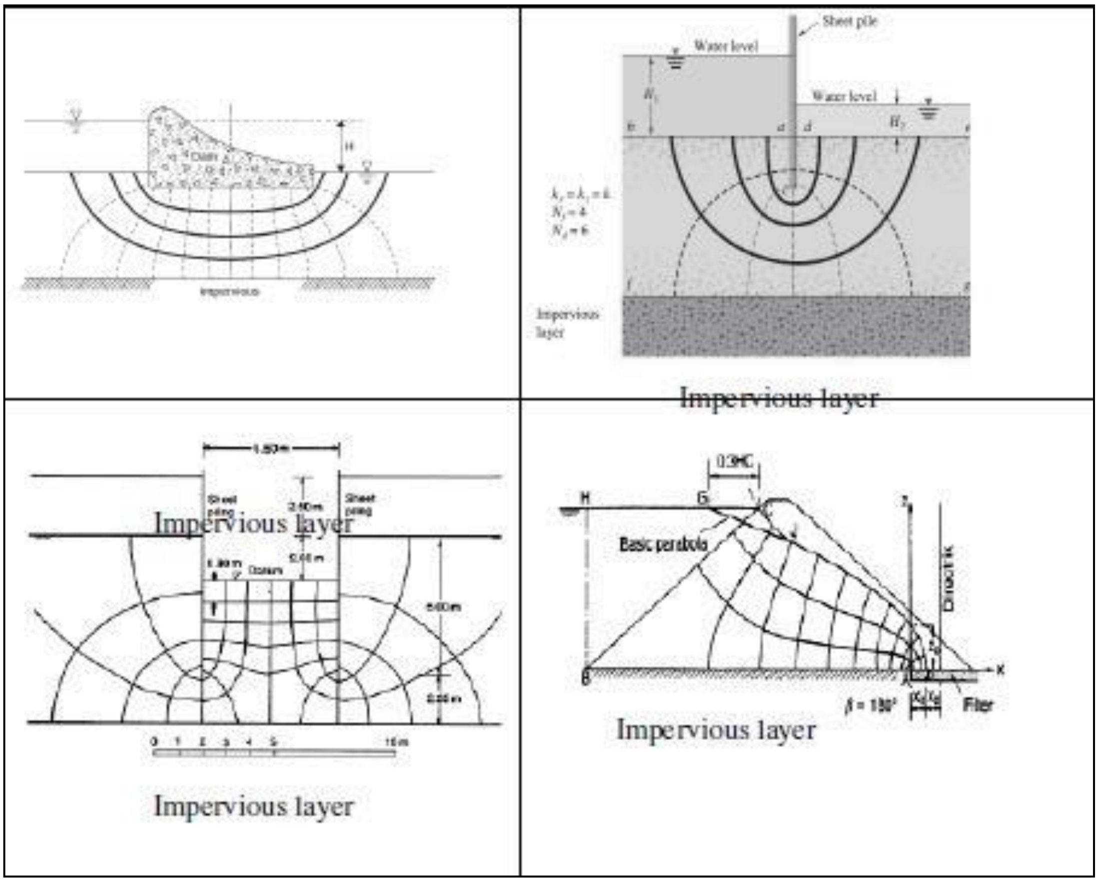
$$= \frac{3}{4} * 10 * (0.2)^3 = 0.06 \text{ kN}$$

Summary of Main Points:

1- In soils $v = ki$
2- There are three heads of importance to flow through porous media: elevation head (h_e), pressure head (h_p)and total head (h_t).
3- Flow depends on difference in total head.
4- The seepage force per a volume of soil is i*$\gamma_w$and acts in the direction of flow.
5- "Quick", refers to a condition where in a cohesion less soil loses its strength because the upward flow of water makes the effective stress become zero.

TWO DIMENSIONAL FLUIDS FLOW

Most problems of flow are two dimensional flows, e.g. are shown in Fig. below:



The purpose of studying the flow in two Dimension are :

1- To find the amount of seepage per meter length (i.e. rate of flow).

2- Pressure distribution (pore water pressure)

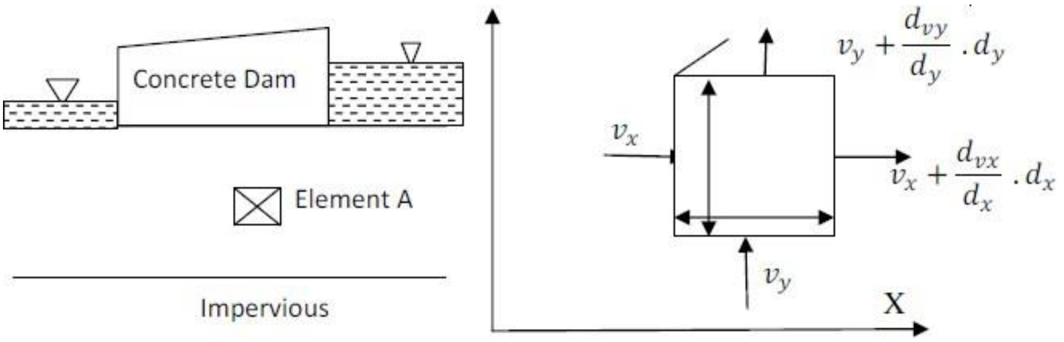
3- Stability against piping or boiling.

4- Pizometer levels of selected point required.

SEEPAGE THEORY:

The general case of seepage in two dimensions will now be considered.

In same principle used in one dimensional problem applied (Darcy's law & Continuity flow state.) Consider the two dimensional steady state flows in the



Take element A with dimension dx , dy and dz

fig.

Rate of flow (q_{in}) the flow entering the element

$$(v_x + \frac{d_{v_x}}{d_x} dx)dydz + (v_y + \frac{d_{v_y}}{d_{..}} dy)dxdz$$

Since the f

$$v_y = -k \frac{dh}{dy}$$

$v_x$dy dz +v

$$\therefore \frac{dv_y}{dy} = -k \frac{d^2h}{dy^2}$$

By simplif

Sub. In equation (1)

$$-k \frac{d^2h}{dx^2} + (-k \frac{d^2h}{dy^2}) = 0$$

$$\frac{d^2h}{dx^2} + \frac{d^2h}{dy^2} = 0 \qquad \textit{Laplace equation}$$

Darcy's la

$$\therefore \frac{dv_x}{dx} = -k \frac{d^2h}{dx^2}$$

Consider a function $\emptyset(x, y)$ so that

$$d\emptyset = \frac{d\emptyset}{dx} . dx + \frac{d\emptyset}{dy} dy$$

$$0 = v_x \, dx + v_y dy$$

$$-v_x dx = v_y \, dy$$

The second function ψ(x,y) called the flow line

$$v_x = -\frac{d\psi}{dy}$$

$$v_y = -\frac{d\psi}{dx}$$

$$\frac{dv_x}{dx} = -\frac{d^2\psi}{dxdy}$$

$$\emptyset(x, y) = -kh(x, y) + c$$

Where c is a constant

Thus if the function $\emptyset(x, y)$ is given a constant value equal to $\emptyset_1$ & it will represent a curve a long which the value of total head (h_1) is constant. If the function $\emptyset(x, y)$ is given a series of constant value $\emptyset_1$, $\emptyset_2$, $\emptyset_3$ etc a family of curves, such curves are called equipotentials and this will corresponding to total head h_1 , h_2 , h_3 ---------h_n from the total differ nation.

$$\frac{dv_y}{dy} = -\frac{d^2\psi}{dydx}$$

$$\frac{dv_x}{dx} + \frac{dv_y}{y} = 0$$

$$\frac{d^2\psi}{dxdy} - \frac{d^2\psi}{dydx} = 0$$

$\therefore$ $\psi(x, y)$ Satisfy the Laplace equation

A gain a series of ψ using $\psi_1, \psi_2, \psi_3 ----\psi_n$

Is selected and this function

$$\psi = \frac{d\psi}{dx}dx + \frac{d\psi}{dy}dy$$

$$0 = v_y dx + (-v_x)dy$$

$$v_y dx = v_x dy$$

$$\frac{dy}{dx} = \frac{v_y}{v_x}$$

Flow net:

The graphical representation of the Laplace equation is represented by the two families of curve:

1-Equipotential lines: A series of lines of equal total head e.g.

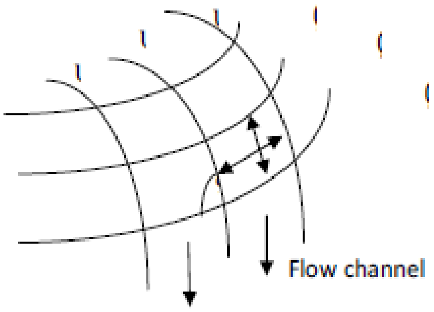
$h1, h2, h3 ------ hn$

2-Flow lines: A family of the rate of flow between any two adjacent flow lines is constant.

For isotropic soil:

The flow net is formed by a mesh of the intersection of two lines with the following limitation

1- Each element is a curvilinear square ψ_1



$\frac{b}{l} \cong 1$

Summary of the main points:

1- Laplace equation govern's the steady state flow in two dimensions
2- The solution is represented by two families of curve

Example:

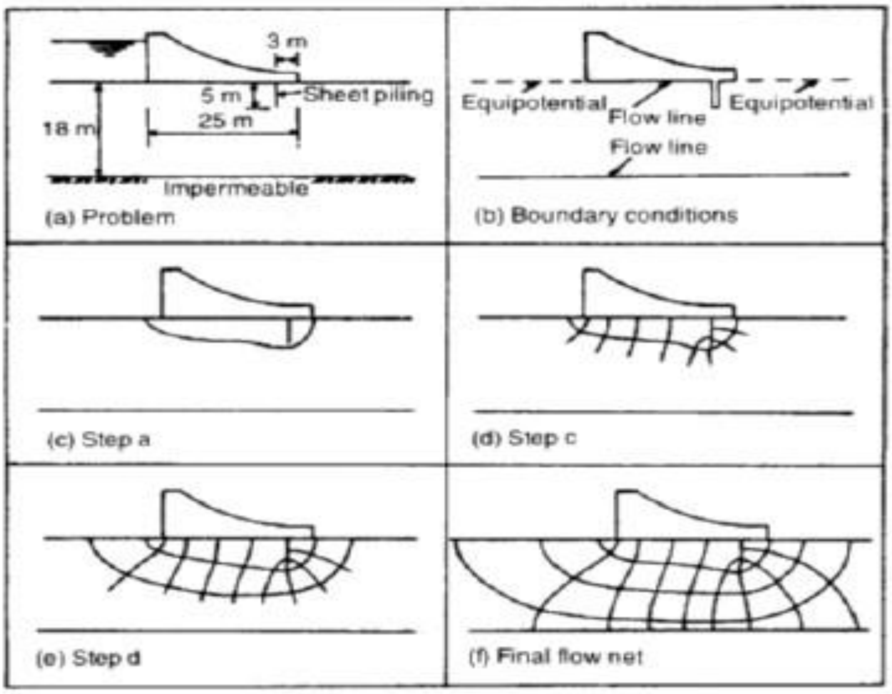


Figure example for flow net construction
Rate of flow

$q = kiA$

$$\Delta q = b * 1 * k * \frac{\Delta_{ht}}{L} \qquad \textit{this is for one flow channel}$$

$$\Delta ht = \frac{H}{nd}$$

$$\Delta q = \frac{b}{L} * k * \frac{H}{Nd} \qquad (b \cong L)$$

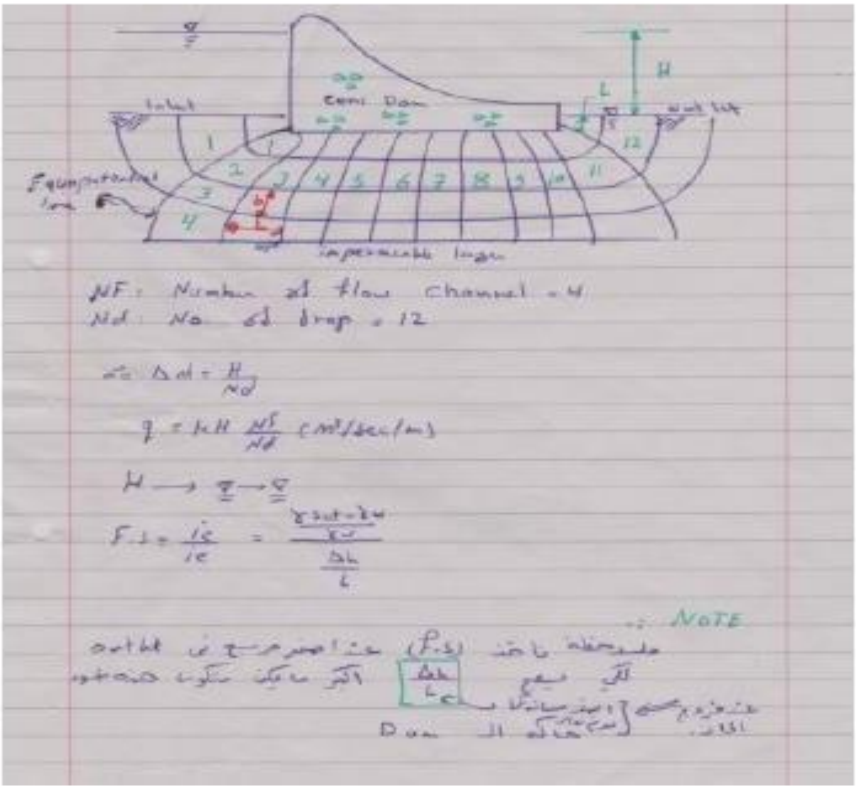
$$\Delta q = k\frac{H}{Nd}$$ this is for one channel

Assume No. of channel = Nf

$$\therefore q = \Delta q Nf = KH\frac{Nf}{Nd}$$

Where H= difference in water level (upstream and downstream.

Example 1:



Example 9

A deposit of cohesion less soil with a permeability of $3*10^{-2}$ cm/sec has a depth of 10 m with an impervious ledge below. A sheet pile wall is driven into deposit to a depth of 7.5 m. The wall extends above the surface of the soil and 2.5 m depth of water acts on one side. Determine the seepage quantity per meter length of the wall.

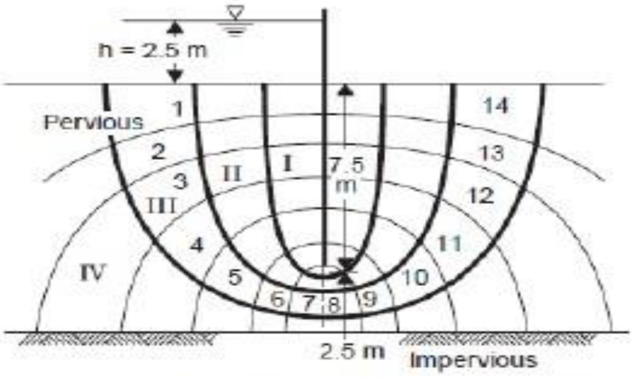


Fig. 6.28 Sheet pile wall (Example 6.6)

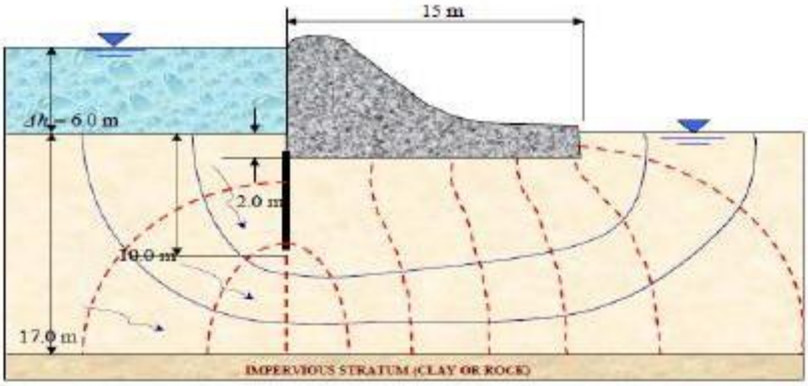
$= 3 \times 10^{-4} \times 2.5 \times \frac{4}{14}$ m^3/sec/metre run

$= 2.143 \times 10^{-4}$ m^3/sec/ meter run

= 214.3 ml/sec/metre run.

Example 10-

For the flow net shown below| includes sheet-pile cutoff wall located at the head water side of the dam in order to reduce the seepage loss. The dam is half kilometer in width and the permeability of the silty sand stratum is 3.5 $*10^{-4}$ cm /sec. Find (a) the total seepage loss under the dam in liters per year , and (b) would the dam be more stable if the cutoff wall was placed under its tail-water side?



Solution:

a) Notice that $\Delta h = 6.0\ m$, the number of flow channels $N_f = 3\ and\ N_d = 10$ by using $q = k\Delta h \frac{N_f}{N_d}$

$q = (3.5*10^{-4}\frac{cm}{sec})\left(\frac{m}{100\ cm}\right)(6.0m)\left(\frac{3}{10}\right)$

$= 6.3 * 10^{-6} m^3/sec/m$

Since the dam is 500 meters wide, the total Q under the dam is

$Q = Lq = 500m\ (6.3*10^{-4} m^3/sec)(\frac{10^3 liters}{1m^3})\left(31.5 * 10^6\ \frac{sec}{year}\right) =$

$100\ \frac{million\ liters}{year}$

b) - No: Placing the cutoff wall at the toe would allow higher uplift hydrostatic pressure to develop beneath the dam.